Comparison of Numerical Results between Related Shapes using a Non-rigid Mapping with Statistical Quantification of Uncertainty

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In the present study, numerical results obtained on different but related shapes are compared by using a non-rigid mapping. Non-rigid registration is employed to obtain mesh representations of different human skull geometries with the same mesh topology. The meshes representing various skull forms are analysed for conditions that approximate mastication as visible in Figure 1(a) through (c).

Numerical tools are increasingly used by evolutionary biologists, paleontologists and anthropologists to investigate the relationship of form and function [2, 3]. The sample study aims to compare the relationship between form and function between human skulls in a novel fashion. Results obtained on different geometric representations of the same mesh are elegantly compared during the post-processing stage [1]. Figure 1(d) illustrates the advanced comparison that becomes possible when utilising a non-rigid mapping between geometries. Contours of the difference in the Von Mises stress results for the same conditions obtained on two separate skull geometries are plotted on the average skull.

When different subject geometries are concerned, it is possible to compare distinguishable landmark locations and force them to map exactly. The challenge lies in accurately mapping between other featureless or non-salient areas.

Figure 1(e) indicates a possible uncertainty in the comparison due to an uncertainty in the mapping between non-salient points. In this study, this uncertainty is further investigated and statistically quantified by generating multiple mappings between analysed skull shapes. This is done by using the method of snapshots [4]. A reduced order model of the mapping and stress differences is set up and principal or statistically significant modes are then investigated.

If a comparison on shape and resulting stress field is done using elastic registration or a similar numerical tool, caution is needed when drawing conclusions

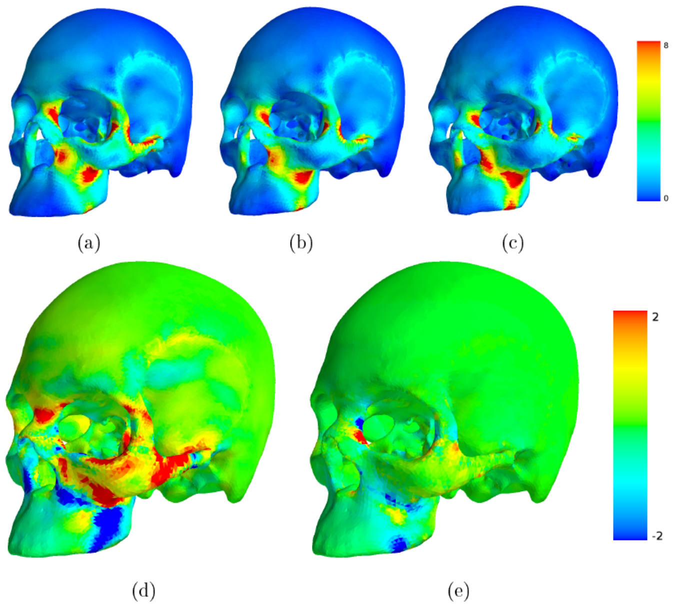

Figure 1: Von Mises stress contours for a molar bite for the range $[0, 8]$ MPa on the (a) first $\left(\sigma^{\mathrm{vM}}_{\mathrm{first}}\right)$, (b) average $\left(\sigma^{\mathrm{vM}}_{\mathrm{aver}}\right)$ and (c) second $\left(\sigma^{\mathrm{vM}}_{\mathrm{second}}\right)$ skull shape. (d) Contours of the difference in Von Mises stress $\left(\sigma^{\mathrm{vM}}_{\mathrm{first}} - \sigma^{\mathrm{vM}}_{\mathrm{second}}\right)$ with (e) variation noticed when comparing results obtained from anonther representation of the second skull. (d) and (e) show contours for the same range of $[-2, 2]$ MPa.

on the significance of a perceived variation. In this study it is shown that due to the uncertainty of the mapping between non-salient areas, this method of comparison does not necessarily report exact differences but rather proves a useful indicator on the difference locations.

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