Parameter-less remote real-time control for the adjustment of pressure in water distribution systems

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Abstract

Reducing pressure in a water distribution system leads to a decrease in water leakage, decreased cracks in pipes and consumption decreases. Pressure management includes an advanced type called *remote real-time control*. Here pressure control valves are controlled in real-time in such a way to provide set optimal pressures mostly at remote consumer locations. A hydraulic valve is expected to minimize problems with transients, although the study also applies to other valves. The control is done by using a controller which typically depends on tunable parameters, which are laborious to determine. A parameter-less controller is simpler because there are no parameters to tune. Such an existing P-controller based on the flow in a pressure control valve being known, is enhanced from assuming constant flow to be valid for variable flow. This novel parameter-less controller performs significantly better than the former P-controller. Several recently proposed controllers, which were studied numerically, are compared: The two parameter-less controllers, and two parameter-dependent controllers. The new parameter-less controller performs either better or worse than the most optimally tuned parameter-dependent controllers.

Keywords: Water distribution system; Pressure management; Remote real-time control; Pressure control valve; Pressure reducing valve; Hydraulic modelling

1 Introduction

Water leakage, pipe deterioration and excessive water usage in a water distribution system (WDS) need to be curtailed. Higher pressures lead to an increase in leakage in pipes, increased damage of pipes and consumption increases [1]. Hence there is a need to adjust the pressure to be lower. The pressure in the WDS can be adjusted via the use of pressure control valves (PCVs) and variable speed pumps (VSPs), in response to real-time pressure measurements at various *remote* nodes. The pressure at these nodes can be set to be low and constant: an advanced form of managing pressure. This is called *remote real-time control* (RRTC) [2], a form of *closed loop pressure control* which is the real-time version of what is known as *remote node-based modulation* [1]. The version which performs control through statistical procedures, as well as time-based and flow-based modulation, are not discussed here [1].

As discussed further below, many optimization methods for PCVs have been proposed. Most of these methods rely on the existence of an accurate hydraulic model of the real-world WDS [3, 4, 5]. However, a subclass of methods, based on proportional integral derivative (PID) controllers [6], are not based on a hydraulic model. These methods typically contain control parameters, which are adjustable quantities in the control algorithm. Pressure management can be attained for a *range* of

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control parameter values; hence the method can match onto a real-world WDS. There has been recent work on proportional (P) controllers (a simplified version of PID control), which forms the context of this work [2, 7, 8, 9, 10]. The P-controllers proposed typically depend on one unknown control parameter. However, it will be argued later that a method which contains no control parameters, called *parameter-less* control, is simpler.

Control is defined here as a closed feedback loop whereby the difference between a measured process variable and a desired set-point is calculated, and this difference is minimised over time by the adjustment of a process setting. Modern industrial programmable logic controllers or remote terminal units can handle user-defined control algorithms [11, 12]. On the other hand, *optimisation* defines an objective with respect to which the WDS must be minimised, subject to various constraints. Common single-objective optimisation studies minimise pressure or leakage in the WDS to determine the number of valves, their locations and their settings [13, 14, 15, 16]. In contrast, control is only concerned with settings. For the settings, control and optimisation studies can be compared [2].

This work considers electrically actuated PCVs, of which there are two types: (1) the *direct acting* one where the *shutter opening* is set directly, or (2) the *hydraulic* type, with has continuous hydraulic feedback, where a target *set-point pressure* is set either upstream or downstream of the PCV. Although the logical structure of the new controller which is developed allows for application to either type of PCV, type 2 is expected to minimize problems with transients [6, 17], while a detailed transient analysis has not been performed for type 1.

A new controller is derived by actively using the theoretical description of a PCV. An established P-controller based on the flow in a PCV being known, is enhanced from assuming constant flow to be valid for variable flow. This novel parameter-less controller performs significantly better than the established parameter-less controller which assumes constant flow; and either better or worse than two established controllers with the luxury of an optimally tuned parameter. Additional novel aspects of this work are highlighted in the Conclusion section.

2 Review of PCV P-controllers

PCVs can be used to maintain a set pressure value at a (remote) control node of the WDS. PCVs maintain the pressure by reducing (pressure reducing valve (PRV)) or sustaining (pressure sustaining valve) the pressure. A PRV is a device which increases/reduces the internal head-loss in order to reduce/increase the pressure at the control node to the set-point. PCVs can be modelled by [7, 8, 10]

$$\tilde{H} = \frac{\xi}{2gA^2}\, Q^2 \tag{1}$$

where $\tilde{H}$ is the head-loss across the PCV, Q is the flow rate in the PCV, g is the gravitational acceleration, A is the cross-sectional area of the port opening within the PCV and ξ is the (dimensionless) head-loss coefficient (the same notation used by [7, 8, 10, 18]).

PCVs, remotely controlled in real-time by using downstream control node pressures, have been proposed. One can seek to control using several *individual* node pressures, or an *average* of node pressures in the WDS [19]. A *critical node* (CN) is a WDS node which is sensitive (defined below), and as far as possible, also has the lowest pressure [2, 20]. For a WDS with one PCV, pressure management will be accomplished by attempting to keep the pressure constant at an individual CN.

The simplest P-controller is a parameter-less one, because of the ease of implementation, even though its controlling ability is expected to be worse than that of a controller with some optimally tuned (and WDS-dependent) control parameter. The ease of implementation stems from the fact that a field test of the WDS, or hydraulic model to simulate the WDS, is not required for the determination of the control parameter. Nor is there a need for tuning rules. Moreover, for a parameter-dependent controller the control parameter is tuned for specific WDS conditions (for example, the water demand and reservoir conditions considered later in this work), so that the controller might not provide satisfactory performance (without extra retuning) for different conditions [11].

Various authors have investigated RRTC algorithms, by numerically assessing the effectiveness the controller. Results were first described for direct shutter opening modulation with a conventional

P-controller called “proportional control” [2], with a parameter (proportional constant) that can be determined either by tuning or from a hydraulic model method [10]. This controller does not require Q to be known; but because it is “conventional”, offers limited robustness with respect to changing WDS conditions.

Recently, controllers that are not just generic, but employ theoretical understanding of the hydraulics of a WDS to enhance their efficacy, were discovered [7, 8]. These controllers require Q to be known, either through a field measurement [7, 8] or a hydraulic model prediction of Q; but offer robustness with respect to changing WDS conditions [8]. Installing a flow meter at the site of the PCV for field measurement would incur additional financial cost. It was realised that control can successfully be performed without introducing any unknown parameter [8] (called “valve resistance” control [7]), instead of the one parameter that was assumed in “proportional control” [2, 10]. In addition to the parameter-less technique a parameter-dependent analogue with one parameter was also introduced [8], which will later be called “DCF”.

Another controller adjusts the head-loss over the PCV [7, 9], a hydraulic variable. It is possible that the “head-loss” controller has excellent modulation [7], and there is also a theoretical argument to support this idea [8].

The “head-loss” controller is now explored in detail. As $\tilde{H}$ is changed, there will be a change in the head at the control node H, characterised by the differential relationship $dH = d\tilde{H}\,/S$; where S is the sensitivity, a function of $\tilde{H}$ at a certain state of the WDS. From this it can be argued that an appropriate controller, called the “head-loss” controller, would be

$$\tilde{H}_{i+1} = \tilde{H}_i - S_i\,(H_i - H_{sp}) \tag{2}$$

where H_{sp} is the target set-point head of the control node. The information at iteration i determines the next iteration $i+1$. The iterations are separated by a control time-step T_c. Define $S_i \equiv S(\tilde{H}_i)$, which has different values for different iterations. When the CN head depends very sensitively on the PCV head-loss, the value of S_i is called the *ideal sensitivity* ($S_i = 1$ or -1). For a PRV, this value is -1. Using ideal sensitivity in Eq. 2 yields a parameter-less controller [7, 9]. In the literature this choice is consistently made without stating the nature of the assumption [7, 8, 9]. In this work, the presence of the sensitivity is explicitly indicated. The point of departure here (as in [8]) is that, from a theoretical point of view, Eq. 2 represents the controller of choice. One reason for this is that it can be derived from the Newton-Raphson numerical method (see Appendix).

The PCV setting is changed at each time-step T_c, with the restriction that the change is limited by some maximum rate of change of ξ. Equivalently, this has conventionally been expressed as a limitation due to a maximum shutter velocity ν_{shut} (for details, see equations 2–3 of [10]). The restriction limits unsteady flow processes [2, 7, 8] and improves convergence of the controller [7]. Note that even if the physical PCV allows a larger ν_{shut}, convergence of the controller will limit the value of ν_{shut} that should be used in the controller.

3 Controllers based on known PCV flow

Using Eq. 1, the “head-loss” controller in Eq. 2 with $Q_{i+1} = Q_i$ (i.e. assuming constant Q) implies

$$\xi_{i+1} = \xi_i - \frac{2gA^2 S_i}{Q_i^2}\,(H_i - H_{sp}) \tag{3}$$

This is often not practical for use in a real-world WDS because the sensitivity S_i needs to be calculated from a hydraulic model of the WDS. Generally, it only makes sense to control a PCV by attempting to set the pressure at a sensitive node; hence the need to set it at a CN. The substitution of the ideal sensitivity is usually made in Eq. 3, because S_i may not be known; and the equation becomes (1) explicitly independent of the WDS (except for A) and (2) parameter-less. The controller in Eq. 3 is accordingly called the “parameter-**l**ess P-controller with known **c**onstant PCV **f**low” (LCF). With ideal sensitivity it is also called “valve resistance” (RES) control [7, 21].

It was formerly noticed that the theoretical derivation just outlined assumes that Q remains *constant* [8]. This is not the case in most WDSs. In an attempt to correct for this and incorporate the missing dynamics, S_i is set to $-K$ (for a PRV) in Eq. 3, where the parameter (proportional constant) K is the notation of [8]. This case is called the "parameter-**d**ependent P-controller with known **c**onstant PCV **f**low" (DCF).

From Eq. 1 the differential relationship

$$d\xi = \frac{2gA^2}{Q^2}\, d\tilde{H} - \frac{2\xi}{Q}\, dQ \tag{4}$$

is obtained. From this, and Eq. 1, a new controller is proposed which does not assume that Q remains constant, called the "parameter-**l**ess controller with known **v**ariable PCV **f**low" (LVF)

$$\xi_{i+1} = \xi_i - \frac{2gA^2 S_i}{Q_i^2}\,(H_i - H_{sp}) - \frac{2\xi_i}{Q_i}\,\Delta Q_i \tag{5}$$

where S_i is set to ideal sensitivity to obtain a parameter-less controller. Here, dQ is approximated by ΔQ_i as defined in the Appendix. All other controllers analysed in this work [2, 7, 8, 9, 10] are based on calculating the change in the setting of the PCV, which is proportional to the difference between the head at the CN and the set-point head (called *P-control*). This is not true for the LVF controller.

Eq. 5 is used as follows to compute the head-loss coefficient ξ at the time of iteration $i+1$ from its known value at the time of iteration i. At the time of iteration i, the known head at the CN H_i, and the known flow rate in the PCV Q_i are used. Also, the known flow in the PCV at a previous time is used in ΔQ_i. Having obtained ξ_{i+1} from Eq. 5, it is then restricted by applying the limitation on the rate of change of ξ. From the resulting ξ_{i+1}, the shutter opening is calculated for a direct acting PRV, or the set-point pressure for a hydraulic PRV.

There can be imprecise readings of the pressure meter [18], yielding incorrect H_i. To decrease the effect of unsteady flow processes, H_i can be measured by adopting a pressure moving average [10]. This can also be done for the flow in the PCV.

4 Numerical modelling

Software packages that can simulate pressure management at a CN via RRTC are available. For example, the "WDNetXL pressure control module" can incorporate leakage and pressure-dependent demand, and can be used with various controllers, including LCF [7]. More conventional packages, like EPANET On-Line, use PID control [22].

An unrelated package was used here [23]. It interacts with an extended-period hydraulic solver, so that the controller can be validated on a hydraulic model of a WDS. The algorithm can read in any WDS specified by an EPANET2-formatted input file. This enables the controller to be validated on hydraulic models generated by various software packages. The time-variation of the demand factor and reservoir levels are read at intervals T_c.

Adopt $T_c = 5$ min (in accordance with [7, 8, 10, 21]), although $T_c = 3$ min is also used in the literature [2, 18]. Also set $\nu_{shut} = 0.0005\ \mathrm{s}^{-1}$ as in [7].

The hydraulic model of the example WDS is chosen to be the Jowitt and Xu WDS [13], specifically as implemented by Araujo et al. [14] (see Figure 1). The same WDS was used in some earlier pivotal P-controller studies [2, 10]. The three reservoirs have time-varying water levels and the demand factor varies substantially between 0.6 and 1.4 (see Figure 2) [13]. As such, the latter variation is found to drive most of the change of Q over time. Leakage is implemented according to [14]. In addition, the effect of pressure-dependent demand is taken into account.

Following the previous results [14, 15, 16] one PRV with diameter 350 mm is installed at the location shown in Figure 1 as the best valve site to control the water losses. It is confirmed that node 22 has the lowest pressure and is sensitive to the PRV head-loss, so that it is chosen as the remote pressure control CN. These choices are consistent with earlier P-controller studies [2, 10]. The target set-point pressure head p_{sp} is taken to be 30 m (also used in [14, 18, 21]).

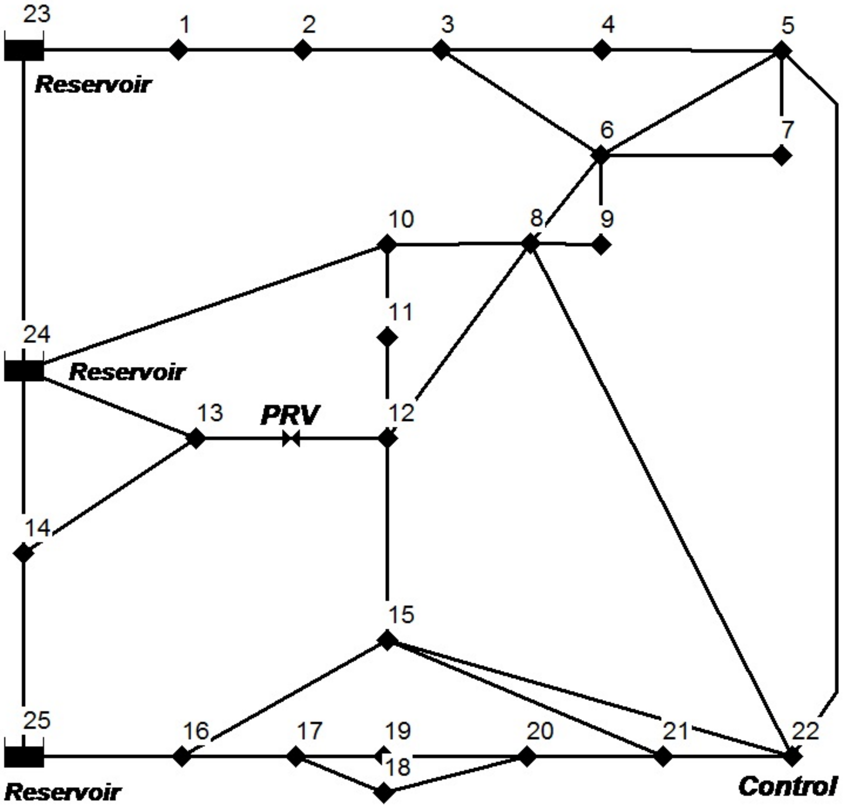

Figure 1: WDS of Jowitt and Xu [13].

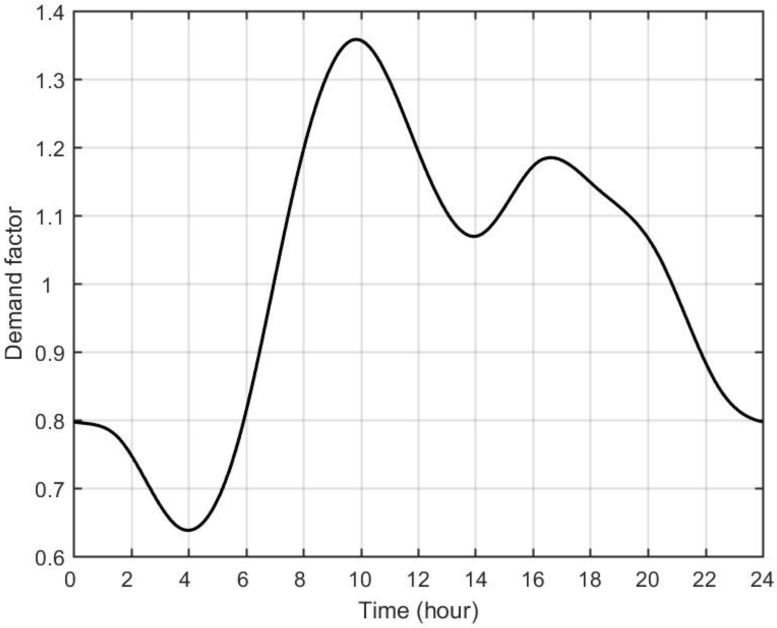

Figure 2: Time dependence of the demand factor.

The calculated sensitivity S_i of the CN when pressure control is performed in the example WDS is in the range -1.080 to -1.235, with an average value of -1.159. All applications of parameter-less controllers to the example WDS are implemented assuming $S_i = -1$.

The model has some limitations, including the use of a very low Hazen-Williams major friction coefficients for pipes 3-2 and 16-25. Although at face value these coefficients are unrealistic, they can

Controller		*Eq.*	*Jowitt and Xu WDS*			*JXWP WDS*		
			Parameter	Δ (m)	δ (m)	*Parameter*	Δ (m)	δ (m)
Conventional	Prop.		$k_c = 0.06$ m^{-1}	0.032	0.096	$k_c = 0.008$ m^{-1}	0.210	0.71
Known Q	DCF	3	$K = 2.2$	0.019	0.057	$K = 1.6$	0.037	0.23
Known Q	LCF	3	Parameter-less	0.102	0.41	Parameter-less	0.262	0.98
Known Q	LVF	5	Parameter-less	0.038	0.18	Parameter-less	0.094	0.39

Table 1: Efficacy of controllers for the two WDSs. For parameter-dependent controllers the optimally tuned case is listed.

be viewed as effectively introducing a source of additional resistance, like an almost closed PCV, in the pipe branches connected to two of the reservoirs [24].

It is found that ν_{shut} as low as 0.00006 s^{-1} can be used for pressure control in the example WDS without changing the results. This corresponds to the closure time from the PCV being fully open to fully closed being 4.6 hours, emphasizing that unsteady flow processes are likely limited.

5 Validation of the variable flow controller LVF

In this and the next section, results obtained for the Jowitt and Xu WDS are discussed. The ratio of the last term to the second last term of LVF in Eq. 5 is found to almost always be positive for the various iterations. The ratio tends to increase as the rate of change of Q increases. The time-averaged value of the ratio is approximately 0.5. Comparing to Eq. 3, it can hence be concluded that the controller is, for the example WDS, effectively similar in character to DCF with $K = 1.5$.

Define Δ as the temporal average of the absolute value of the difference between the head at the control node and H_{sp}, according to Eq. 2 of [8]. Δ is a measure of how near the control node pressures are to the set-point pressure, and hence how well the controller controls the pressure. Define δ as the maximum over time of the absolute value of the difference between the head at the control node and H_{sp}. The quantities Δ and δ quantify the mean and maximum deviations respectively. The results are in Table 1.

For an uncontrolled near-open PRV the pressure variation at the CN from minimum to maximum over a 24 hour period is found to be 7.2 m [23]. For a controlled PRV, the variation is much smaller, as can be seen by comparing δ.

LCF has small Δ and δ. Moreover, DCF performs optimally for K near 2.2, with tiny Δ and δ. K in the range 1.6 to 3.2 yields values of Δ within a factor of two of its minimal value, and the shape of Δ as a function of K is fairly flat [23]. This range of K is the *effective range* for the controller to operate in [10].

The new controller LVF obtains values of the CN pressure very near to the set-point, with its Δ a factor of 2.7 improvement on the parameter-less LCF. In addition, the Δ for LVF it is just within Δ's obtained in the effective range for DCF, which has the luxury of a tunable parameter. When LVF is used, the times with the smallest pressure deviations in Figure 3 approximately coincide with the times when the demand in Figure 2, and hence Q, changes the slowest. The performance of the controller appears to worsen when Q changes faster. The maximum deviation occurs before the sixth hour.

For the remainder of this section issues related to the shutter opening are considered. Manufacturers provide mathematical curves that allow for the calculation of ξ as a power-law function of the (dimension-less) normalised shutter opening α, using two constants commonly denoted k_1 and k_2 (the same notation used by [7, 18]). The constants vary from one PCV to the next. Here α is the ratio of the shutter opening and the maximum stroke of the PCV [7, 8, 10]. It varies between $\alpha = 0$ (PCV fully closed) and $\alpha = 1$ (PCV fully open) (the convention used by [7, 10]). Assume $k_1 = 2.8$ and $k_2 = 1.5$ (used in [7, 10]).

The "proportional control" method for a direct acting PCV adjusts $\alpha_{i+1} = \alpha_i - k_c\,(H_i - H_{sp})$ [2, 7, 8, 10]. Here k_c is a dimension-full constant [7] which is defined as the ratio of the dimension-less proportional constant K_p (used in [2, 8, 10]) and the maximum stroke of the PCV. LVF compares

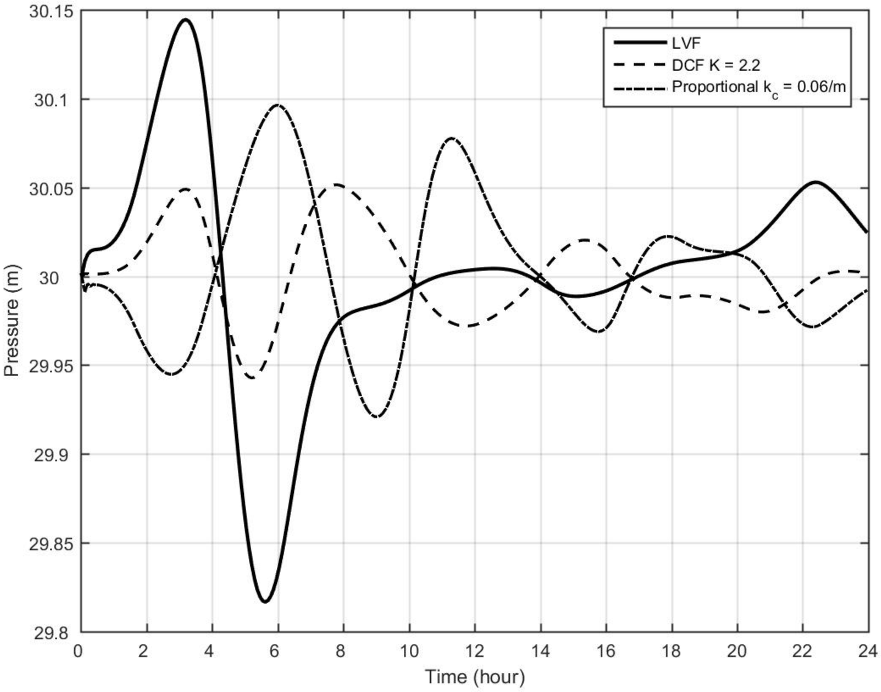

Figure 3: Time-dependent pressure head at the CN for LVF and the two optimally tuned P-controllers with tunable parameters.

well with the optimally tuned "proportional control" shown in Table 1, which has the luxury of a tunable parameter; and performs better for some k_c in the effective range of the "proportional controller". LVF is compared to the two optimally tuned P-controllers with tunable parameters in Figure 3. A side observation is that the evidence from the Central-Northern Italy WDS that the optimally tuned "proportional control" performs worse than the optimally tuned DCF (where the ratio of Δ's is $1.5 - 2.6$ [8]) is confirmed for the Jowitt and Xu WDS where the ratio is 1.7.

The time-dependence of the shutter opening can be studied for the various controllers. Particularly, the temporal average of the absolute value of the difference between the normalised shutter openings α for a controller and the $K = 2.2$ DCF, denoted $\langle \Delta\alpha \rangle$, is a measure of the mean deviation of a controller from the most optimally tuned controller. The time-dependence of the shutter opening for LCF, LVF, and "proportional control" with optimally tuned k_c, is almost identical to the optimally converged controller (DCF with $K = 2.2$). The mean deviation $\langle \Delta\alpha \rangle$ is respectively a small 0.0060 [23], and a tiny 0.0024 and 0.0032.

6 Commercial and experimental environments

Experimental progress on water demand measurement via a smart meter system has been reported [25, 26], paving the way for the present project on smart water infrastructure [27], of which this work is a part.

Significant advances in the area of pressure control will come from field experiments in real WDSs. Control techniques are being implemented for pressure management at a CN in a WDS via RRTC both in commercial and experimental environments. A growing number of commercial manufacturers are providing control technology based on the real-time transmission of the actual pressure values at a CN at a given moment [1]. Laboratory scale experimental test-bed set-ups that model a WDS include employing an extended PID-type control (active disturbance rejection control) for modulating one VSP [11]; and using fuzzy logic [28] or generalised minimum variance self-tuning methods [29, 30] for modulating an experimental set-up with one PCV and one VSP.

It is interesting to note that an experimental maximum error on pressure control of 3.11–3.47% [28] and 2.12% [30], and an average error of 1.02% [28], were reported, with the freedom to use tunable parameters. In comparison, the maximum errors for the parameter-less LCF and LVF controllers (δ/p_{sp}) are respectively 1.4% and 0.6%, and the average error (Δ/p_{sp}) respectively 0.34% and 0.13%, although these do not take uncertainty in meter measurements and other real-world issues into account.

7 JXWP WDS

In view of the limitations of the example WDS, some changes are made to it in accordance with [24] to make it more realistic, yielding a WDS which is called the "JXWP WDS". The primary controlled PRV is chosen to separate the WDS into inlet and outlet zones respectively upstream and downstream of the PRV; by closing all open bypasses, and installing a secondary classic hydraulic type PRV that is usually closed and only opens when there is significant demand. In addition, elevation differences between the reservoirs and the rest of the WDS are substantially increased, enabling a large amount of excess pressure to be removed by using pressure management.

Specifically, the following changes to the Jowitt and Xu WDS are made to obtain the JXWP WDS. Both Pipes 10-24 and 16-25 are closed. The very low Hazen-Williams coefficients for pipes 3-2 and 16-25 are set to a more realistic value of 100. PRVs have the same k_1 and k_2 as before. The secondary PRV is inserted in pipe 23-1; with an elevation the same as that of node 1, a diameter the same as before, and a downstream PRV pressure head setting of 31 m. The primary PRV is located at the same place as before with a diameter of 440 mm. The reservoir levels are 53 m above their previous levels, and the demand factor is twice as large as before. The elevations of nodes 1 to 3 are lowered to the minimum in the Jowitt and Xu WDS, i.e. 7 m.

Node 22 is found to have the lowest pressure and to be sensitive to the primary PRV head-loss, so that it is chosen as the CN. The results are in Table 1. For the parameter-less LCF and LVF, the values of Δ for the JXWP WDS are respectively a factor of 2.6 and 2.5 times larger than for the Jowitt and Xu WDS. It is likely that worse performance of the parameter-less controllers in the JXWP WDS is related to the larger head-loss over the controlled PRV, due to larger elevation differences in the WDS. The performance of the "proportional controller" versus DCF (comparing the optimally tuned parameter-dependent controllers), is significantly worse for the JXWP WDS than for the Jowitt and Xu WDS.

8 Conclusion

The following aspects of this work are novel. (1) The role of the sensitivity in the controllers is explicitly indicated. (2) A novel parameter-less controller LVF is proposed, which shows a factor of 2.7 to 2.8 improvement over the parameter-less LCF for two WDSs studied. The parameter-less LVF performs worse than the optimally tuned DCF with a tunable parameter for the two WDSs. (When optimally tuned, DCF is overall the best performing controller). LVF performs either better or worse than the optimally tuned "proportional controller", depending on the WDS. Since optimal tuning is usually not obtained in the real-world, a more realistic comparison is to compare LVF with these controllers when the parameter is in the effective range. Hence LVF performs better than stated. The efficacy of the new LVF compared to previous controllers is intrinsically a property of the controller itself. It is not an artifact of the WDS used, as can be seen by its success in *two* example WDSs considered. (3) The LCF and LVF controllers are derived by actively using the theoretical description of a PCV. The derivation for LCF is closely related to the derivation in [8]. (4) The conditions under which the "head-loss" controller is equivalent to either the parameter-less LCF or LVF are derived in the Appendix. (5) In an example WDS, the time-dependence of the PCV setting is similar for LCF, LVF, the optimally tuned "proportional controller", and the optimally tuned DCF.

The efficacy of parameter-less control is pointed out. Considering the prospect of not having to tune any parameter, the controller becomes particularly easy to use. The new LVF considerably improves on the former LCF, and the fact that the performance is comparable to the best parameter-dependent controllers makes it viable for adoption in commercial and experimental environments, justifying the additional cost incurred to replace a conventional control system [11]. In contrast to most controllers, the LCF and LVF (and DCF) controllers have the ability to respond to changing WDS conditions, through their dependence on the PCV flow which is required to be known.

The value of research into parameter-less control partially lies in the identification of the preferable form of the algorithm, because there is no freedom to introduce arbitrary parameters. Increased efficacy beyond a parameter-less controller can always be attained by constructing new parameter-dependent controllers by adding tunable parameters.

LVF appears to perform the poorest when the flow in the PCV changes the fastest. Moreover, the efficacy of the new LVF has only been verified for a single PCV, and when there are no tanks. Because of these limitations, ongoing research should be conducted in the area of parameter-less control.

A Appendix: Derivation of equivalence of controllers and details of LVF

Let $t_{c\,i}$ be a time period which differs from iteration to iteration; and $t_{c\,i} < T_c$. At time t_i the PCV head-loss coefficient is ξ_i; and the head-loss, head, flow, and sensitivity respectively $\tilde{H}_i$, H_i, Q_i and S_i. Soon after that the adjustment process starts, continuing up to time $t_i + t_{c\,i}$, when the PCV is fully adjusted to the new coefficient ξ_{i+1} with corresponding flow $\tilde{Q}_{i+1}$. At time $t_{i+1} \equiv t_i + T_c$ the coefficient is still ξ_{i+1}; and the other variables $\tilde{H}_{i+1}$, H_{i+1}, Q_{i+1} and S_{i+1}. Let $q(t)$ be an interpolated function of the function $Q(t)$ which is smooth within an iteration and from iteration to iteration.

The "head-loss" controller and LCF are equivalent if and only if $Q_{i+1} = Q_i$. This can be seen by requiring that Eqs. 2 and 3 hold at the same time. The lack of equivalence is characterised by

$$Q_{i+1} - Q_i \approx \frac{dq(t_i)}{dt}\, T_c \tag{6}$$

where the approximation uses the lowest order in the Taylor expansion with respect to time.

When Eq. 1 is differentiated, $d\tilde{H}$ can be written in terms of ξ, Q, $d\xi$ and dQ, where $d\xi$ and dQ are *independent* changes, because ξ and Q are independent variables. Rewriting this yields Eq. 4. dQ is interpreted as the expected change of the flow from the current time t_i to a future time t_{i+1}. This change is independent of $d\xi$, i.e. should assume that ξ remain unchanged. dQ is approximated by ΔQ_i for the LVF controller in Eq. 5. Since dQ is an expected change, one way to estimate it from currently known flow information is to approximate it, as was done for the numerical simulations, by $\Delta Q_i \equiv Q_i - \tilde{Q}_i$. Note that Q_i and $\tilde{Q}_i$ are measured while the coefficient has *the same* value ξ_i. A less preferable choice would be to approximate dQ by $\Delta Q_i \equiv Q_i - Q_{i-1}$, where Q_i and Q_{i-1} are measured for *different* coefficients. Simulation yields inferior results for this choice, with $\Delta = 0.050$ m and $\delta = 0.21$ m for the Jowitt and Xu WDS.

dQ can be estimated from a demand prediction algorithm. Otherwise, in general the same estimate as above can be made with one modification. If it is not the case that $t_{c\,(i-1)} \ll T_c$, the flow difference $Q_i - \tilde{Q}_i$ takes place during a time interval $T_c - t_{c\,(i-1)}$, while dQ should be a flow change during a time interval T_c. Hence dQ can be estimated by

$$\Delta Q_i \equiv (Q_i - \tilde{Q}_i)\,\frac{T_c}{T_c - t_{c\,(i-1)}} \tag{7}$$

The "head-loss" controller and LVF are equivalent if and only if $Q_{i+1} = Q_i$ and $\Delta Q_i = 0$. This can be seen by requiring that Eqs. 2 and 5 hold at the same time. The lack of equivalence is characterised by $Q_{i+1} - Q_i$ and ΔQ_i.

The statements about the equivalence, and lack of equivalence, of LCF and LVF to the "head-loss" controller make no assumption about S_i.

The concepts used for the derivation of the “head-loss”, LCF, DCF and LVF controllers can inspire the construction of analogous controllers for a VSP [31]. An argument presented there can be used to derive Eq. 2 from the Newton-Raphson numerical method under certain assumptions.

References

[1] D. J. Vicente, L. Garrote, R. Sánchez, and D. Santillán. Pressure management in water distribution systems: Current status, proposals, and future trends. *Journal of Water Resources Planning and Management*, 142(2):04015061, 2015.

[2] A. Campisano, E. Creaco, and C. Modica. RTC of valves for leakage reduction in water supply networks. *Journal of Water Resources Planning and Management*, 136(1):138–141, 2010.

[3] P. Sivakumar and R. K. Prasad. Extended period simulation of pressure-deficient networks using pressure reducing valves. *Water Resources Management*, 29(5):1713–1730, 2015.

[4] S. Yoyo, P. R. Page, S. Zulu, and F. A’Bear. Addressing water incidents by using pipe network models. In *WISA Biennial 2016 Conference and Exhibition*, page 130. WISA, Johannesburg, South Africa, 2016. Held 15-19 May 2016, Durban, South Africa. ISBN 978-0-620-70953-8.

[5] M. S. Osman, A. M. Abu-Mahfouz, P. R. Page, and S. Yoyo. Real-time dynamic hydraulic model for water distribution networks: steady state modelling. In *Proceedings of Sixth IASTED International Conference: Environment and Water Resource Management (AfricaEWRM 2016)*, pages 142–147. ACTA Press, Calgary, Canada, 2016. Held 5-7 September 2016, Gaborone, Botswana. ISBN 978-0-88986-984-4.

[6] S. L. Prescott and B. Ulanicki. Improved control of pressure reducing valves in water distribution networks. *Journal of Hydraulic Engineering*, 134(1):56–65, 2008.

[7] O. Giustolisi, A. Campisano, R. Ugarelli, D. Laucelli, and L. Berardi. Leakage management: WDNetXL pressure control module. In *13th Computer Control for Water Industry Conference, CCWI 2015*, volume 119 of *Procedia Engineering*, pages 82–90. Elsevier Ltd., 2015.

[8] E. Creaco and M. Franchini. A new algorithm for real-time pressure control in water distribution networks. *Water Science & Technology: Water Supply*, 13(4):875–882, 2013.

[9] E. Sanz, R. Pérez, and R. Sánchez. Pressure control of a large scale water network using integral action. In 2^{nd} *IFAC Conference on Advances in PID Control*, volume 45(3) of *IFAC Proceedings Volumes*, pages 270–275. Elsevier Science Direct, 2012.

[10] A. Campisano, C. Modica, and L. Vetrano. Calibration of proportional controllers for the RTC of pressures to reduce leakage in water distribution networks. *Journal of Water Resources Planning and Management*, 138(4):377–384, 2012.

[11] R. Madonski, M. Nowicki, and P. Herman. Application of active disturbance rejection controller to water supply system. In *Control Conference (CCC), 2014 33rd Chinese*, pages 4401–4405. IEEE, New York, 2014. Held 28-30 July, 2014, Nanjing, China. ISBN 978-9-8815-6387-3.

[12] A. Campisano, J. Cabot Ple, D. Muschalla, M. Pleau, and P. A. Vanrolleghem. Potential and limitations of modern equipment for real time control of urban wastewater systems. *Urban Water Journal*, 10(5):300–311, 2013.

[13] P. W. Jowitt and C. Xu. Optimal valve control in water distribution networks. *Journal of Water Resources Planning and Management*, 116(4):455–472, 1990.

[14] L. S. Araujo, H. Ramos, and S. T. Coelho. Pressure control for leakage minimisation in water distribution systems management. *Water Resources Management*, 20(1):133–149, 2006.

[15] M. Nicolini and L. Zovatto. Optimal location and control of pressure reducing valves in water networks. *Journal of Water Resources Planning and Management*, 135(3):178–187, 2009.

[16] E. Creaco and G. Pezzinga. Multiobjective optimization of pipe replacements and control valve installations for leakage attenuation in water distribution networks. *Journal of Water Resources Planning and Management*, 141(3):04014059, 2014.

[17] B. Ulanicki and P. Skworcow. Why PRVs tends to oscillate at low flows. In *16th Conference on Water Distribution System Analysis, WDSA 2014*, volume 89 of *Procedia Engineering*, pages 378–385. Elsevier Ltd., 2014.

[18] A. Campisano, C. Modica, S. Reitano, R. Ugarelli, and S. Bagherian. Field-oriented methodology for real-time pressure control to reduce leakage in water distribution networks. *Journal of Water Resources Planning and Management*, 142(12):04016057, 2016.

[19] P. R. Page. Smart optimisation and sensitivity analysis in water distribution systems. In J. Gibberd and D. C. U. Conradie, editors, *Smart and Sustainable Built Environments (SASBE) 2015: Proceedings*, pages 101–108. CIB, CSIR, University of Pretoria, 2015. Held 9–11 December 2015, Pretoria, South Africa. ISBN 978-0-7988-5624-9.

[20] L. Berardi, D. Laucelli, R. Ugarelli, and O. Giustolisi. Leakage management: planning remote real time controlled pressure reduction in Oppegård municipality. In *13th Computer Control for Water Industry Conference, CCWI 2015*, volume 119 of *Procedia Engineering*, pages 72–81. Elsevier Ltd., 2015.

[21] D. Laucelli, L. Berardi, R. Ugarelli, A. Simone, and O. Giustolisi. Supporting real-time pressure control in Oppegård Municipality with WDNetXL. In *12th International Conference on Hydroinformatics, HIC 2016*, volume 154 of *Procedia Engineering*, pages 71–79. Elsevier Ltd., 2016.

[22] P. Ingeduld. Real-time forecasting with EPANET. In K. C. Kabbes, editor, *World Environmental and Water Resources Congress 2007: Restoring Our Natural Habitat*, pages 1–9. American Society of Civil Engineers, 2007. Held 15-19 May 2007, Tampa, Florida, USA. ISBN 978-0-7844-0927-5.

[23] P. R. Page, A. M. Abu-Mahfouz, and S. Yoyo. Real-time adjustment of pressure to demand in water distribution systems: Parameter-less P-controller algorithm. In *12th International Conference on Hydroinformatics, HIC 2016*, volume 154 of *Procedia Engineering*, pages 391–397. Elsevier Ltd., 2016.

[24] T. M. Walski. Discussion of "Knowledge-based optimization model for control valve locations in water distribution networks" by Mohammed E. Ali. *Journal of Water Resources Planning and Management*, 141(5):07015001, 2015.

[25] M. J. Mudumbe and A. M. Abu-Mahfouz. Smart water meter system for user-centric consumption measurement. In *Industrial Informatics (INDIN), 2015 IEEE 13th International Conference on*, pages 993–998. IEEE, New York, 2015. Held 22-24 July 2015, Cambridge, UK.

[26] C. P. Kruger, A. M. Abu-Mahfouz, and G. P. Hancke. Rapid prototyping of a wireless sensor network gateway for the Internet of Things using off-the-shelf components. In *Industrial Technology (ICIT), 2015 IEEE International Conference on*, pages 1926–1931. IEEE, New York, 2015. Held 17-19 March 2015, Seville, Spain. ISBN 978-1-4799-7800-7.

[27] A. M. Abu-Mahfouz, Y. Hamam, P. R. Page, K. Djouani, and A. Kurien. Real-time dynamic hydraulic model for potable water loss reduction. In *12th International Conference on Hydroinformatics, HIC 2016*, volume 154 of *Procedia Engineering*, pages 99–106. Elsevier Ltd., 2016.

[28] S. T. M. Bezerra, S. A. Silva, and H. P. Gomes. Operational optimisation of water supply networks using a fuzzy system. *Water SA*, 38(4):565–572, 2012.

[29] M. J. G. Silva, C. S. Araújo, S. T. M. Bezerra, C. R. Souto, S. A. Silva, and H. P. Gomes. Generalized minimum variance control for water distribution system. *IEEE Latin America Transactions*, 13(3):651–658, 2015.

[30] M. J. G. Silva, C. S. Araújo, S. T. M. Bezerra, S. A. Silva, C. R. Souto, and H. P. Gomes. Adaptive control system applied in water distribution system with emphasis on energy efficiency. *Engenharia Sanitaria e Ambiental*, 20(3):405–413, 2015.

[31] P. R. Page, A. M. Abu-Mahfouz, and M. L. Mothetha. Pressure management of water distribution systems via the remote real-time control of variable speed pumps. *Journal of Water Resources Planning and Management*, 143(8):04017045, 2017. Published online, DOI: 10.1061/(ASCE)WR.1943-5452.0000807.