Revealing the Radial Characteristics of Q-Plate Generated Vortex Beams

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ABSTRACT

Q-plates (QP) have become ubiquitous in experiments requiring the generation of vortex beams [1]. It consequently follows that it is important to characterize the vortex beams created by this geometric-phase optical element. Operation of a q = 0.5 QP is experimentally demonstrated along with characterization of the beam in terms of a special case of Hypergeometric-Gaussian (HyGG) modes through decomposition into Laguerre-Gaussian (LG) modes [2].

WHAT IS GEOMETRIC PHASE?

When a property of light undergoes an adiabatic change that results in it returning to the initial state, an additional phase is incurred, known as a *geometric phase* [3]. Its value can be related to half the enclosed surface area of the traversed path in its parameter space [3]. Figure 1 illustrates this for such a change in right circular (RC) polarization whereby the geometric phase incurred is then half the enclosed surface area, $\Omega/2$.

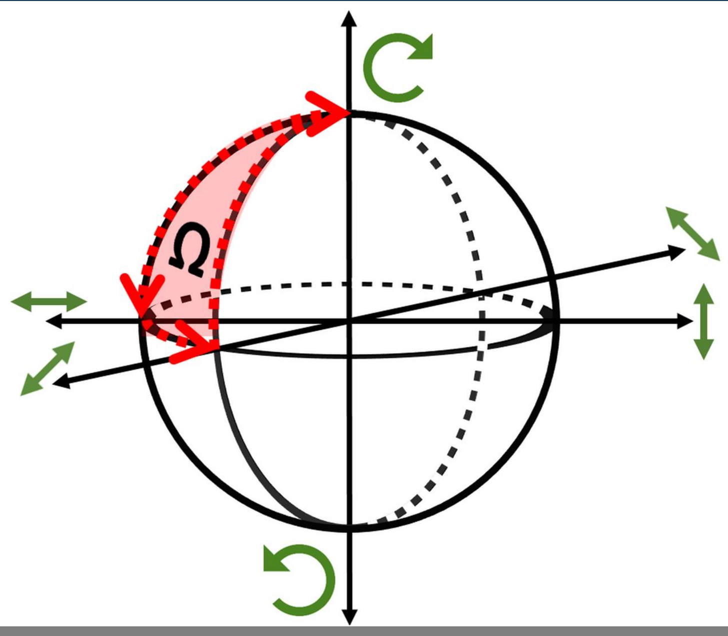
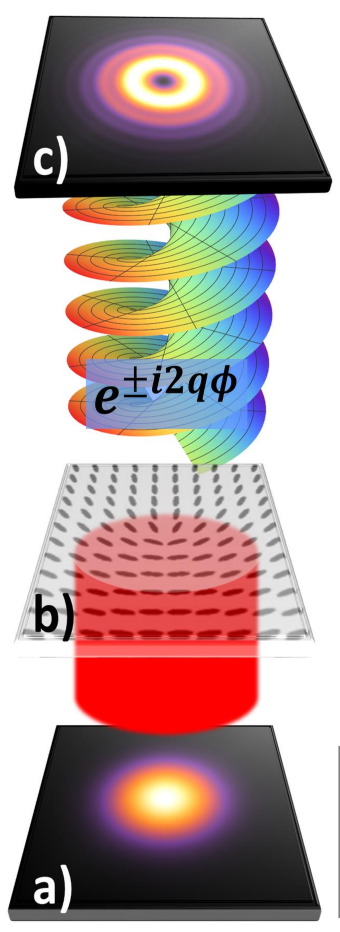


Fig.1. Closed path traversed in polarization-space, resulting in an additional geometric phase of $\Omega/2$.

THE QP AS A GEOMETRIC-PHASE OPTICAL ELEMENT



Inhomogeneous, anisotropic arrangement of the optical media comprising the QP gives rise to space-variant polarization changes across the wavefront [4]. Subsequent variation of the circular polarization (CP) from left to right and *vice verse*, indicates spin angular momentum (SAM) variation [1]. This gives rise to a position dependent geometric phase ($e^{i2q\phi}$) which produces a corkscrew rotation of the wavefront that carries orbital angular momentum (OAM) of $2q\hbar$ per photon where q is the topological charge of the QP [1]. This is illustrated in Fig. 2. The operation is described by the selection rules:

$$i)\ |RCP, l\rangle \rightarrow |LCP, l-2q\rangle \qquad ii)\ |LCP, l\rangle \rightarrow |RCP, l+2q\rangle \qquad (1)$$

Where ϕ is the transverse plane azimuthal angle, OAM of $2q$ is added (subtracted) to LCP (RCP) light and the polarization is reversed due to the spin-to-orbital angular momentum conversion (STOC) taking place.

Fig. 2. Illustration of QP STOC process for an input Gaussian beam. Insets: Transverse image of **(a)** Gaussian beam **(b)** Schlieren diagram of a q = 0.5 QP and **(c)** a QP generated beam.

QP GENERATED MODES

QP generated modes may be expressed as an azimuthal phase variation with a Gaussian envelope such as LG modes without OAM modulated amplitude [2]:

$$LG_l = \left(\frac{r\sqrt{2}}{w_0}\right)^{|l|} \sqrt{\frac{2}{\pi {w_0}^2 (|l|)!}}\, e^{-\left(\frac{r}{w_0}\right)^2} e^{-il\phi} \qquad (2)$$

MISSING AMPLITUDE — QP MODE STRUCTURE

where r is the radial coordinate, w_0 the beam waist, l the OAM index and the radial index, p = 0. The amplitude term modulates the radial power distribution [2]. Subsequent omission (such as in QP modes) forces natural modulation in the form of radial power dispersion through additional excited radial fields [2]. These modes may be interpreted as HyGG, which are linear combinations of LG radial modes (p) [5]:

$$HyGG_l = \sum_p c_p LG_{p,l} \qquad (3)$$

EXPERIMENTAL SETUP

Experimental setups used to characterize the QP are illustrated in Fig. 3.

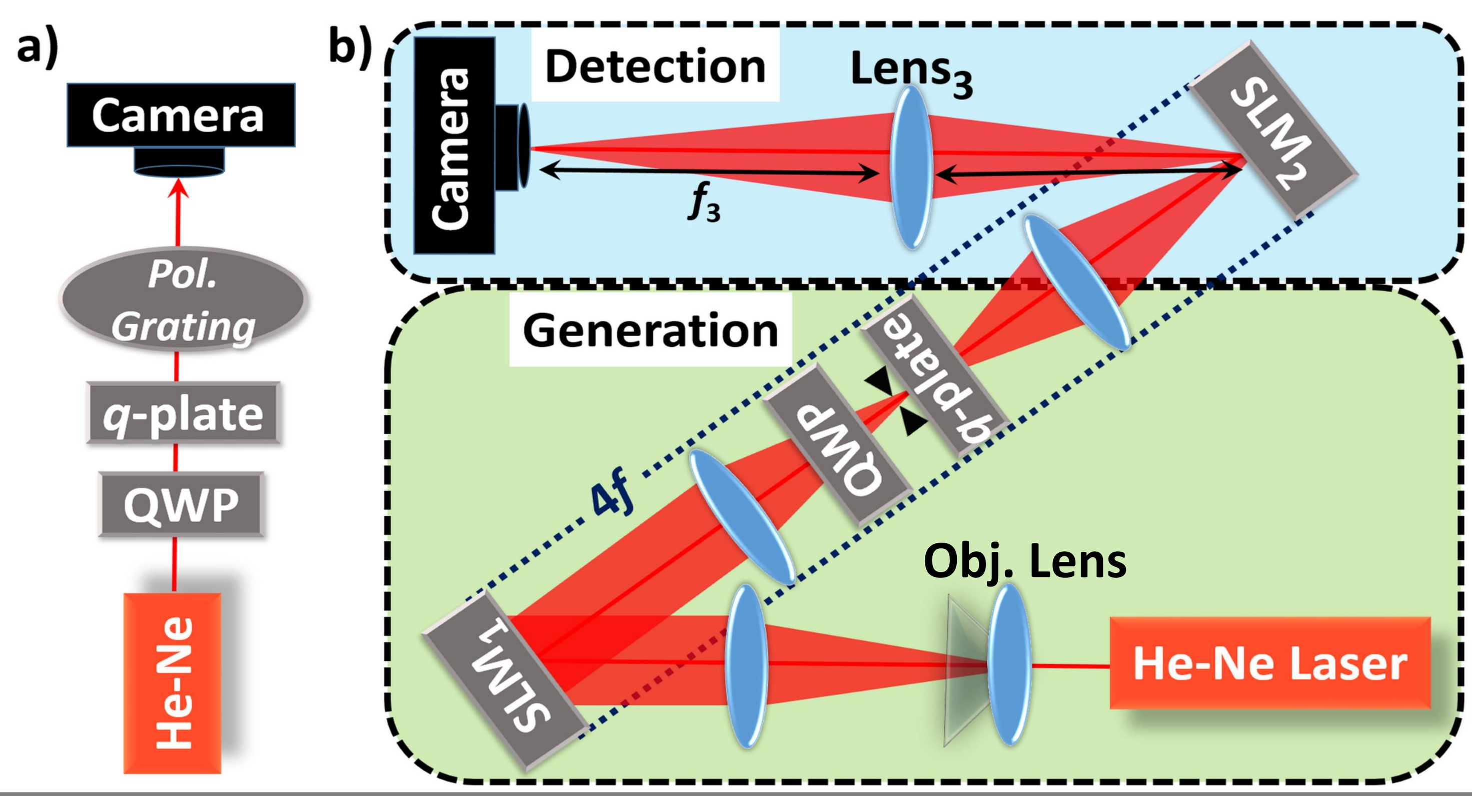


Fig. 3. Schematic of the experimental setup for (a) detecting the change in SAM (b) decomposition into LG basis modes and OAM detection.

In Fig. 3(a) a horizontally polarized He-Ne laser beam was passed through a quarter-wave plate (QWP) and polarization grating. Left circular polarization (LCP) intensity was measured by the camera for a range of QWP orientations, before the measurements were repeated with a QP inserted as seen. In Fig. 3(b), a digital hologram of an LG beam was displayed on a reflective phase-only spatial light modulator (SLM1). The SLM was illuminated with a He-Ne laser beam which was expanded and collimated. The beam reflected off of the SLM was converted to circularly polarized light by a QWP, spatially filtered with a pinhole in the first diffraction order and propagated through a QP in the 4f system. The resulting beam was reflected off a match filter (SLM2) and the Fourier transform thereof detected in the Fourier plane of lens 3.

RESULTS

QP Operation

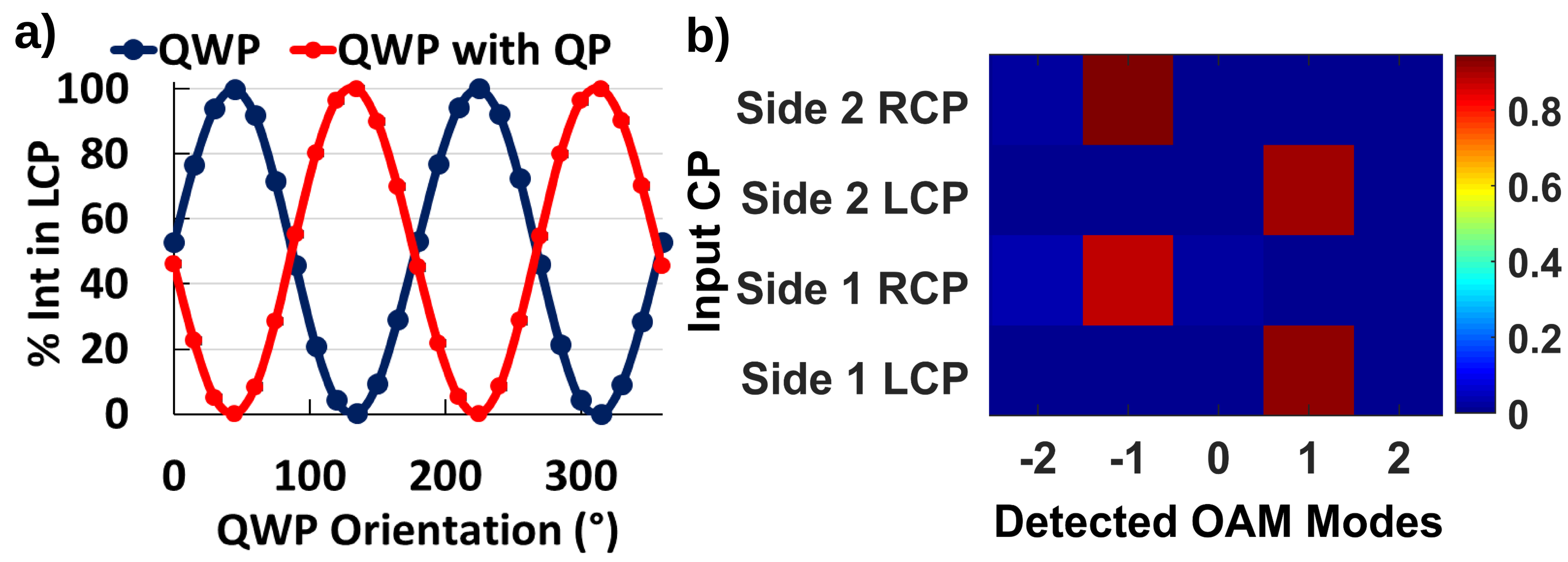


Fig. 4. (a) CP inversion induced by a q = 0.5 QP as measured by the percentage of total power present in LCP plane as a function of QWP orientation. **(b)** Change in OAM induced by the QP for LCP and RCP, measured for both QP transverse surfaces.

QP and HyGG Beams in LG Basis

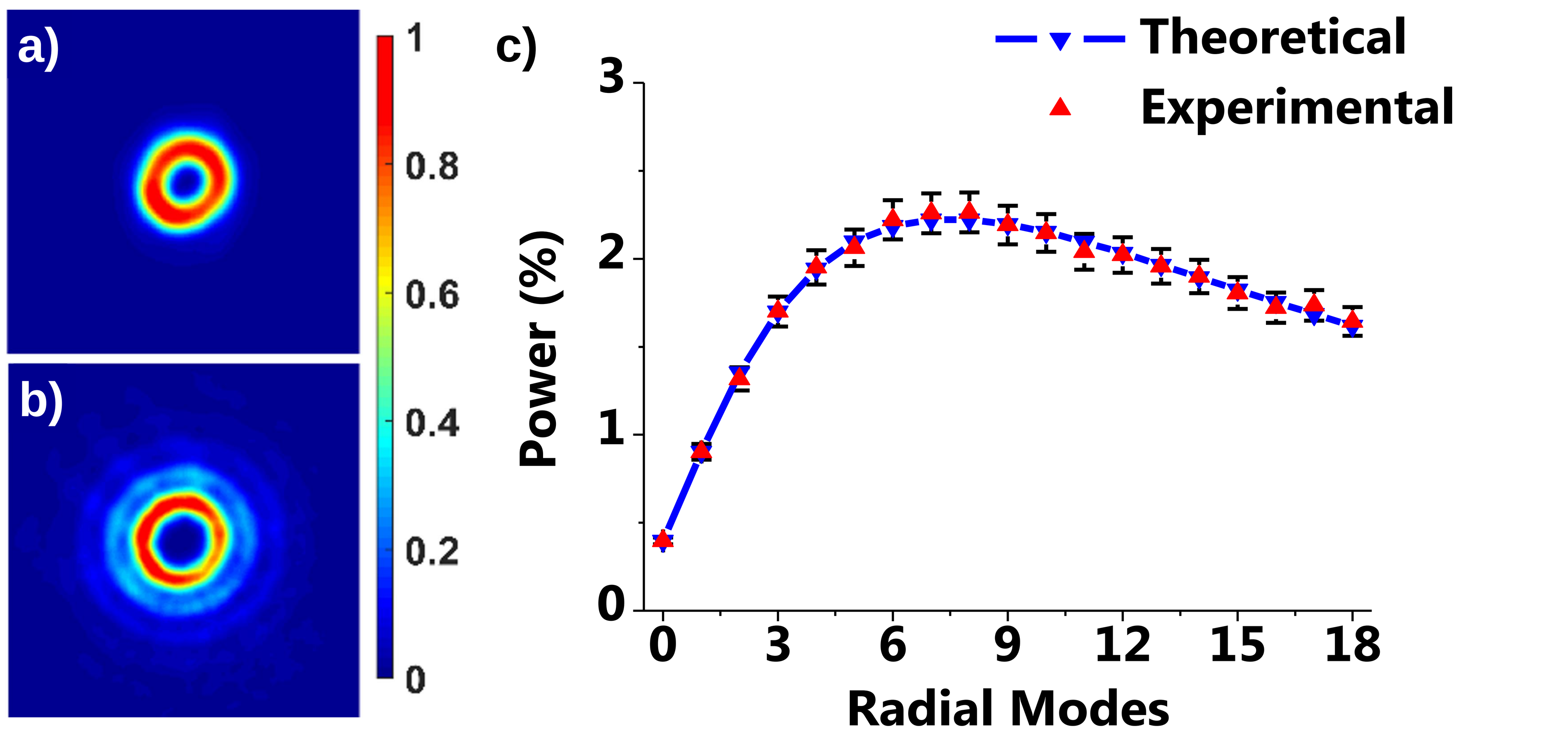


Fig. 5. Experimental transverse intensity profiles of **a)** a pure LG beam and **b)** an imperfect vortex beam with increased radial power distribution. **c)** Measured power distribution into radial (p) modes of the LG basis for an l = 20 generated beam and the corresponding theoretical HyGG beam, illustrating increased radial power distribution and confirmation of the generated modes as HyGG. Similar agreement was found for other OAM modes.

CONCLUSION

- **Circular polarization of the QP generated beam is inverted, indicating change of the SAM.**
- **QP increases (decreases) OAM by 2q in LCP (RCP) light by imparting geometric phase through STOC.**
- **Good agreement between measured and theoretical radial power distributions in the LG basis for QP and HyGG modes confirm the QP generated modes are HyGG.**
- **Imperfect beams are created by the QP whereby radial divergence of the beam power occurs with free-space propagation as seen by their description as HyGG modes.**

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