The Wigner distribution function in modal characterisation

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Abstract

Optical field characterisation often requires various isolated experiments to obtain characterisation parameters, we investigate a novel approach to characterise an optical field employing a Wigner distribution function with a modal decomposition technique to obtain parameters in a single set of experimental measurements. We demonstrate a mathematical representation of the approach and highlight the procedure in modal characterisation using Laguerre-Gaussian modes.

Introduction

Modal characterisation has been an area of interest in photonics for several decades with a diverse range of studies. The determination of the parameters of an optical field allows us to understand the propagation and interaction of the field with optical elements. Techniques of modal characterisation have been developed and has seen many advances such as modal decomposition being one of the more recent techniques. Characterisation focuses on the determination of various field parameters including beam size, far-field divergence, beam quality(M^2) and wave-front. The knife-edge method is used in the determination of divergence by measuring the beam size [1], and for wave-front determination, the gradient-measurement technique is used together with a Shack-Hartmann sensor (SHS) [2]. Modal decomposition is used to decompose an optical field into its constituent modes with each mode weighted with a complex coefficient [3]. We introduce a novel approach to modal characterisation by incorporating a Wigner distribution function (WDF) with modal decomposition. The (WDF) provides the representation of a field in the dual phase-space i.e. space and spatial frequency. Applying the WDF gives rise to an interference term which distorts the linearity of the decomposition in the spatial frequency domain. We explore techniques to successfully apply the WDF with modal decomposition linearly and without the effects of the interference term for beam characterization. We apply and test the WDF using Laguerre-Gaussian (LG) modes to determine beam size.

Modal decomposition

Modal decomposition is a technique used to characterise an optical field, the field is decomposed and represented as a superposition of its constituent modes [3]. Each mode is weighted with a complex coefficient in the expansion which is found by an inner product. Determining these modal weights is the main aim of the decomposition, the coefficients yield information about the amplitude and phase of the orthogonal modes. Using this, we can reconstruct the entire field.

The modal decomposition of an arbitrary field $\mathbf{U}(r)$ is given by:

$$\mathbf{U}(r) = \sum_P \sum_l a_{p,l} \Psi_{p,l}(r)]$$

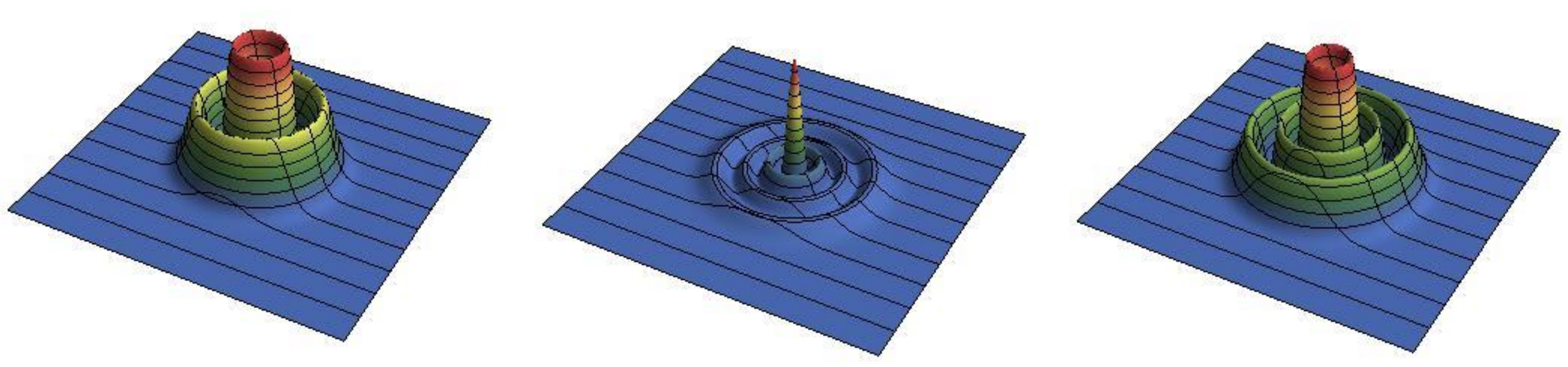

Figure 1: The LG modes $\Psi_{2,1}$, $\Psi_{3,0}$ and $\Psi_{2,2}$ (from left to right).

Wigner distribution function

We introduce the WDF for application in modal decomposition characterisation. The WDF describes an optical field simultaneously in the spatial and frequency domains [4]. The optical field Wigner function also resembles the description of a ray in geometrical optics. The Wigner distribution function is a bilinear function, thus a sum of optical signals produces a complex cross term [5]. However, various integrals of the Wigner function produce the same results as integrals of the optical signal itself, notably the integrals for the beam moments which are used to determine beam width, divergence and the beam quality factor for both anti-and symmetric beams [1]. We use these relations to determine these parameters without the interference of the complex cross term.

Let $f(x)$ be a field function in the spatial domain, the Wigner distribution of $f(x)$ is an integral of the correlation function $\left(f\left(x+\frac{1}{2}x'\right)f^*\left(x+\frac{1}{2}x'\right)\right)$ represented as:

$$W_f(x,e) = \int_{-\infty}^{\infty} f\left(x+\frac{1}{2}x'\right) f^*\left(x+\frac{1}{2}x'\right) e^{-iex'}dx'.$$

The WDF of a sum of field functions $f(x)$ and $g(x)$ is given by:

$$W_{f+g}(x,u) = Wf(x,u) + Wg(x,u) + 2Re[Wf_{,g}(x,u)].$$

WDF in determining beam width

The WDF in determining beam width is carried out using Laguerre-Gaussian modes as a description of the field, the WDF may be applied to any set of orthogonal modes. The individual LG modes are generated using a generation function. We demonstrate the WDF intensity plots with LG modes as the optical field functions in the space domain in Fig.2.

$$\mathbf{W_U}(r,u) = \sum_P \sum_l a_{p,l} W_\Psi(r,u)_{p,l}$$

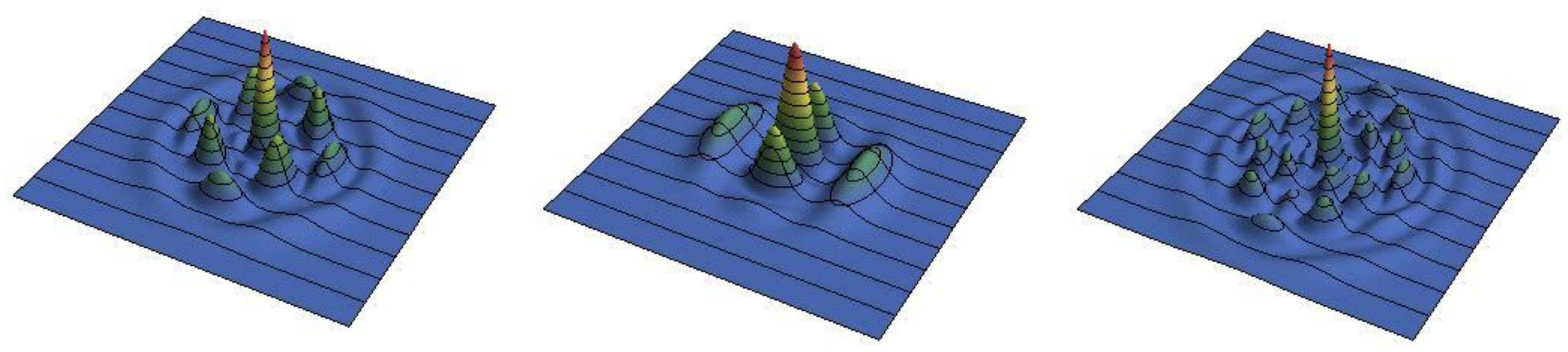

Figure 2: The WDF modes of the LG modes $\Psi_{2,1}$, $\Psi_{3,0}$ and $\Psi_{2,2}$ (from left to right).

The beam size is determined using the equivalence relation between an optical field function and its WDF:

$$m_{xx} = \frac{\frac{1}{2\pi}\int\int (u-mx)^2 W_\Psi(r,u)_{p,l}]drdu}{\frac{1}{2\pi}\int\int W_\Psi(r,u)_{p,l}drdu} = \frac{\int (u-mx)^2 |\Psi_{p,l}(r)|^2 dr}{\int |\Psi_{p,l}(r)|^2 dr}$$

WDF Field function

A simulation for the lower order modes was carried out to determine the beam size, the results are presented in Table 3. We demonstrate the relationship between the field function and WDF. The beam size was verified for both the WDF and field function integrals:

Table 1: Beam size for various LG modes

l	p	m_{xx}
0	0	$\frac{d^2(2+\pi)}{4\pi}$
0	1	$\frac{d^2(32+21\pi)}{36\pi}$
1	1	$\frac{d^2(512+231\pi)}{196\pi}$
1	2	$\frac{d^2(22768+16371\pi)}{10404\pi}$

The beam size may be experimentally determined using a similar setup to that of a modal decomposition [3] incorporating multiplexing and photo diodes for real time sampling.

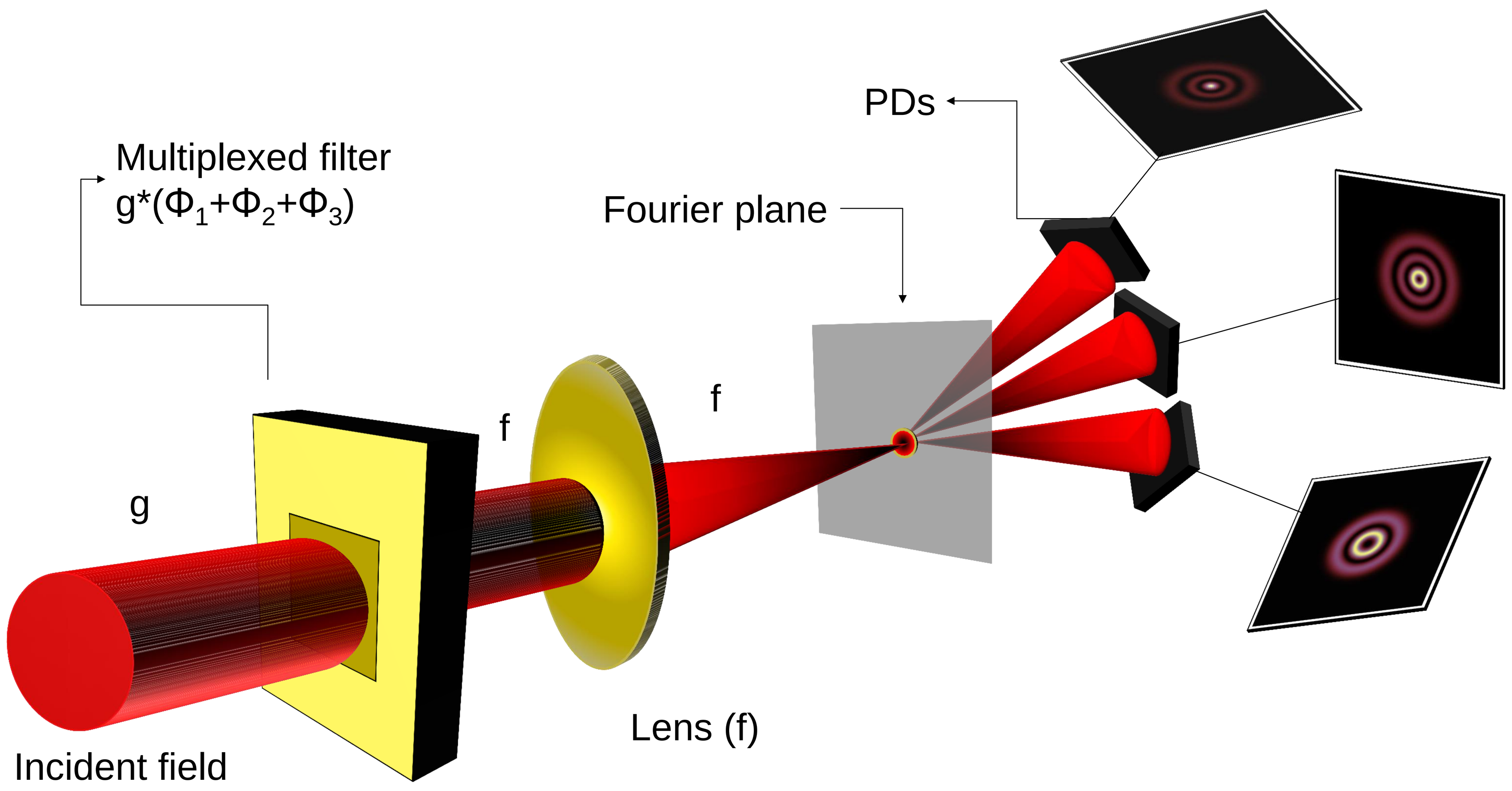

Figure 3: Modal decomposition with multiplexing and PDs.

Conclusion

We have demonstrated the WDF in determining beam width without the interference of the cross term in the spatial domain and effectively shown the relationship between a field function and its WDF through the integral relations. To further characterise optical fields, we are developing a full characterisation method extended to include the frequency domain.

References

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