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Interface information transfer between non-matching, nonconforming interfaces using radial basis function interpolation

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ABSTRACT

In this paper we outline the use of radial basis function interpolation (RBF) to transfer information across non-matching and nonconforming interface meshes, with particular focus to partitioned fluid-structure interactions (FSI). In general, transferring information across a non-matching interface presents itself as a nontrivial problem. RBF interpolation, which requires no global connectivity information, provides an elegant means by which to negate any geometric discrepancies along the interface. The aim is to investigate the feasibility of RBF interpolation, with a strong focus on a comparison between a conservative and consistent formulation.

KEYWORDS

radial basis function interpolation, non-matching interface mesh, FVM-FEM coupling, consistent, conservative

INTRODUCTION

Fluid-structure interactions is the two-way coupled analysis of deformable structures and the corresponding interactions with fluid flow. A few common examples include flutter analysis of aero-elastic structures (Farhat et al. [1], Rifai et al. [2]) or blood flow through the cardiovascular system (Torii et al. [3], Wolters et al. [4]). To couple the two domains requires the transfer of information across the shared interface. The information to be transferred typically include the interface displacements transferred from the solid domain to the fluid domain, and interface stresses to be transferred from the fluid domain in the form of interface pressures and wall shear stresses.

The numerical properties between the fluid domain and the solid domain differ sufficiently that they are naturally solved with very different geometric discretisation requirements. For example, the fluid solver typically requires far more degrees of freedom than the shared structural problem. Or conversely, the structural domain, either around sharp edges or in vicinities of high stresses, may require localised mesh refinement. Consider for example Figure 1, illustrating the potential mismatch along a curved interface. The interface meshes are both mismatched and nonconforming, where due to the curvature there are both gaps and overlaps. The problem may further be complicated by the choice of using different numerical schemes on each of the sub-problems. The solid domain is typically solved using the finite element method (FEM) where interface information is located at element nodal coordinates with well defined internal interpolation shape functions. The fluid field solver on the other hand, typically based on the finite volume method (FVM), defines quantities at face centres where the internal interpolation function can at best be described as a face-constant step function.

Accurate information transfer is critical to the accuracy and stability of partitioned FSI solution schemes. There are a large number of interface information transfer schemes available in literature; many are based either on physical arguments (Cebral and Lohner [5], Farhat et al. [6], Jiao and Heath [7]) or approaches based on mathematical arguments (Beckert and Wendland [8], Lombardi et al. [9], Quaranta et al. [10], Smith et al. [11]).

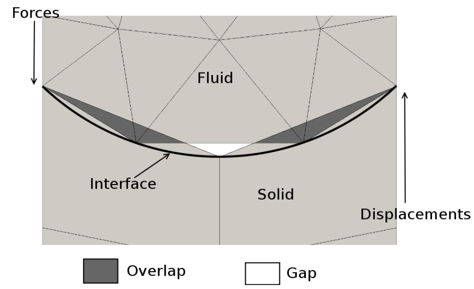


Figure 1: Illustrative example of a non-matching mesh along a curved interface.

In this paper we aim to outline the use of radial basis function (RBF) interpolation to transfer information. Multivariate interface transfer methods have become very popular, primarily because they require no mesh connectivity information and is therefore well suited to coupling FVM-FEM discretisation schemes, and further negates any potential geometric discrepancy. In general when considering RBF interpolation, there is a trade-off between transferring the information in a conservative or in a consistent sense, with no way of satisfying both conditions. By conservative transfer, we imply that the sum of forces are transferred exactly, or that equal work is done along the interface. For consistent information transfer, we adopt the definition to imply that information is transferred in such a manner that a constant stress state can be transferred exactly (or patch test is satisfied).

Many researchers stipulate that conservative information transfer is crucial to the overall stability and accuracy of FSI simulations (Lombardi et al. [9], Piperno et al. [12]). To enforce conservatism using multivariate transfer, the interpolation/projection functions are often constructed on the basis of satisfying virtual work along the interface. In this paper, we aim to demonstrate that this approach in essence results in a zero-order scheme, with the possibility of transferring unrealistic and oscillatory stress states (similar observations have also been noted in de Boer et al. [13]). Furthermore, the conservative formulation can not be used to transfer information between discretisation schemes of different orders. We further aim to show, that while a consistent formulation is not provably conservative, it is convergent, and hence conservative, within the limit of mesh refinement.

The outline for the remainder of the paper is as follows. We start by outlining the mathematical notation and formulation for consistent and conservative interface information transfer. We provide a brief overview of RBF interface information transfer and aim to compare conservative and consistent interface transfer via a patch test and transfer of an analytical function across a curved interface.

INTERFACE CONDITIONS

The coupled FSI problem is a two-field problem, with a fluid domain Ω_f and a solid domain Ω_s which share a common interface Γ_{FSI}. By allowing each of the two domains to be discretised independently, each of the two domains have distinct interfaces, namely Γ_f and Γ_s, where $\Gamma_f \neq \Gamma_s$. The partitioned FSI problem can be viewed as a two-field problem with jump conditions along the interface which need to be satisfied in the form of the kinematic and dynamic continuity conditions. Each FSI cycle requires satisfying the equilibrium of interface tractions $\boldsymbol{t}_f$ and $\boldsymbol{t}_s$, and compatibility of interface velocities, i.e.

$$\boldsymbol{t}_s = \boldsymbol{t}_f, \quad \frac{\partial \boldsymbol{d}_s}{\partial t} = \boldsymbol{u}_f \quad \text{along}\, \Gamma_{\text{FSI}},$$

where $\boldsymbol{t}_f = p_f \boldsymbol{n}_f - \boldsymbol{\sigma}_f \cdot \boldsymbol{n}_f$ and $\boldsymbol{t}_s = \boldsymbol{\sigma}_s \cdot \boldsymbol{n}_s$. p_f denotes the fluid pressure along the interface, $\boldsymbol{\sigma}_f$ the fluid viscous stress tensor and $\boldsymbol{\sigma}_s$ the solid stress tensor; the outward pointing normals along Γ_s and Γ_f are $\boldsymbol{n}_s$ and $\boldsymbol{n}_f$ respectively.

In the event that the meshes are both matching, and the numerical schemes and order of internal shape functions are exactly the same, the discrete nodal quantities can simply be transferred, i.e. $\boldsymbol{U}_f = \boldsymbol{U}_s$ and $\boldsymbol{T}_s = \boldsymbol{T}_f$. For non-matching meshes, an intermediate projection or interpolation step is required:

$$\boldsymbol{U}_f = H_{fs}\boldsymbol{U}_s, \text{ and } \boldsymbol{T}_s = H_{sf}\boldsymbol{T}_f. \tag{1}$$

Here H_{AB} represents the transformation matrix to transfer information from mesh B to mesh A. $\boldsymbol{U}$ and $\boldsymbol{T}$ are the vectors of discrete values at the interface points and can be defined by the approximations

$$\boldsymbol{u}(\boldsymbol{x}) \approx \sum_{i=1}^{n^u} N_u^i(\boldsymbol{x})\boldsymbol{U}^i, \quad \text{and} \quad \boldsymbol{t}(\boldsymbol{x}) \approx \sum_{j=1}^{n^t} N_t(\boldsymbol{x})_t^j \boldsymbol{T}^j. \tag{2}$$

$N_{u,t}$ here represents the spatial interpolation function used for the displacement and tractions respectively. Typically for the FEM, $N(\boldsymbol{x})$ is the internal basis/shape functions and a step function for the FVM method; n^u and n^t are the number of degrees of freedom (DOF) along the interface where the discrete displacement and traction quantities are known.

CONSERVATIVE INFORMATION TRANSFER

The general consensus in literature is that the information transfer should be conservative (see for example Beckert and Wendland [8], Farhat et al. [6], Jaiman et al. [14, 15], Lombardi et al. [9], Quaranta et al. [10]). By the strictest of definitions this would imply that the integrated quantities should be equal on both Γ_f and Γ_s. The concentrated loads, typically located at the nodal coordinates for the FEM and at face centres for the FVM method, can be defined as

$$\boldsymbol{F}_f = \int_{\Gamma_f} \boldsymbol{t}_f d\Gamma, \text{ and } \boldsymbol{F}_s = \int_{\Gamma_s} \boldsymbol{t}_s d\Gamma.$$

Conservation of forces can then be expressed as

$$\boldsymbol{F}_s = \sum_{i=1}^{n_s} \boldsymbol{F}_s^i = \sum_{j=1}^{n_f} \boldsymbol{F}_f^j = \boldsymbol{F}_f. \tag{3}$$

Along a non-matching interface, there are an infinite number of nodal load vectors which will satisfy conservation. A convenient argument often used to enforce conservation is the definition of virtual work. For steady state (or for very small time steps), energy can be stated to be globally conserved over the interface if

$$\int_{\Gamma_f} \boldsymbol{u}_f \cdot \boldsymbol{t}_f \boldsymbol{n}_f d\Gamma_f = \int_{\Gamma_s} \boldsymbol{u}_s \cdot \boldsymbol{t}_s \boldsymbol{n}_s d\Gamma_s. \tag{4}$$

Using the approximations in equation (2), the semi-discrete form of equation (4) becomes

$$[M_{ff}\boldsymbol{U}_f]^T \boldsymbol{T}_f = [M_{ss}\boldsymbol{U}_s]^T \boldsymbol{T}_s, \tag{5}$$

where matrices M_{ff} and M_{ss} are defined as

$$M_{ff}^{ij} = \int_{\Gamma_f} N_f^i N_f^j d\Gamma, \quad \text{and} \quad M_{ss}^{ij} = \int_{\Gamma_s} N_s^i N_s^j d\Gamma.$$

While not strictly correct, matrices M_{ss} and M_{ff} are often referred to in this context as mass matrices (Jiao and Heath [7]).

Given some displacement transformation matrix H_{fs} such that $\boldsymbol{U}_f = H_{fs}\boldsymbol{U}_s$, and substituting this into equation (5), it is possible to construct a global traction transformation matrix,

$$\boldsymbol{T}_s = \left[M_{ff}H_{fs}M_{ss}^{-1}\right]^T \boldsymbol{T}_f. \tag{6}$$

Choosing $H_{sf} = \left[M_{ff} H_{fs} M_{ss}^{-1} \right]^T$, will result in global conservation of interface stress states. If we further recognise that the discrete concentrated forces are defined by $\boldsymbol{F}_s = M_{ss}^T \boldsymbol{T}_s$ and $\boldsymbol{F}_f = M_{ff}^T \boldsymbol{T}_f$, then equation (6) can be rewritten as

$$\boldsymbol{F}_s = H_{fs}^T \boldsymbol{F}_f. \tag{7}$$

Energy along the interface will therefore provably be conserved if the transpose of the displacement interpolation matrix is used to project the concentrated nodal forces. It is perhaps important to note, that the force interpolation presented in equation (7) contains no information regarding the solid domain's internal interpolation and integration schemes. This immediately imposes a limitation, as it restricts passing information between equal order schemes only (i.e. linear FVM to linear FEM).

CONSISTENT INFORMATION TRANSFER

In this paper the definition of consistent transfer is adopted from de Boer et al. [13], which states that a constant stress state should be transferred exactly . The terminology of consistency stems from the requirement that the interpolation and integration of quantities along the interface should be consistent with the sub-domain schemes. In order to transfer a constant quantity requires that the row-sum of both H_{sf} and H_{fs} be equal to 1. Consistent information transfer therefore requires that the displacement and force transformation matrices be constructed independently of each other. In other words

$$\boldsymbol{U}_f = H_{fs} \boldsymbol{U}_s, \;\; \boldsymbol{T}_s = H_{sf} \boldsymbol{T}_f, \;\;\; H_{sf} \neq H_{fs}^T. \tag{8}$$

The consistent approach, when using RBF interpolation, translates to transferring the field quantities in the form of pressures and shear stresses directly. These field quantities are then integrated along the solid interface using integration rules consistent with the solid domain. For arbitrary nonlinear interface fields, with large gaps between the two sub-domain interfaces, there is no guarantee that such a consistent scheme will be energy conserving. The fact that the consistent scheme is not provably conservative, does not, as mentioned in Farhat et al. [1], imply that the transfer of information will lead to inaccurate or unstable FSI simulations. Transient partitioned FSI solvers are by construction non-conservative due to the time lag between the solid and flow solvers within a given time step. The primary concern should then rather be whether the accuracy and errors introduced by the information transfer are less than those already present. In the limit case of a constant stress state (patch test), provided the constant is exactly transferred, the system will be in static equilibrium, and hence energy conserving. We will further demonstrate that the error in energy, introduced for nonlinear stress states, disappears in the limit of mesh refinement.

RADIAL BASIS FUNCTION INTERPOLATION

The class of multivariate interpolation is based on the idea of using a global interpolation function to transfer information. Multivariate interpolation requires no connectivity information and is therefore well suited to the transfer of information between interface meshes with arbitrary geometric mismatches.

Radial basis functions (RBF) have in particular gained large popularity in the field of multivariate approximation theory. RBF is comparatively simple to implement, the underlying mathematical properties are well understood and provides good interpolation properties (for a numerical comparison to Kriging and the moving least-squares method, see Krishnamurthy [16]). RBF interpolation is based on fitting a series of splines, or basis functions to interpolate information from one point cloud to another.

Let us assume we wish to transfer information, $\boldsymbol{s}(\boldsymbol{x})$, from mesh A to mesh B, where $\boldsymbol{s}(\boldsymbol{x}) = \{\boldsymbol{u}, \boldsymbol{t}\}$. At the centres, $x_1, x_2, ..., x_{n_A}$ (either nodal coordinates or integration points) of mesh A, we know the discrete values of $\boldsymbol{s}(\boldsymbol{x})$, which we denote here as $g_1, g_2, ..., g_{n_A}$, where n_A is the number of DOFs at which information is known along the interface of mesh A. We now wish to construct a continuous function which allows us to interpolate the known values, at known locations, from A to the centres of B. The interpolation function, using RBF with an additional linear polynomial, has the following form:

$$\boldsymbol{s}(\boldsymbol{x}) = \sum_{i=1}^{n_A} \alpha_i \phi\left(||\boldsymbol{x} - \boldsymbol{x}_{A_i}||\right) + \sum_{j=1}^{m} p_j(\boldsymbol{x})\,\beta_j, \tag{9}$$

where ϕ is the chosen basis function, $||\cdot||$ refers to the Euclidean distance (in 3 dimensions, $||\boldsymbol{x} - \boldsymbol{x}_{A_i}|| = \sqrt{(x - x_{A_i})^2 + (y - y_{A_i})^2 + (z - z_{A_i})^2}$) and the coefficients α_i are to be solved so that the condition

$$\boldsymbol{s}(x_{A_i}) = g_i, \quad \text{for } i = 1, ..., n_A \tag{10}$$

is satisfied. $p_j(\boldsymbol{x})$ are the monomial terms of a polynomial of degree m, and β_j are m additional constants introduced due to the additional polynomial terms. The m additional constants are obtained by including an additional m constraints in the form

$$\sum_{i=1}^{n_A} p_i(\boldsymbol{x})\,\alpha_i = 0. \tag{11}$$

The inclusion of the polynomial has an important consequence. If the function $\boldsymbol{s}(\boldsymbol{x})$ can be described exactly by a linear polynomial, and the included polynomial $p(\boldsymbol{x})$ is linear, then the polynomial will be exactly reproduced. This follows from, given that ϕ is a conditionally positive definite function, there provably exists a unique function $\boldsymbol{s}(\boldsymbol{x})$ which satisfies both equations (10) and (11) (see Wendland [17] for a proof of this, and Beckert and Wendland [8] for a more detailed discussion). In this paper a linear polynomial of the form $p(x) = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 z$ is used. By using a linear polynomial we can provably transfer constant information as well as rigid body motion (when transferring displacement). The inclusion of the linear polynomial does place some mild restrictions on our choice of information centres. While no additional sampling points are necessary, it does mean that at least 4 points must not fall along a plane. If Γ_{FSI} is in fact a flat face, then a linear polynomial cannot be used.

Using equations (10) and (11) the following matrix problem can be defined to solve for the RBF coefficients:

$$\begin{bmatrix} \boldsymbol{g}_A \\ \boldsymbol{0} \end{bmatrix} = \begin{bmatrix} \boldsymbol{M}_{AA} & \boldsymbol{P}_A \\ \boldsymbol{P}_A^T & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix}. \tag{12}$$

Here $\boldsymbol{\alpha}$ is the vector containing the coefficient sets $\boldsymbol{\alpha}_i$, and $\boldsymbol{\beta}$ is the vector containing the polynomial constants, both to be solved for. For a linear polynomial $p(\boldsymbol{x})$, $\boldsymbol{P}_A$ is an $n_A \times 4$ matrix where each row i is given by $\{1, x_{A_i}, y_{A_i}, z_{A_i}\}$, for $i = 1, 2, ..., n_A$. Finally, $\boldsymbol{g}_A$ refers to the matrix of known values to be interpolated to mesh B. $\boldsymbol{M}_{AA}$ is an $n_A \times n_A$ matrix containing the evaluations of the RBF basis functions

$$\boldsymbol{M}_{AA} = \begin{bmatrix} \phi_{A_1A_1} & \phi_{A_1A_2} & \cdots & \phi_{A_1A_{n_A}} \\ \vdots & \vdots & \ddots & \vdots \\ \phi_{A_{n_A}A_1} & \phi_{A_{n_A}A_2} & \cdots & \phi_{A_{n_A}A_{n_A}} \end{bmatrix}, \tag{13}$$

where $\phi_{A_1A_2} = \phi\left(||\boldsymbol{x}_{A_1} - \boldsymbol{x}_{A_2}||\right)$. Once the coefficients $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ have been solved, the interpolated quantities on mesh B, $\boldsymbol{g}_B$ can then be found by

$$[\boldsymbol{g}_B] = \begin{bmatrix} \phi_{BA} & \boldsymbol{P}_B \end{bmatrix} \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix}. \tag{14}$$

In other words,

$$\boldsymbol{g}_B = \begin{bmatrix} \phi_{BA} & \boldsymbol{P}_B \end{bmatrix} \begin{bmatrix} \boldsymbol{M}_{AA} & \boldsymbol{P}_A \\ \boldsymbol{P}_A^T & \boldsymbol{0} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{g}_A \\ \boldsymbol{0} \end{bmatrix}. \tag{15}$$

NAME:	DEFINITION
C^0 compactly supported piecewise polynomial (C^0):	$(1-(\|\|x\|\|/r))_+^2$
C^2 compactly supported piecewise polynomial (C^2):	$(1-(\|\|x\|\|/r))_+^4\,(4\,(\|\|x\|\|/r)+1)$
Thin-plate spline (TPS):	$\|\|x\|\|^2 \ln \|\|x\|\|$
Multi-quadratic biharmonic (MQ):	$\sqrt{\|\|x\|\|^2 + a}$
Cubic:	$\|\|x\|\|^3$
Quintic:	$\|\|x\|\|^5$

Table 1: List of RBF basis functions used in this study. Basis functions obtained from [8, 13, 19, 20].

The transformation matrix H_{BA} is therefore the first n_B rows and n_A columns of the matrix

$$\begin{bmatrix} \phi_{BA} & \boldsymbol{P}_B \end{bmatrix} \begin{bmatrix} \boldsymbol{M}_{AA} & \boldsymbol{P}_A \\ \boldsymbol{P}_A^T & \boldsymbol{0} \end{bmatrix}^{-1}. \tag{16}$$

The matrix inverse in equation (15) is usually not computed explicitly. We are only interested in $\boldsymbol{g}_B$, which can be found by solving equation (12) and performing the dot product in equation (14).

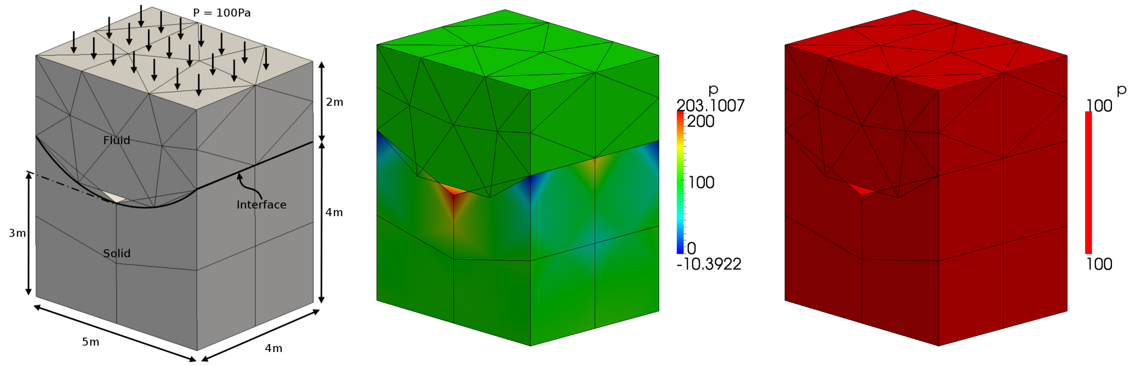
The list of RBF basis functions employed in this study are shown in Table 1. r in the C^0 and C^2 basis functions refer to the choice of support radius and the subscript $+$ indicates that only positive quantities are taken into account. How to choose r is important to the overall behaviour and interpolation quality of compactly supported functions. A larger value of r typically leads to very good interpolation results. Choosing r too large however, leads to ill-conditioned systems. Equally, smaller values of r leads to a sparsely populated banded matrix which is beneficial for efficient linear system solutions. For good interpolation results, r is typically recommended to be set to $r = 2r_{\text{max}}$, where r_{max} is the radius which includes all points. For the MQ RBF function, the shape of the spline is controlled via the parameter a. Small choices of a lead to sharp cone-like splines which flatten out as a is increased. De Boer *et al.* [13, 18], suggest values of a in the range $10^{-5} - 10^{-3}$ for a domain of unit length.

NUMERICAL ANALYSIS

PATCH TEST

The purpose of the patch test is to determine whether the transfer scheme can exactly represent a constant stress state. The patch test geometry used in this analysis is shown in Figure 2(a), where the discretised meshes along the curved interface are incompatible, and contain gaps and overlapping regions (see Bathe and Ledezma [21] for an alternative proposal of a patch test benchmark problem). We apply a constant, normal pressure along the top of the fluid domain of $p = 100$Pa. The fluid domain is prescribed with a density and viscosity of $\rho = 1$kg/m^3 and $\mu = 10$kg/ms respectively. For the solid domain, Young's modulus is set to E $= 200$GPa with a Poisson's ratio of $\nu = 0.49999$ to represent an incompressible material. The material and problem descriptions are chosen such that the solid domain displacements are very small. The resulting displacements are small enough to be near negligible, but because of the coupled nature of an FSI problem, inaccuracies, even at such small levels remain fundamentally important to the overall solution of the coupled system.

In Figure 2, we show the pressure state for both the conservative and consistent information transfer. The consistent approach exactly transfers the pressure state, and the FSI system is in perfect equilibrium (to illustrate this we summarise the norm of the transferred forces in Table 2). By contrast, the interface stress state arising from the conservative approach is highly oscillatory. The conservative approach transfers the magnitude of forces in each distinct direction exactly without accounting for the slight differences in surface areas and norms and therefore, despite being exactly conservative, the results deriving from the conservative scheme are incorrect. The inaccuracy of the stress state further manifests itself in an oscillatory displacement field (not shown here).



(a) (b) (c)

Figure 2: (a) Constant pressure patch test problem description solved using (b) conservative information transfer and (c) consistent information transfer.

	Conservative		Consistent	
	Fluid	Solid	Fluid	Solid
$\sum F_x$	-1.34e-07	-1.34e-07	-1.07e-10	-8.84e-11
$\sum F_y$	-2000.00	-2000.00	-2000.00	-2000.0 0
$\sum F_z$	-0.332	-0.332	-0.332	1.016e-17
$\lvert\sum F\rvert$	2000.00	2000.00	2000.00	2000.00

Table 2: Comparison of the sum of the transferred forces for the constant pressure patch test.

ANALYTICAL TEST FUNCTION

In this section, we compare the accuracy of the consistent and conservative approaches by transferring an analytical function across a curved interface, for different levels of grid refinement. We make use of the smooth, nonlinear function

$$\boldsymbol{s}\left(x,z\right)=\sqrt{\cos\left(x^2+z^2\right)}. \tag{17}$$

To measure the accuracy of the transfer schemes we make use of the relative L^2 error defined by

$$\boldsymbol{s}_{\text{error}}=\sqrt{\frac{\sum_{i=1}^{n}\left(\boldsymbol{s}^{i}_{\text{exact}}-\boldsymbol{s}^{i}_{\text{interpolated}}\right)^2}{\sum_{i=1}^{n}\left(\boldsymbol{s}_{\text{exact}}\right)^2}}. \tag{18}$$

The interface mesh used for the analysis (depicted in Figure 3) is fairly typical of FSI simulations. The fluid interface is described with 2D triangular elements coming from a FVM domain, and the solid interface consists of quadrilateral elements with 4 quadrature points. It is important to mention that we limit our analysis to linear FE meshes for the structural domain. We already mentioned that the conservative scheme cannot be used to transfer information between linear to quadratic fields. Therefore, rather than biasing the results to the order incompatibility of the internal interpolation and quadrature rules, we wish to demonstrate that the conservative approach is a zero-order method even when transferring between two linear discretisation schemes.

In Figure 4(a) we report the error in displacement as a function of the choice of RBF basis functions for the simultaneous refinement of both the solid and fluid interfaces. These results are further compared to the error incurred using a linear FVM method. All the basis functions results in displacement errors and convergence rates below those already present within the fluid domain solver.

In Figure 4(b) we show the relative L^2 norm pressure error. The conservative and consistent schemes are indicated with dashed (- · -) and solid lines (–) respectively. For comparison, we include the pressure error for both linear

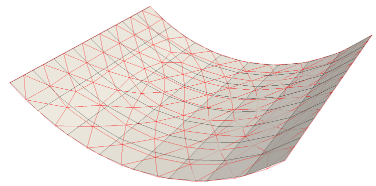


Figure 3: Illustrative example of the typical surface mesh mismatch using triangular fluid and four-noded solid surface elements (red: fluid mesh, black: solid mesh).

and quadratic FEM interpolation. The analysis most importantly shows that the conservative approach results in a non-convergent pressure transfer. This implies that, while the conservative formulation provably transfers the correct magnitude of forces, the spatial distribution and orientation of these forces are incorrect. The same trend has been observed in de Boer et al. [13]. This is a fairly concerning result: it implies that no matter how fine the fluid and solid mesh along a curved interface, the interface stress states will never be correct. As a side note, the conservative pressures are computed from the transferred concentrated forces by $P_s = [M_{ss}]^{-1} H_{fs}^T F_f$.

Since the consistent approach cannot provably conserve energy, we show the difference in work done along the interface using the consistent formulation in Figure 4(c). For a scheme to be globally conservative, this error should be exactly 0. This is indeed the case when using the conservative approach (by construction), and therefore is not shown. All the basis functions result in an energy error convergence rate of approximately 3.5, almost an order higher than the displacement and pressure convergence rates. Therefore, while the consistent approach is not provably conservative, the incurred energy errors decrease consistently with mesh refinement. Finally in Figure 4(d), we show the pressure error incurred by the consistent formulation as a function of mesh mismatch, illustrated by modifying the ratio of fluid to solid elements along the interface. The error in pressure transfer reaches a minimum at roughly twice as many fluid to solid elements with the error remaining fairly constant as the number of fluid elements is further increased.

CONCLUSION

In this paper, we outlined the use of radial basis function interpolation for interface information transfer. RBF interpolation requires no global connectivity information, and is therefore an elegant means by which to transfer information across a non-matching interface between FVM-FEM schemes. We focused, in particular, on a comparison between conservative and consistent formulations. We demonstrated that the conservative formulation is a zero-order method, where the error made in the spatial distribution of the transferred stress state does not reduce with the simultaneous refinement of the interface meshes. We further demonstrated that the consistent formulation, while not provably conservative, provides consistently decreasing errors in work done along the interface with mesh refinement. Furthermore, the rate of convergence of transfer errors (for both displacement and pressure) are higher than the associated FVM errors. The quintic function provided the highest rates of convergence but should be used with caution, as it rapidly leads to poorly conditioned matrices. Overall, the cubic function appears to provide a good compromise between accuracy and stability, with the C^2 and C^0 basis functions also providing promising results.

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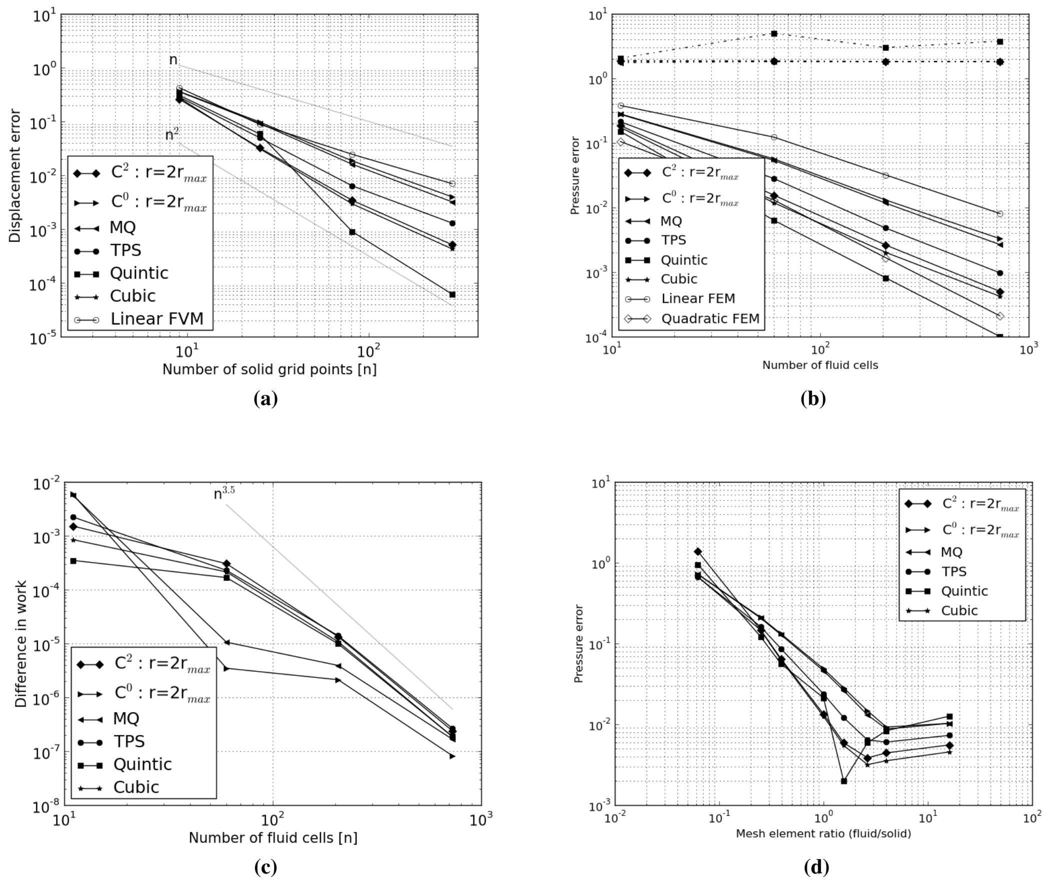

Figure 4: Results obtained from the transfer of the analytical test function. (a) Plot of displacement error for simultaneous refinement of the fluid and solid meshes (same for both conservative and consistent formulations). (b) Comparison of pressure error for the simultaneous refinement of fluid and solid interface discretisation. Solid lines (—) represent consistent transfer and (- · -) dashed lines represent conservative transfer. (c) Difference in work done across the interface. Only consistent transfer is shown, as the conservative transfer by construction is energy preserving. (d) Plot of L^2 relative pressure error using consistent transfer as a function of the ratio of the number of fluid to solid elements.

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