

See discussions, stats, and author profiles for this publication at: <https://www.researchgate.net/publication/304106801>

Systemic Banking Crisis Early Warning Systems Using Dynamic Bayesian Networks

Article in *Expert Systems with Applications* · June 2016

DOI: 10.1016/j.eswa.2016.06.024

CITATION

1

READS

85

3 authors, including:



[Joel Janek Dabrowski](#)

University of Pretoria

3 PUBLICATIONS 11 CITATIONS

[SEE PROFILE](#)



[Johan Pieter De Villiers](#)

Council for Scientific and Industrial Research...

39 PUBLICATIONS 156 CITATIONS

[SEE PROFILE](#)

Some of the authors of this publication are also working on these related projects:



Evaluation Techniques for Uncertainty Representation and Reasoning [View project](#)

All content following this page was uploaded by [Joel Janek Dabrowski](#) on 25 June 2016.

The user has requested enhancement of the downloaded file. All in-text references [underlined in blue](#) are added to the original document and are linked to publications on ResearchGate, letting you access and read them immediately.

Systemic Banking Crisis Early Warning Systems Using Dynamic Bayesian Networks

Joel Janek Dabrowski^{a,*}, Conrad Beyers^{b,**}, Johan Pieter de Villiers^{c,a}

^aDepartment of Electrical, Electronic and Computer Engineering, University of Pretoria, 2 Lynnwood Rd, Pretoria, South Africa

^bDepartment of Insurance and Actuarial Science, University of Pretoria, 2 Lynnwood Rd, Pretoria, South Africa

^cCouncil for Scientific and Industrial Research, Meiring Naudé Rd, Lynnwood, Pretoria, South Africa

Abstract

For decades, the literature on banking crisis early-warning systems has been dominated by two methods, namely, the signal extraction and the logit model methods. However, these methods, do not model the dynamics of the systemic banking system. In this study, dynamic Bayesian networks are applied as systemic banking crisis early-warning systems. In particular, the hidden Markov model, the switching linear dynamic system and the naïve Bayes switching linear dynamic system models are considered. These dynamic Bayesian networks provide the means to model system dynamics using the Markovian framework. Given the dynamics, the probability of an impending crisis can be calculated. A unique approach to measuring the ability of a model to predict a crisis is utilised. The results indicate that the dynamic Bayesian network models can provide precise early-warnings compared with the signal extraction and the logit methods.

Keywords: Hidden Markov Model, Switching Linear Dynamic System, Naive Bayes Switching Linear Dynamic System, Time Series, Regime

1. Introduction

The purpose of an early warning system (EWS) is to provide an indication of an imminent crisis. In this study, EWSs for banking crises are considered. A systemic banking crisis could cost a significant portion of a country's gross domestic product (GDP) (Davis and Karim, 2008). An EWS could assist policy makers in avoiding or reducing the effects of such a crisis.

The most common EWS methods are the logit model and signal extraction methods (Davis and Karim, 2008; Demirgüç-Kunt and Detragiache, 2005). In addition, various other static classifiers have been applied, which include classification trees, neural networks, and random forests (Alessi et al., 2015). Dynamic methods are scarce in EWS literature. In this study, it is argued that dynamic methods are preferable for crisis detection. The temporal dynamics of a banking system change before and during a crisis. It is hypothesised that crises could be identified by modelling these dynamics.

The dynamic Bayesian network is proposed for modelling the banking system dynamics. The switching linear dynamic system (SLDS) and the naïve Bayes switching linear dynamic system (NB-SLDS) are two methods that have not been considered before in the literature. These methods provide the means to model dynamics as indicator variables that are tracked through time. This is performed using state space models.

Based on the dynamics, a tranquil or crisis regime could be inferred.

In this study, the SLDS and NB-SLDS are compared with the logit, signal extraction, and hidden Markov models (HMM). The methods are implemented on a recent European dataset consisting of various countries and indicators. The results demonstrate that the SLDS and NB-SLDS are superior the other models in terms of both accuracy and pre-crisis detection. The novelty of the current work includes proposing the SLDS and NB-SLDS models as systemic banking crisis EWSs. Furthermore, a unique approach to measuring the ability of the EWS to predict a banking crisis is applied.

This manuscript is organised as follows: Section 2 provides a discussion on related work. An introduction to the general approach to EWSs is subsequently presented in Section 3. Sections 4, 5, 6, 7, and 8, the methods considered in this study are discussed. The results are documented and discussed in Section 9. Future work are provided in Section 10. A summary and conclusion are provided to conclude the study in Section 11. Appendix A provides more detailed results.

2. Related Work

In the years following World War II, the global economy was relatively stable (Demirgüç-Kunt and Detragiache, 2005). In the early 1980s the liberalisation of the credit markets started. Several financial crises occurred in developing countries. These were often accompanied by banking crises. By the 1990s, banking crises became more widespread, prompting research on the development of EWSs. The purpose of these EWSs was to identify variables that could provide indicators for banking crisis, as

*Corresponding author.

**Alternative contact details via co-author (other than email): tel: +27 12 420 4419; Department of Insurance and Actuarial Science, University of Pretoria, Private Bag X20 Hatfield, 0028, Pretoria, South Africa.

Email addresses: dabrowski.joel@gmail.com (Joel Janek Dabrowski), conrad.beyers@up.ac.za (Conrad Beyers), jdvilliers1@csir.co.za (Johan Pieter de Villiers)

well as identify an actual imminent crisis. The banking crises tended to diminish during the early 2000's; however, in 2008, a financial crisis erupted that affected banks globally.

In order to develop an EWS, a definition of a banking crisis is required. Various definitions of banking crises have been formulated ([Gaytán and Johnson, 2002](#)). In this study, the definition that is selected corresponds with the dataset utilised. Accordingly, a systemic banking crisis is defined as “the occurrence of simultaneous failures in the banking sector that significantly impairs the capital of the banking system as a whole, which mostly results in large economic effects and government intervention” ([Lainà et al., 2015](#)). This definition is chosen, as the dataset contains a list of crises, with corresponding crisis periods and dates. These crisis periods are in accordance with the definition. The models are trained according to the crisis dates. This study is therefore constrained to the particular definition associated with the dataset.

A leading indicator is a variable that exhibits unusual behaviour in the periods preceding a crisis ([Graciela Kaminsky, 1998](#)). Leading indicators are used in an EWS for providing a warning of an imminent crisis. According to the literature, various leading indicators have been utilised. These include credit levels, asset prices, financial regulations, interest rates, exchange rates, and GDP ([Lainà et al., 2015](#)). Additionally, other variables such as political factors could be considered ([Graciela Kaminsky, 1998](#)). It is important to note that different sources of banking distress will have different indicators ([Gaytán and Johnson, 2002](#)). Honohan and Honohan (1997) indicate three classes of banking crises, namely, macroeconomic epidemics, microeconomic deficiencies, and endemic crises. In this study, the leading indicators utilised are macro-financial factors and the transforms thereof. Banking crises typically relating to the macroeconomic epidemic class are investigated.

The logit model ([Demirgüç-Kunt and Detragiache, 1998](#)) and the signal extraction method ([Kaminsky and Reinhart, 1999](#)) are the two most commonly used methods in banking crisis EWSs ([Davis and Karim, 2008](#)). The logit model comprises a logistic regression that classifies indicator data into a *tranquil* or *crisis* state. The signal extraction method is a heuristic method that defines a threshold. If an indicator variable exceeds the threshold within a window, a warning signal is issued. A drawback of this method is that it is univariate. Furthermore, it does not provide an indication of the severity of a crisis.

Various expert systems have been proposed as EWSs. These include expert systems based on self-organising maps ([Jagric et al., 2015](#)), neural network models ([Celik and Karatepe, 2007](#); [Iturriaga and Sanz, 2015](#); [Nag and Mitra, 1999](#); [Sevim et al., 2014](#)), regression trees ([Manasse and Roubini, 2009](#)), binary classification trees ([Duttagupta and Cashin, 2008](#)), hybrid models ([Lin, 2009](#); [Lin et al., 2008](#)), grey rational analysis ([Lin and Wu, 2011](#)), support vector machines ([Ahn et al., 2011](#); [Feki et al., 2012](#)), random forests ([Alessi and Detken, 2014](#)), and the multiple-indicator-multiple-cause method ([Rose and Spiegel, 2012](#)). Comparisons between such methods have been conducted by [Boyacioglu et al. \(2009\)](#). These expert systems are based on static classifier methods. They do not consider the changes in the system over time.

The work of [Cerchiello and Giudici \(2016\)](#) is more closely related with the present study. These authors propose an expert system that makes use of a graphical model for systemic risk estimation. In this system, conditional dependencies such as those between financial institutions are modelled. The aim is to identify institutions that pose risk to the banking system. The proposed graphical model does not model dynamics directly.

In dynamic methods, temporal changes in the system are taken into account. The autoregressive process (AR) and moving average process (MA) are two traditional time-series analysis methods. These models have been combined and extended to form the autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models ([Kirchgässner et al., 2012](#)). These models are referred to as the Box Jenkins models. The ARMA model has been used as an EWS in financial crises ([Faranda et al., 2015](#)). The Markov switching model is more closely related with the present study. This model is also referred to as the Hidden Markov Model (HMM) ([Murphy, 2012](#)). It is a dynamic Bayesian network (DBN) that samples over time by using the Markov assumption. The Markov switching model has been used as an EWS in currency crises ([Abiad, 2003](#)) and in speculative attacks ([Martinez Peria, 2002](#)). In this study, the literature is extended to include the switching linear dynamic system (SLDS) and the naïve Bayes switching linear dynamic system (NB-SLDS). These models use a state space representation that allows for indicator variables to be ‘tracked’ over time.

3. General Approach to the Banking Crisis EWS

Several methods are considered as banking crisis EWSs in this study. These include the signal extraction, the logit model, the hidden Markov model (HMM), the switching linear dynamic system (SLDS) and the naïve Bayes switching linear dynamic system (NB-SLDS). A large volume of literature exists on the application of the signal extraction and logit models as banking crisis EWSs. The application of the DBN models to time series data generally comprises a two-step process. First, the model parameters are learned, given the data. Second, given the model, its parameters and a new dataset, the latent variables are inferred. In the instance of a banking crisis, the latent variable of interest is a random variable indicating the probability of a crisis. For an EWS, the probability of a crisis provides an indication of an imminent crisis.

4. Signal Extraction Method

The signal extraction method for banking crisis detection was pioneered by [Kaminsky and Reinhart \(1999\)](#). The method is univariate and non-parametric. A threshold is defined for a particular feature or indicator. A feature or indicator is a variable that is assumed to capture vulnerabilities and imbalances in the domestic economy. For example this could be an asset, a credit, a micro economic or a macroeconomic variable. If the data within a specified window crosses the threshold, a signal is issued, indicating that a crisis is imminent. To determine the

Algorithm 1 Signal extraction method.

Require: the dataset, the specified window size, k .

- 1: **for** $\theta = 1\% \rightarrow 100\%$ **do**
 - 2: Initialise a contingency (confusion) matrix to zeros
 - 3: **for** each country in the dataset **do**
 - 4: Compute the threshold for the indexed θ
 - 5: **for** each sample $t = k \rightarrow T$ **do**
 - 6: Extract a data window of size k that ends at t from the referenced countries data sequence.
 - 7: Emit crisis signal if any sample in the window crosses the threshold.
 - 8: Update the contingency (confusion) matrix according to the result and true state.
 - 9: **end for**
 - 10: Compute noise-to-signal ratio from the contingency matrix using (4.1)
 - 11: **end for**
 - 12: **end for**
 - 13: Select the threshold that produces the lowest signal-to-noise ratio
-

threshold, a brute force method is used to minimise a performance measure, referred to as the noise-to-signal ratio (Lainà et al., 2015; Borio and Drehmann, 2009). An EWS could use the signal directly; when a signal is issued, a warning is raised.

The noise-to-signal ratio is defined according to the contingency matrix (known as a confusion matrix in machine learning literature). The contingency matrix $\Lambda = [\Lambda_{jk}]$ is defined such that an element, Λ_{jk} contains the number of samples whose true state is k and which are classified to the state j (Theodoridis and Koutroumbas, 2009). The columns describe the true state. The rows describe the inferred state. In a banking crisis, there are two states, namely *tranquil* and *crisis*. With two states, the contingency matrix has a size of 2×2 . The number of correctly identified *tranquil* samples is identified by Λ_{11} . The number of correctly identified *crisis* samples is identified by Λ_{22} . The number of *crisis* samples incorrectly identified as *tranquil* samples is identified by Λ_{12} . The number of *tranquil* samples incorrectly identified as *crisis* samples is identified by Λ_{21} . The noise-to-signal ratio is given by (Lainà et al., 2015; Borio and Drehmann, 2009)

$$\text{Noise-to-signal ratio} = \frac{\Lambda_{12}/(\Lambda_{12} + \Lambda_{22})}{\Lambda_{11}/(\Lambda_{11} + \Lambda_{21})}. \quad (4.1)$$

A threshold θ is defined as a particular percentile of a given data sequence. A search is performed over a fine range of thresholds. The threshold that produces the minimum noise-to-signal ratio is selected. The signal extraction method is presented in Algorithm 1.

5. Logit Model

The logit model comprises the performing of logistic regression on a set of features or indicators. Logistic regression makes use of the model (Barber, 2012; Murphy, 2012)

$$p(c = 1|\mathbf{x}) = \text{sigm}(b + \mathbf{x}^T \mathbf{w}), \quad (5.1)$$

where c is a class variable to be determined, \mathbf{x} is the input training data, sigm is the sigmoid function, b is a scalar, and \mathbf{w} is a vector. The variables b and \mathbf{w} are the parameters of the model. In the banking crisis application, (5.1) provides an indication of the probability of a crisis. The purpose of logistic regression is to determine a boundary that separates two sets of data. The boundary is given by the hyperplane (Barber, 2012)

$$b + \mathbf{x}^T \mathbf{w} = 0. \quad (5.2)$$

Note that the assumption of the logit model is that the data are linearly separable.

To train the model, a gradient ascent method could be used to maximise the likelihood (Barber, 2012)

$$\mathcal{L}(\mathbf{w}, b) = \sum_{n=1}^N c^n \log(\text{sigm}(b + \mathbf{w}^T \mathbf{x}^n)) + (1 - c^n) \log(1 - \text{sigm}(b + \mathbf{w}^T \mathbf{x}^n)). \quad (5.3)$$

Once the model is trained, a set of features or indicators could be presented to the model. Using (5.1), the probability of a crisis will be the output. The probability of a crisis provides an indication of an imminent crisis in an EWS.

6. Hidden Markov Model

The hidden Markov model (HMM) is a dynamic Bayesian network that consists of a Markov chain emitting observable measurements. The model has been successfully used in a variety of applications. Two of the most successful applications are speech recognition and gene finding (Murphy, 2012). No specific applications to systemic banking crises have been found in the literature, and particularly not for the approach to the HMM as presented in this study.

The HMM consists of a set of latent variables $h_{1:T}$ and a set of observed variables $v_{1:T}$. The latent variables form a first order Markov chain, with transition probability $p(h_t|h_{t-1})$. The observed variables are conditionally dependent on the latent variables through an emission probability $p(v_t|h_t)$. The latent variables are discrete and the observable variables could be discrete or continuous. The graphical model of the HMM is presented in Figure 1. The joint distribution of the HMM is given by (Barber, 2012; Murphy, 2012)

$$p(h_{1:T}, v_{1:T}) = p(v_1|h_1)p(h_1) \prod_{t=2}^T p(v_t|h_t)p(h_t|h_{t-1}).$$

For the banking crisis EWS, the latent variable h_t is a discrete variable describing either a *tranquil* or *crisis* state. Given the nature of the latent variable, the transition probability is a discrete distribution. The distribution is represented by using a transition matrix Φ , where $\Phi_{ij} = p(h_t = j|h_{t-1} = i)$, which is the probability of transitioning from state i to state j .

The observed variable is assigned to describe a continuous banking crisis indicator variable. Given the nature of the observed variable, the emission distribution is continuous. It is

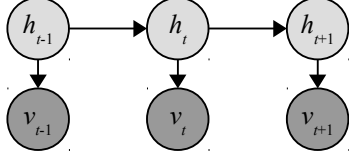


Figure 1: The hidden Markov model. The latent variables $h_{1:T}$ form a Markov chain. Each observable variable v_t is conditionally dependent on its corresponding latent variable h_t . Observable variable nodes are illustrated in a darker shade of grey.

assumed that $p(v_t|h_t)$ takes the form of a Gaussian distribution parametrised by mean μ and covariance Σ . That is, for state j , the observation model is given by the conditional Gaussian distribution (Murphy, 2012)

$$p(v_t|h_t = j) \sim \mathcal{N}(v_t|\mu_j, \Sigma_j).$$

In a crisis, an indicator variable could rise or fall to a particular mean during the crisis. The volatility of the indicator variables could increase during a crisis. The HMM models such behaviour by using the mean and covariance, respectively.

In the banking crisis application, a sequence of samples for a set of indicator variables $v_{1:T}$ are presented to a trained model. Using inference, the probability of a crisis at each time t is determined, given the sequence of samples. That is, $p(h_t = \text{crisis}|v_{1:T})$ is inferred. For an EWS, the probability of a crisis indicates an imminent crisis.

6.1. Inference

Inference comprises computing the latent variables, given a data sequence of the observed variable and the HMM parameters. Filtering comprises determining the belief state $p(h_t|v_{1:t})$. Note that this is up to time t . Filtering is performed using the forwards algorithm. Smoothing comprises computing $p(h_t|v_{1:T})$, given the complete sequence of evidence up to time T .

Smoothing is performed using the forwards-backwards algorithm. This algorithm consists of two steps, namely, a prediction step and an update step. In the prediction step, the following is computed (Murphy, 2012)

$$p(h_t = j|v_{1:t-1}) = \sum_i p(h_t = j|h_{t-1} = i)p(h_{t-1} = i|v_{1:t-1}). \quad (6.1)$$

The update step involves computing (Murphy, 2012)

$$\alpha_t(j) = p(h_t = j|v_{1:t}) \propto p(v_t|h_t = j)p(h_t = j|v_{1:t-1}). \quad (6.2)$$

At time $t = 1$, α_1 is given by (Murphy, 2012)

$$\alpha_1(j) = p(h_1|v_1) \propto p(v_1|h_1 = j)p(h_1 = j). \quad (6.3)$$

Together, (6.1) and (6.2) form the so-called α -recursion.

The forward-backward algorithm consists of a forward recursion and a backward recursion. The forward recursion is the

Algorithm 2 HMM forward-backward smoothing algorithm. Note that it is recommended to work with logarithms to avoid numerical underflow.

Require: the transition matrix, the dataset (local evidence), and the initial state distribution $p(h_1)$.

- 1: Perform initial update step using (6.3).
 - 2: **for** $t = 2 \rightarrow T$ **do**
 - 3: Perform the prediction step using (6.1).
 - 4: Compute α_t up to a normalisation constant in the update step using (6.2).
 - 5: Normalise the α_t computed in the previous step.
 - 6: **end for**
 - 7: Initialise $\beta_T = \alpha_T$.
 - 8: **for** $t = T - 1 \rightarrow 1$ **do**
 - 9: Compute β_t up to a normalisation constant using (6.4).
 - 10: Normalise the β_t computed in the previous step.
 - 11: **end for**
 - 12: **for** $t = 1 \rightarrow T$ **do**
 - 13: Compute γ_t using (6.5).
 - 14: **end for**
-

filtering method described above. The backward recursion is computed as a β -recursion, where (Murphy, 2012)

$$\begin{aligned} \beta_{t-1}(i) &= p(v_{1:T}|h_{t-1} = i) \\ &= \sum_j \beta_t(j)p(v_t|h_t = j)p(h_t = j|h_{t-1} = i). \end{aligned} \quad (6.4)$$

The forward and backward recursion results are combined to form the posterior given by (Murphy, 2012; Barber, 2012)

$$\gamma_t(j) = p(h_t = j|v_{1:T}) = \frac{\alpha_t(j)\beta_t(j)}{\sum_j \alpha_t(j)\beta_t(j)}. \quad (6.5)$$

The forward-backward smoothing algorithm is presented in Algorithm 2.

6.2. Learning

The expectation maximisation (EM) algorithm is applied for learning in the HMM from a given dataset. The algorithm comprises a two-step process. In the expectation step, the latent variables are inferred by using an inference algorithm, such as presented in Section 6.1. Inference produces a fully observable HMM. Given the fully observable HMM, the second step comprises learning the parameters in a maximisation step. These two steps are repeated until a likelihood measure converges.

The maximisation step comprises computing the values of the HMM parameters. For a set of N training sequences, the transition matrix is updated by using (Murphy, 2012)

$$\hat{\Phi}_{jk} = \frac{\sum_i^N \sum_{t=2}^T p^{old}(h_{i,t} = j, h_{i,t-1} = k|v_{i,1:T})}{\sum_i^N \sum_{t=2}^T p^{old}(h_{i,t-1} = k|v_{i,1:T})} \quad (6.6)$$

where $p^{old}()$ denotes the inferred conditional distribution using the parameters determined in the previous iteration of the EM algorithm.

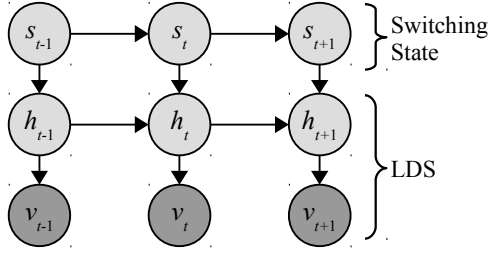


Figure 2: The switching linear dynamic system (SLDS). The lower section of the SLDS consisting of $h_{1:T}$ and $v_{1:T}$ forms a linear dynamic system (LDS). Observable variable nodes are illustrated in a darker shade of grey.

The emission distribution mean and covariance are updated using (Murphy, 2012)

$$\hat{\mu}_k = \frac{\sum_i^N \sum_{t=1}^T p^{old}(h_{i,t} = k | v_{i,1:T}) v_{i,t}}{\sum_i^N \sum_{t=1}^T p^{old}(h_{i,t} = k | v_{i,1:T})} \quad (6.7)$$

and

$$\hat{\Sigma}_k = \frac{\sum_i^N \sum_{t=1}^T p^{old}(h_{i,t} = k | v_{i,1:T}) (v_{i,t} - \hat{\mu}_k)(v_{i,t} - \hat{\mu}_k)^T}{\sum_i^N \sum_{t=1}^T p^{old}(h_{i,t} = k | v_{i,1:T})}. \quad (6.8)$$

At each iteration the HMM likelihood is computed for assessing the terminating conditions. The HMM likelihood is given by (Barber, 2012)

$$p(v_{1:T}) = \sum_j p(h_t = j, v_{1:T}) = \sum_j \alpha_t(j). \quad (6.9)$$

where $\alpha_t(j)$ is given in (6.2).

7. Switching Linear Dynamic System

The switching linear dynamic system (SLDS) models a system with a continuous underlying process that jumps from one regime to another (Bar-Shalom and Li, 1993; Barber, 2012; Murphy, 2002). The system is associated with various names in the literature, such as the switching Kalman filter and the switching state space model (Murphy, 2002). The SLDS has been used in applications such as econometric (Kim, 1994), speech recognition (Mesot and Barber, 2007) human behaviour detection (Turaga et al., 2008), and human-figure tracking problems (Pavlovic et al., 1999).

The SLDS consists of a discrete switching state variable s_t , a latent continuous variable h_t and an observable continuous variable v_t . The graphical model of the SLDS is illustrated in Figure 2. The latent and observable variables form a linear dynamic system (LDS). The LDS is a state space model (Barber, 2012).

The SLDS is modelled according to the following state space equations

$$h_t = A(s_t)h_{t-1} + \eta_t^h(s_t), \quad (7.1)$$

$$v_t = B(s_t)h_t + \eta_t^v(s_t). \quad (7.2)$$

The matrix $A(s_t)$ is referred to as the state transition matrix or the system matrix. The matrix $B(s_t)$ is referred to as the measurement matrix. The variables η_t^h and η_t^v are the state noise process and the measurement noise, respectively. These variables are assumed to be white Gaussian noise, parametrised by zero mean and covariance Σ_h and Σ_v , respectively. Equation (7.1) describes the transition probability distribution

$$p(h_t | h_{t-1}, s_t) = \mathcal{N}(h_t | \bar{h}_t(s_t) + A(s_t)h_{t-1}, \Sigma_h(s_t)),$$

where $\bar{h}_t(s_t)$ is the hidden bias (Barber, 2012). Equation (7.2) describes the emission probability distribution

$$p(v_t | h_t, s_t) = \mathcal{N}(v_t | \bar{v}_t(s_t) + B(s_t)h_t, \Sigma_v(s_t)),$$

where $\bar{v}_t(s_t)$ is the output bias (Barber, 2012). A zero hidden and output bias is assumed in this study. The switching state variable s_t could be viewed as a variable that selects a particular set of LDS parameters to model the dynamics of a specific regime.

The switching state variable s_t describes the regimes, or states the system transitions between. In this banking crisis application, s_t is identical in operation to the HMM discrete latent variable h_t . That is, s_t describes either a *tranquil* state or a *crisis* state. The LDS portion of the SLDS is used to model the dynamics of indicator variables. The LDS is a state space model described by a set of first-order difference equations in matrix form. Equations (7.1) and (7.2) describe the state space equations for a particular state s_t . In the banking crisis application, h_t models an indicator variable as well as its first and its second derivative in the form of difference equations. Similar to the HMM, the objective is to infer $p(s_t = crisis | v_{1:T})$, which provides the indication of an imminent crisis in an EWS.

The operation of the LDS is easily understood in the context of a target tracking application. The observable variable could represent the target location that is obtained from a noisy sensor such as a radar. The latent variable describes the target state. This could be its true position, its velocity (first derivative) and its acceleration (second derivative). Equation (7.1) predicts the target's current state h_t given its previous state h_{t-1} . Equation (7.2) models the sensor output, taking into account the uncertainty owing to sensor noise.

7.1. Inference

As for the HMM, a filtering and a smoothing method could be derived for the SLDS. Unlike the HMM, however, exact inference in the SLDS is intractable (Barber, 2012; Murphy, 2002). The system is propagated forward according to equation (7.1). At each time, the system could be propagated to S different states. At the succeeding time, each of these states is propagated to a further S states. The number of probability distribution components thus increases by a factor of S as time progresses. To manage this exponentially increasing state space, the posterior distribution is represented as a mixture of D Gaussians of the form (Barber, 2012)

$$p(x) = \sum_{i=1}^D p(x | \mu_i, \Sigma_i) p(i), \quad (7.3)$$

where $p(x|\mu_i, \Sigma_i) \sim \mathcal{N}(\mu_i, \Sigma_i)$ is the i^{th} component Gaussian distribution and $p(i)$ is the weight of the component. After each iteration of the inference algorithm, the number of components is pruned to a fixed size. A distribution that is approximated by a mixture of Gaussians is denoted by $q(\cdot)$.

7.1.1. Filtering

The filtering method used in this study is the Gaussian sum filtering (GSF) algorithm. Filtering comprises the inference of the latent variables, given the sequence observed variables up to time t . In the SLDS, the posterior distribution $p(s_t, h_t|v_{1:t})$ is computed. The inference progresses up the network from v_t to h_t to s_t . The posterior distribution can be split into a discrete and a continuous factor respectively, by using the chain rule of probability (Barber, 2012)

$$p(s_t, h_t|v_{1:t}) = p(s_t|v_{1:t})p(h_t|s_t, v_{1:t}) \quad (7.4)$$

The discrete factor is the probability of the switching state given the observed data. The second factor is a continuous probability distribution describing the h_t . Each factor is determined in the filtering algorithm.

The continuous factor is computed by using the forward LDS algorithm. This algorithm is commonly referred to as the Kalman filter (Murphy, 2012). To determine this factor, the parameters of the distribution are computed. The latent variable is represented by a Gaussian distribution, parametrised by mean $f(s_t)$ and covariance $F(s_t)$. This Gaussian distribution is propagated forwards using the LDS dynamics given by equations (7.1) and (7.2). Propagating the LDS for all S switching states results in an $S \times S$ component Gaussian mixture. Each component in the mixture is parametrised by mean and covariance (Barber, 2012)

$$\mu_{h|v}(s_t, s_{t-1}) = \mu_h + \Sigma_{hv}\Sigma_{vv}^{-1}(v_t - \mu_v), \quad (7.5)$$

$$\Sigma_{h|v}(s_t, s_{t-1}) = \Sigma_{hh} - \Sigma_{hv}\Sigma_{vv}^{-1}\Sigma_{vh}, \quad (7.6)$$

where

$$\mu_h(s_t, s_{t-1}) = A(s_t)f(s_{t-1}), \quad (7.7)$$

$$\mu_v(s_t, s_{t-1}) = B(s_t)A(s_t)f(s_{t-1}), \quad (7.8)$$

$$\Sigma_{hh}(s_t, s_{t-1}) = A(s_t)F(s_{t-1})A^T(s_t) + \Sigma^h(s_{t-1}), \quad (7.9)$$

$$\Sigma_{vv}(s_t, s_{t-1}) = B(s_t)\Sigma_{hh}B^T(s_t) + \Sigma^v(s_{t-1}), \quad (7.10)$$

$$\Sigma_{vh}(s_t, s_{t-1}) = B(s_t)F(s_{t-1}) = \Sigma_{hv}^T. \quad (7.11)$$

Note that the (s_t, s_{t-1}) dependence notation is dropped in (7.5) and (7.6) for the terms given in (7.7) to (7.11) for readability.

The mixture weights are given by (Barber, 2012)

$$w_t(s_t, s_{t-1}) = q(s_{t-1}|s_t, v_{1:t-1}, v_t) \propto \mathcal{N}(\mu_v, \Sigma_{vv})p(s_t|s_{t-1})\alpha_{t-1}(s_{t-1}) \quad (7.12)$$

At each time step, the $S \times S$ Gaussian mixture is collapsed back to an S component Gaussian mixture, with mean $f(s_t)$ and covariance $F(s_t)$. The mixture could be collapsed by merging the

Algorithm 3 Gaussian sum filtering (GSF) algorithm for the SLDS model.

Require: the dataset, the linear dynamic system parameters, the switching transition probabilities, and the initial state probability $p_0(s)$.

- 1: Initialise $\alpha_0(s) = p_0(s)$, and the initial Gaussian parameters $f(s_t)$ and $F(s_t)$.
 - 2: **for** $t = 1 \rightarrow T$ **do**
 - 3: **for** $s_t = 1 \rightarrow S$ **do**
 - 4: **for** $s_{t-1} = 1 \rightarrow S$ **do**
 - 5: Use (7.7) through (7.11) to compute $\mu_{h|v}$ in (7.5), $\Sigma_{h|v}$ in (7.8) and $\mathcal{N}(\mu_v, \Sigma_{vv})$.
 - 6: Compute w_t using (7.12)
 - 7: **end for**
 - 8: Compute $\alpha_t(s_t)$ using (7.13)
 - 9: Collapse the mixture of Gaussians parametrised by $\mu_{h|v}$, $\Sigma_{h|v}$ and w_t to a single Gaussian distribution, parametrised by $f(s_t)$ and $F(s_t)$
 - 10: **end for**
 - 11: Normalise $\alpha_t(s_t)$
 - 12: **end for**
-

components with the lowest weights into a single Gaussian distribution parametrised by

$$\mu = \sum_i w_i \mu_i$$

$$\Sigma = \sum_i w_i (\Sigma_i + \mu_i \mu_i^T) - \mu \mu^T,$$

where w_i is the mixture weight of the i^{th} component given by (7.12), μ_i is the mean of the i^{th} component given by (7.5) and Σ_i is the covariance of the i^{th} component given by (7.6).

The discrete component in (7.4) is computed as

$$\alpha_t(s_t) = p(s_t|v_{1:t}) \propto \sum_{s_{t-1}} w_t(s_t, s_{t-1}) \quad (7.13)$$

The algorithm for the Gaussian sum filtering is presented in Algorithm 3.

7.1.2. Smoothing

The smoothing method considered in this study is the generalised pseudo Bayes (GPB) algorithm (Kim, 1994; Bar-Shalom and Li, 1993; Barber, 2012). Smoothing comprises the inference of the unobservable variables at time t , given the complete sequence observed variables up to time T . In the SLDS, the posterior distribution $p(s_t, h_t|v_{1:T})$ is determined. This distribution could be expressed as (Barber, 2012)

$$p(s_t, h_t|v_{1:T}) \quad (7.14)$$

$$= \sum_{s_{t+1}} p(s_{t+1}|v_{1:T})p(s_t|s_{t+1}, v_{1:T})$$

$$\cdot \int_{h_{t+1}} p(h_t|h_{t+1}, s_t, s_{t+1}, v_{1:t})p(h_{t+1}|s_t, s_{t+1}, v_{1:T}).$$

Each of these factors are computed in the smoothing algorithm.

The first factor is the previous smoothed posterior denoted by $\beta_{t+1}(s_t + 1)$. The update for the smoothed posterior in the current iteration is given by (Barber, 2012)

$$\beta_t(s_t) = p(s_t|v_{1:T}) = \sum_{s_{t+1}} p(s_{t+1}|v_{1:T})p(s_t|s_{t+1}, v_{1:t}) \quad (7.15)$$

The first factor is the previous $\beta_{t+1}(s_t + 1)$. The second factor is equivalent to the second factor in (7.14). This second factor is approximated by the filtered result (Barber, 2012)

$$p(s_t|s_{t+1}, v_{1:t}) = \frac{p(s_{t+1}|s_t)p(s_t|v_{1:t})}{\sum_{s_t} p(s_{t+1}|s_t)p(s_t|v_{1:t})} \quad (7.16)$$

The integral in (7.14) is the marginal distribution $p(h_t|s_t, s_{t+1}, v_{1:T})$. This distribution is parametrised by mean and covariance (Barber, 2012)

$$\mu_t(s_t, s_{t+1}) = \overleftarrow{A}(s_t, s_{t+1})g(s_{t+1}) + \overleftarrow{m}(s_t, s_{t+1}) \quad (7.17)$$

$$\Sigma_{t,t}(s_t, s_{t+1}) = \overleftarrow{A}(s_t, s_{t+1})G(s_{t+1})\overleftarrow{A}^T(s_t, s_{t+1}) + \overleftarrow{\Sigma}(s_t, s_{t+1}), \quad (7.18)$$

where

$$\overleftarrow{A}(s_t, s_{t+1}) = \Sigma_{t+1,t}/\Sigma_{t+1,t+1} \quad (7.19)$$

$$\overleftarrow{m}(s_t, s_{t+1}) = f(s_t) - \overleftarrow{A}(s_t, s_{t+1})\mu_h \quad (7.20)$$

$$\overleftarrow{\Sigma}(s_t, s_{t+1}) = F(s_t) - \overleftarrow{A}(s_t, s_{t+1})\Sigma_{t+1,t}. \quad (7.21)$$

$$\Sigma_{t+1,t} = A(s_{t+1})F(s_t). \quad (7.22)$$

$$\Sigma_{t+1,t+1} = A(s_{t+1})F(s_t)A^T(s_{t+1}) + \Sigma^h(s_{t+1}) \quad (7.23)$$

$F(s_t)$ and $f(s_t)$ are the filtered covariance and mean, respectively. $\overleftarrow{A}(s_t, s_{t+1})$ and $\overleftarrow{\eta}(s_t, s_{t+1}) \sim \mathcal{N}(\overleftarrow{m}(s_t, s_{t+1}), \overleftarrow{\Sigma}(s_t, s_{t+1}))$ are the parameters for the reversed LDS dynamics. These are the Rauch-Tung-Striebel equations for the LDS backward procedure (Barber, 2012, 2006). The smoothed posterior $q(h_{t+1}|s_{t+1}, v_{1:T})$ is parametrised by mean $g(s_{t+1})$ and covariance $G(s_{t+1})$. These are computed in the previous iteration of the backward procedure.

The smoothed posterior distribution is represented by a mixture of Gaussians, $q(\cdot)$. The mean $g(s_t)$ and covariance $G(s_t)$ in the current iteration are computed in the marginalisation (Barber, 2012)

$$q(h_t|s_t, v_{1:T}) = \sum_{s_{t+1}} p(s_{t+1}|s_t, v_{1:T})q(h_t|s_t, s_{t+1}, v_{1:T}). \quad (7.24)$$

This marginalisation is approximated by collapsing the mixture. The second factor in (7.24) describes the mixture components that are parametrised by (7.17) and (7.18). The first factor describes the mixture weights given by (Barber, 2012)

$$p(s_{t+1}|s_t, v_{1:T}) = \frac{p(s_t|s_{t+1}, v_{1:T})p(s_{t+1}|v_{1:T})}{\sum_{s_{t+1}} p(s_t|s_{t+1}, v_{1:T})p(s_{t+1}|v_{1:T})} \quad (7.25)$$

The smoothed posterior for the state vector, $p(h_t|v_{1:T})$ could be optionally computed by the following marginalisation (Barber, 2012)

$$q(h_t|v_{1:T}) = \sum_{s_t} p(s_t|v_{1:T})q(h_t|s_t, v_{1:T}). \quad (7.26)$$

Algorithm 4 Generalised pseudo Bayesian (GPB) smoothing algorithm for the SLDS model.

Require: The filtering results of Algorithm 3, the linear dynamic system parameters and the switching transition probabilities.

- 1: Set $g(s_T) = f(s_T)$, $G(s_T) = F(s_T)$ and $\gamma_T = \alpha_T(s_T)$.
- 2: Find $p(h_T|v_{1:T})$ by collapsing the Gaussian mixture given by (7.24).
- 3: **for** $t = T - 1 \rightarrow 1$ **do**
- 4: **for** $s_t = 1 \rightarrow S$ **do**
- 5: **for** $s_{t+1} = 1 \rightarrow S$ **do**
- 6: Determine $\mu_t(s_t, s_{t+1})$ in (7.17) and $\Sigma_t(s_t, s_{t+1})$ in (7.17) using (7.18) through (7.23).
- 7: Determine $p(s_t|s_{t+1}, v_{1:T})$ using (7.16).
- 8: **end for**
- 9: **end for**
- 10: Determine $\beta_t(s_t) = p(s_t|v_{1:T})$ using (7.15).
- 11: Determine $p(s_{t+1}|s_t, v_{1:T})$ using (7.24).
- 12: **for** $s_t = 1 \rightarrow S$ **do**
- 13: Find $p(h_t|s_t, v_{1:T})$ by collapsing the Gaussian mixture given by (7.24).
- 14: **end for**
- 15: Find $p(h_t|v_{1:T})$ by collapsing the Gaussian mixture given by (7.25).
- 16: **end for**

The marginalisation is approximated by collapsing the Gaussian mixture parametrised by mean $g(s_t)$, covariance $G(s_t)$, and weights $\beta_t(s_t)$.

The generalised Pseudo Bayes smoothing algorithm is presented in Algorithm 4.

7.2. Learning

As for the HMM, learning of the parameters for the SLDS could be performed by using the EM algorithm. The expectation step could be performed using the GPB smoothing algorithm. Maximisation is performed by updating the parameters at each iteration.

The SLDS is parametrised by the set $\Theta = \{\mu_0(s_t), \sigma_0^2(s_t), A(s_t), \Sigma_H(s_t), B(s_t), \Sigma_V(s_t), p(s_{t=0}), \Phi\}$. The initial mean $\mu_0(s_t)^{(n)}$ relates to the prior mean $f(s_t)^{(n)}$ in the filtering algorithm. The initial variance $\sigma_0(s_t)^{2(n)}$ relates to the prior covariance $F(s_t)^{(n)}$ in the filtering algorithm. $A(s_t)^{(n)}$, $\Sigma_H(s_t)^{(n)}$, $B(s_t)^{(n)}$, and $\Sigma_V(s_t)^{(n)}$ are the LDS parameters. The prior switching state is given by $p(s_{t=0})$. The switching state transition matrix is denoted by Φ . The following set of equations provides approximations for the LDS parameters. The i subscript indicates $s_t = i$.

The initial mean is given by (Murphy, 1998)

$$\hat{\mu}_{0_i} = \mathbb{E}_{p^{old}(h_1|v)}[h_1], \quad (7.27)$$

where $\mathbb{E}[\cdot]$ denotes the expected value. The initial covariance is given by (Murphy, 1998)

$$\begin{aligned} \hat{\sigma}_{0_i} &= \mathbb{E}_{p^{old}(h_1|v)}[(h_1 - \mu_0)(h_1 - \mu_0)^T] \\ &= \mathbb{E}_{p^{old}(h_1|v)}[h_1 h_1^T] - \mu_{0_i} \mu_{0_i}^T. \end{aligned} \quad (7.28)$$

The state transition matrix is given by (Murphy, 1998)

$$\hat{A}_i = \frac{\sum_{t=2}^T \mathbb{E}_{p^{old}(h_t|V)} [h_t h_{t-1}^T] p^{old}(s_t = i | v_{1:T})}{\sum_{t=2}^T \mathbb{E}_{p^{old}(h_t|V)} [h_{t-1} h_{t-1}^T] p^{old}(s_t = i | v_{1:T})}. \quad (7.29)$$

The state noise covariance is given by (Murphy, 1998)

$$\hat{\Sigma}_{H_i} = \left[\begin{array}{c} \sum_{t=2}^T \mathbb{E}_{p^{old}(h_t|V)} [h_t h_t^T] p^{old}(s_t = i | v_{1:T}) \\ - \sum_{t=2}^T A_i \mathbb{E}_{p^{old}(h_t|V)} [h_{t-1} h_t^T] p^{old}(s_t = i | v_{1:T}) \\ \cdot \left[\frac{1}{\sum_{t=2}^T p^{old}(s_t = i | v_{1:T})} \right] \end{array} \right] \quad (7.30)$$

The measurement matrix is given by (Murphy, 1998)

$$\hat{B}_i = \frac{\sum_{t=1}^T v_t \mathbb{E}_{p^{old}(h_t|V)} [h_t^T] p^{old}(s_t = i | v_{1:T})}{\sum_{t=1}^T \mathbb{E}_{p^{old}(h_t|V)} [h_t h_t^T] p^{old}(s_t = i | v_{1:T})}. \quad (7.31)$$

The measurement noise covariance is given by (Murphy, 1998)

$$\hat{\Sigma}_{V_i} = \left[\begin{array}{c} \sum_{t=1}^T (v_t v_t^T) p^{old}(s_t = i | v_{1:T}) \\ - \sum_{t=1}^T B_i \mathbb{E}_{p^{old}(h_t|V)} [h_t] v_t^T p^{old}(s_t = i | v_{1:T}) \\ \cdot \left[\frac{1}{\sum_{t=1}^T p^{old}(s_t = i | v_{1:T})} \right] \end{array} \right] \quad (7.32)$$

The initial switching state is given by (Murphy, 1998)

$$p(s_1 = i) = \frac{p^{old}(s_1 = i | v_{1:T})}{\sum_{i=1}^S p^{old}(s_1 = i | v_{1:T})}. \quad (7.33)$$

The state transition probability is given by (Murphy, 1998)

$$\hat{\Phi}_{ij} = p(s_t = i | s_{t-1} = j) = \frac{\sum_{t=2}^T p^{old}(s_t = i, s_{t-1} = j | v_{1:T})}{\sum_{t=2}^T p^{old}(s_{t-1} = j | v_{1:T})}. \quad (7.34)$$

The state transition matrix Φ is a square $S \times S$ matrix. It consists of $p(s_t = i | s_{t-1} = j)$ for each i along the rows and each j along the columns.

The likelihood used in the EM algorithm is given by (Barber, 2012)

$$p(v_{1:T}) = \prod_{t=0}^{T-1} \sum_{s_t, s_{t-1}} p(v_t | s_t, s_{t-1}, v_{1:t-1}) \cdot p(s_t | s_{t-1}, v_{1:t-1}) \cdot p(s_{t-1} | v_{1:t-1}). \quad (7.35)$$

After each iteration of the EM algorithm, the likelihood is computed. If the learned parameters produce a higher likelihood than that in the previous iteration, the parameters have improved the model's performance. The algorithm is repeated until the likelihood converges.

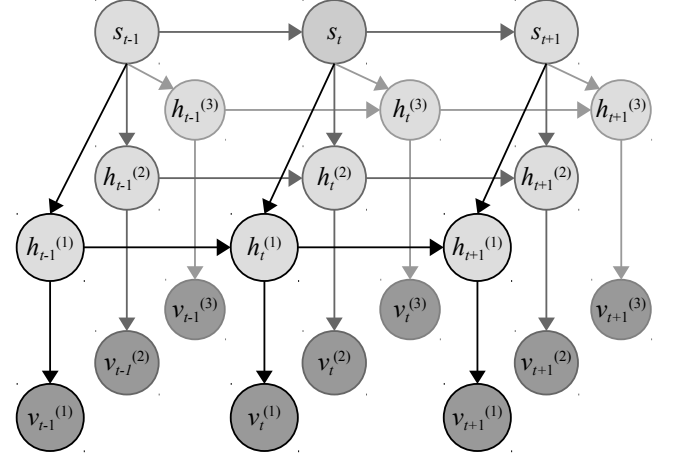


Figure 3: The dynamic Bayesian network structure of the naïve Bayes switching linear dynamic system (NB-SLDS) for three input variables (Dabrowski et al., 2016). Each input variable is associated with a separate LDS. In general, the NB-SLDS could consist of N visible nodes ($v_t^{(1)} \dots v_t^{(N)}$), N hidden nodes ($h_t^{(1)} \dots h_t^{(N)}$), and a single switching state node (s_t) at each time t . Note that the augmented NB-SLDS contains links from s_t to $v_t^{(n)}$. Observable variable nodes are illustrated in a darker shade of grey.

8. Naïve Bayes Switching Linear Dynamic System

The naïve Bayes switching linear dynamic system (NB-SLDS) is a variant of the SLDS (Dabrowski et al., 2016). The graphical model of the NB-SLDS is presented in Figure 3. The NB-SLDS uses a set of N variables as input. An LDS is associated with each variable. It is assumed that the LDSs are conditionally independent, given the switching state s_t . The benefit of this assumption is that the model can handle missing and unsynchronised data. The SLDS allows for modelling multiple variables by combining them into a single state space. However, during learning and inference, a sample from all variables at each time step is required. In the NB-SLDS, if a sample from a variable is missing, the variable could be ignored. Only variables with samples contribute. Furthermore, variables combined in a single state space in the SLDS are assumed conditionally dependent, given s_t . Each variable that is added to the state space results in an exponential increase in the dimensionality of the system. Adding variables to the NB-SLDS results in a linear increase in the dimensionality of the system.

For a set of N variables, the state equations for the n^{th} LDS in the NB-SLDS are given by (Dabrowski et al., 2016)

$$h_t^{(n)} = \mathbf{A}(s_t)^{(n)} h_{t-1}^{(n)} + \eta_t^h(s_t)^{(n)}, \quad (8.1)$$

$$v_t^{(n)} = \mathbf{B}(s_t)^{(n)} h_t^{(n)} + \eta_t^v(s_t)^{(n)}. \quad (8.2)$$

where $n \in \{1, \dots, N\}$. The variables presented in these equations are described in Section 7. Equation (8.2) describes the augmented NB-SLDS that contains a link from s_t to $v_t^{(n)}$. In the case of the standard NB-SLDS presented in Figure 3, B is not dependent on s_t .

8.1. Inference

Inference in the NB-SLDS is similar to inference in the SLDS. In the NB-SLDS, however, there are N LDSs that are

processed independently. The derivations for the inference algorithms are presented in (Dabrowski et al., 2016). The GSF method is applied for filtering and the GPB algorithm is applied for smoothing. As for the HMM and the SLDS, the objective of inference is to infer the probability of a crisis given a sequence of samples for a set of indicator variables. This probability provides an indication of an imminent crisis in an EWS.

8.1.1. Filtering

In the NB-SLDS, filtering comprises computing the posterior distribution $p(s_t, h_t^{(1)}, \dots, h_t^{(N)} | v_{1:t}^{(1)}, \dots, v_{1:t}^{(N)})$. For notational convenience, the posterior distribution can be written as $p(s_t, h_t^{(1:N)} | v_{1:t}^{(1:N)})$. Using the chain rule of probability, this distribution can be split into two factors

$$p(s_t, h_t^{(1:N)} | v_{1:t}^{(1:N)}) = p(s_t | v_{1:t}^{(1:N)}) \prod_{n=1}^N p(h_t^{(n)} | s_t, v_{1:t}^{(n)}) \quad (8.3)$$

The first factor is a discrete probability distribution. The second factor in the product is a continuous probability distribution. This factor relates to the n^{th} LDS. These factors are computed by the GSF algorithm.

The factor within the product in (8.3) is computed by using the forward LDS algorithm. For each LDS and switching state s_t , the posterior distribution is represented by a Gaussian mixture, parametrised by mean $f(s_t)^{(n)}$ and covariance $F(s_t)^{(n)}$. This Gaussian mixture is propagated forward using the LDS dynamics given by equations (8.1) and (8.2). The propagation for all S switching states produces an $S \times S$ component Gaussian mixture. Each component in the mixture is parametrised by the following mean and covariance (Dabrowski et al., 2016)

$$\mu_{hv}(s_t, s_{t-1})^{(n)} = \mu_h^{(n)} + \Sigma_{hv}^{(n)} \Sigma_{vv}^{-1(n)} (v_t^{(n)} - \mu_v^{(n)}), \quad (8.4)$$

$$\Sigma_{hv}(s_t, s_{t-1})^{(n)} = \Sigma_{hh}^{(n)} - \Sigma_{hv}^{(n)} \Sigma_{vv}^{-1(n)} \Sigma_{vh}^{(n)}. \quad (8.5)$$

where (Dabrowski et al., 2016)

$$\mu_h(s_t, s_{t-1})^{(n)} = A(s_t)^{(n)} f(s_{t-1})^{(n)}, \quad (8.6)$$

$$\mu_v(s_t, s_{t-1})^{(n)} = B(s_t)^{(n)} A(s_t)^{(n)} f(s_{t-1})^{(n)}, \quad (8.7)$$

$$\Sigma_{hh}(s_t, s_{t-1})^{(n)} = A(s_t)^{(n)} F(s_{t-1})^{(n)} A^T(s_t)^{(n)} + \Sigma^h(s_{t-1})^{(n)}, \quad (8.8)$$

$$\Sigma_{vv}(s_t, s_{t-1})^{(n)} = B(s_t)^{(n)} \Sigma_{hh}^{(n)} B^T(s_t)^{(n)} + \Sigma^v(s_{t-1})^{(n)}, \quad (8.9)$$

$$\Sigma_{vh}(s_t, s_{t-1})^{(n)} = B(s_t)^{(n)} F(s_{t-1})^{(n)} = \Sigma_{hv}^T(s_t, s_{t-1})^{(n)}. \quad (8.10)$$

For readability, the (s_t, s_{t-1}) dependence notation in (8.4) and (8.5) is omitted from the terms given in (8.6) to (8.10).

The mixture weights are given by (Dabrowski et al., 2016)

$$w_t(s_t, s_{t-1}) = q(s_{t-1} | s_t, v_{1:t-1}^{(1:N)}, v_t^{(1:N)}) \propto \mathcal{N}(\mu_v^{(n)}, \Sigma_{vv}^{(n)}) p(s_t | s_{t-1}) \alpha_{t-1}(s_{t-1}) \quad (8.11)$$

At each time step, the $S \times S$ Gaussian mixture is collapsed back to an S component Gaussian mixture, with mean $f(s_t)^{(n)}$ and covariance $F(s_t)^{(n)}$. The method used for the SLDS, as described in Section 7.1.1 could be used for the NB-SLDS.

Algorithm 5 Gaussian sum filtering (GSF) algorithm for the NB-SLDS model (Dabrowski et al., 2016).

Require: the dataset, the linear dynamic system parameters and the switching transition probabilities and the initial state probability $p_0(s)$.

- 1: Initialise $\alpha_0(s) = p_0(s)$, and the initial Gaussian parameters $f(s_t)^{(n)}$ and $F(s_t)^{(n)}$.
 - 2: **for** $t = 1 \rightarrow T$ **do**
 - 3: **for** $s_t = 1 \rightarrow S$ **do**
 - 4: **for** $s_{t-1} = 1 \rightarrow S$ **do**
 - 5: **for** $n = 1 \rightarrow N$ **do**
 - 6: Use (8.6) through (8.10) to compute $\mu_{hv}^{(n)}$ in (8.4), $\Sigma_{hv}^{(n)}$ in (8.5) and $\mathcal{N}(\mu_v^{(n)}, \Sigma_{vv}^{(n)})$.
 - 7: **end for**
 - 8: Compute w_t using (8.11)
 - 9: **end for**
 - 10: Compute $\alpha_t(s_t)$ using (8.12)
 - 11: **for** $n = 1 \rightarrow N$ **do**
 - 12: Collapse the mixture of Gaussians parametrised by $\mu_{hv}^{(n)}, \Sigma_{hv}^{(n)}$ and w_t to a single Gaussian parametrised by $f(s_t)^{(n)}$ and $F(s_t)^{(n)}$
 - 13: **end for**
 - 14: **end for**
 - 15: Normalise $\alpha_t(s_t)$
 - 16: **end for**
-

The first factor in (8.3) is computed as (Dabrowski et al., 2016)

$$\alpha_t(s_t) = p(s_t | v_{1:t}^{(1:N)}) \propto \sum_{s_{t-1}} w_t(s_t, s_{t-1}) \quad (8.12)$$

The algorithm for the Gaussian sum filtering for the NB-SLDS is presented in Algorithm 5.

8.2. Smoothing

Smoothing comprises computing the conditional distribution $p(s_t, h_t^{(1:N)} | v_{1:T}^{(1:N)})$. This distribution could be expressed as (Dabrowski et al., 2016)

$$p(s_t, h_t^{(1:N)} | v_{1:T}^{(1:N)}) \quad (8.13) \\ = \sum_{s_{t+1}} p(s_{t+1} | v_{1:T}^{(1:N)}) p(s_t | s_{t+1}, v_{1:T}^{(1:N)}) \\ \cdot \prod_{n=1}^N \int_{h_{t+1}^{(n)}} p(h_t^{(n)} | h_{t+1}^{(n)}, s_t, s_{t+1}, v_{1:t}^{(n)}) p(h_{t+1}^{(n)} | s_t, s_{t+1}, v_{1:T}^{(n)}).$$

The smoothing algorithm comprises computing each of these factors. The algorithm assumes that the filtered results are available. It progresses backward from the last sample at time T to the first sample.

The first factor is the previous smoothing posterior denoted by $\beta_{t+1}(s_t + 1)$. In the current iteration, the smoothed posterior is updated according to (Dabrowski et al., 2016)

$$\beta_t(s_t) = p(s_t | v_{1:T}^{(1:N)}) = \sum_{s_{t+1}} p(s_{t+1} | v_{1:T}^{(1:N)}) p(s_t | s_{t+1}, v_{1:t}^{(1:N)}) \quad (8.14)$$

The first factor is the previous posterior $\beta_{t+1}(s_t + 1)$. The second factor in (8.14) is equivalent to the second factor in (8.13). It is approximated by the filtered result (Dabrowski et al., 2016)

$$p(s_t|s_{t+1}, v_{1:t}^{(1:N)}) = \frac{p(s_{t+1}|s_t)p(s_t|v_{1:t}^{(1:N)})}{\sum_{s_t} p(s_{t+1}|s_t)p(s_t|v_{1:t}^{(1:N)})} \quad (8.15)$$

The integral within the product in (8.13) is the marginal distribution $p(h_t^{(n)}|s_t, s_{t+1}, v_{1:T}^{(n)})$. This marginal distribution is parametrised by mean and covariance (Dabrowski et al., 2016)

$$\mu_t(s_t, s_{t+1})^{(n)} = \overleftarrow{A}(s_t, s_{t+1})^{(n)} g(s_{t+1})^{(n)} + \overleftarrow{m}(s_t, s_{t+1})^{(n)} \quad (8.16)$$

$$\Sigma_{t,t}(s_t, s_{t+1})^{(n)} = \overleftarrow{A}(s_t, s_{t+1})^{(n)} G(s_{t+1})^{(n)} \overleftarrow{A}^T(s_t, s_{t+1})^{(n)} + \overleftarrow{\Sigma}(s_t, s_{t+1})^{(n)}, \quad (8.17)$$

where (Dabrowski et al., 2016)

$$\overleftarrow{A}(s_t, s_{t+1})^{(n)} = \Sigma_{t+1,t}^{(n)} / \Sigma_{t+1,t+1}^{(n)} \quad (8.18)$$

$$\overleftarrow{m}(s_t, s_{t+1})^{(n)} = f(s_t)^{(n)} - \overleftarrow{A}(s_t, s_{t+1})^{(n)} \mu_h^{(n)} \quad (8.19)$$

$$\overleftarrow{\Sigma}(s_t, s_{t+1})^{(n)} = F(s_t)^{(n)} - \overleftarrow{A}(s_t, s_{t+1})^{(n)} \Sigma_{t+1,t}^{(n)}. \quad (8.20)$$

$$\Sigma_{t+1,t}^{(n)} = A(s_{t+1})^{(n)} F(s_t)^{(n)}. \quad (8.21)$$

$$\Sigma_{t+1,t+1}^{(n)} = A(s_{t+1})^{(n)} F(s_t)^{(n)} A^T(s_{t+1})^{(n)} + \Sigma^h(s_{t+1})^{(n)} \quad (8.22)$$

where the mean $f(s_t)^{(n)}$ and covariance $F(s_t)^{(n)}$ are parameters of the filtered posterior. The parameters for the reversed LDS dynamics are $\overleftarrow{A}(s_t, s_{t+1})^{(n)}$ and $\overleftarrow{\eta}(s_t, s_{t+1})^{(n)} \sim \mathcal{N}(\overleftarrow{m}(s_t, s_{t+1})^{(n)}, \overleftarrow{\Sigma}(s_t, s_{t+1})^{(n)})$. The mean $g(s_{t+1})^{(n)}$ and covariance $G(s_{t+1})^{(n)}$ are the parameters for the smoothed posterior $q(h_{t+1}^{(n)}|s_{t+1}, v_{1:T}^{(n)})$. These are computed in the previous iteration of the backward procedure.

The posterior distribution is approximated as a mixture of Gaussians $q(\cdot)$. The mean $g(s_t)^{(n)}$ and covariance $G(s_t)^{(n)}$ in the current iteration are computed by collapsing a mixture of Gaussians given by (Dabrowski et al., 2016)

$$q(h_t^{(n)}|s_t, v_{1:T}^{(n)}) = \sum_{s_{t+1}} p(s_{t+1}|s_t, v_{1:T}^{(1:N)}) q(h_t^{(n)}|s_t, s_{t+1}, v_{1:T}^{(n)}). \quad (8.23)$$

The second factor is a Gaussian mixture component that is parametrised by (8.16) and (8.17). The first factor describes the mixture weights given by (Dabrowski et al., 2016)

$$p(s_{t+1}|s_t, v_{1:T}) = \frac{p(s_t|s_{t+1}, v_{1:T}^{(1:N)})p(s_{t+1}|v_{1:T}^{(1:N)})}{\sum_{s_{t+1}} p(s_t|s_{t+1}, v_{1:T}^{(1:N)})p(s_{t+1}|v_{1:T}^{(1:N)})} \quad (8.24)$$

Optionally, the smoothed posterior for the state vector, $p(h_t^{(n)}|v_{1:T}^{(n)})$ could be computed in the following marginalisation (Dabrowski et al., 2016)

$$q(h_t^{(n)}|v_{1:T}^{(n)}) = \sum_{s_t} p(s_t|v_{1:T}^{(1:N)}) q(h_t^{(n)}|s_t, v_{1:T}^{(n)}). \quad (8.25)$$

The marginalisation is approximated by collapsing the Gaussian mixture parametrised by mean $g(s_t)^{(n)}$, covariance $G(s_t)^{(n)}$, and weights given in (8.14).

The generalised Pseudo Bayes smoothing algorithm is presented in Algorithm 6.

Algorithm 6 Generalised pseudo Bayesian (GPB) smoothing algorithm for NB-SLDS model (Dabrowski et al., 2016).

Require: The filtering results of Algorithm 5, the linear dynamic system parameters, and the switching transition probabilities.

- 1: Set $g(s_T)^{(n)} = f(s_T)^{(n)}$, $G(s_T)^{(n)} = F(s_T)^{(n)}$ and $\gamma_T = \alpha_T(s_T)$.
 - 2: Find $p(h_T^{(n)}|v_{1:T}^{(n)})$ by collapsing the Gaussian mixture given by (8.23).
 - 3: **for** $t = T - 1 \rightarrow 1$ **do**
 - 4: **for** $s_t = 1 \rightarrow S$ **do**
 - 5: **for** $s_{t+1} = 1 \rightarrow S$ **do**
 - 6: **for** $n = 1 \rightarrow N$ **do**
 - 7: Determine $\mu_t(s_t, s_{t+1})^{(n)}$ in (8.16) and $\Sigma_t(s_t, s_{t+1})^{(n)}$ in (8.17) using (8.18) through (8.22).
 - 8: **end for**
 - 9: Determine $p(s_t|s_{t+1}, v_{1:T})$ using (8.15).
 - 10: **end for**
 - 11: **end for**
 - 12: Determine $\beta_t(s_t) = p(s_t|v_{1:T}^{(1:N)})$ using (8.14).
 - 13: Determine $p(s_{t+1}|s_t, v_{1:T})$ using (8.24).
 - 14: **for** $n = 1 \rightarrow N$ **do**
 - 15: **for** $s_t = 1 \rightarrow S$ **do**
 - 16: Find $p(h_t^{(n)}|s_t, v_{1:T}^{(n)})$ by collapsing the Gaussian mixture given by (8.23).
 - 17: **end for**
 - 18: Find $p(h_t^{(n)}|v_{1:T}^{(n)})$ by collapsing the Gaussian mixture given by (8.25).
 - 19: **end for**
 - 20: **end for**
-

8.3. Learning

As for the HMM and the SLDS, the EM algorithm is used for parameter learning. The NB-SLDS is parametrised by the set $\Theta = \{\mu_0(s_t)^{(1:N)}, \sigma_0(s_t)^{2(1:N)}, A(s_t)^{(1:N)}, \Sigma_H(s_t)^{(1:N)}, B(s_t)^{(1:N)}, \Sigma_V(s_t)^{(1:N)}, p(s_{t=0}), \Phi\}$. The initial mean $\mu_0(s_t)^{(n)}$ relates to the prior mean $f(s_t)^{(n)}$. The initial variance $\sigma_0(s_t)^{2(n)}$ relates to the prior covariance $F(s_t)^{(n)}$. The parameters $A(s_t)^{(n)}$, $\Sigma_H(s_t)^{(n)}$, $B(s_t)^{(n)}$ and $\Sigma_V(s_t)^{(n)}$ are the parameters of the n^{th} LDS. The prior switching state is given by $p(s_{t=0})$. The switching state transition matrix is denoted by Φ .

The following set of equations provides approximations for the NB-SLDS parameters. The i subscript indicates $s_t = i$. The (n) superscript indicates the n^{th} LDS within the NB-SLDS.

The initial mean update is given by (Dabrowski et al., 2016)

$$\mu_{0_i}^{(n)} = \mathbb{E}_{p^{old}(h_1^{(n)}|v^{(n)})}[h_1^{(n)}]. \quad (8.26)$$

The initial covariance update is given by (Dabrowski et al., 2016)

$$\begin{aligned} \sigma_{0_i}^{(n)} &= \mathbb{E}_{p^{old}(h_1^{(n)}|v^{(n)})}[(h_1^{(n)} - \mu_{0_i}^{(n)})(h_1^{(n)} - \mu_{0_i}^{(n)})^T] \\ &= \mathbb{E}_{p^{old}(h_1^{(n)}|v^{(n)})}[h_1^{(n)} h_1^{T(n)}] - \mu_{0_i} \mu_{0_i}^T. \end{aligned} \quad (8.27)$$

The state transition matrix update is given by (Dabrowski et al., 2016)

$$A_i^{(n)} = \frac{\sum_{t=2}^T \mathbb{E}_{p^{old}(h_t^{(n)}|V^{(n)})} [h_t^{(n)} h_{t-1}^{T(n)}] p^{old}(s_t = i|v_{1:T}^{(n)})}{\sum_{t=2}^T \mathbb{E}_{p^{old}(h_t^{(n)}|V^{(n)})} [h_{t-1}^{(n)} h_{t-1}^{T(n)}] p^{old}(s_t = i|v_{1:T}^{(n)})}. \quad (8.28)$$

The state noise covariance update is given by (Dabrowski et al., 2016)

$$\Sigma_{H_i}^{(n)} = \left[\sum_{t=2}^T \mathbb{E}_{p^{old}(h_t|V)} [h_t h_t^T] p^{old}(s_t = i|v_{1:T}) - \sum_{t=2}^T A_i^{(n)} \mathbb{E}_{p^{old}(h_t|V)} [h_{t-1} h_t^T] p^{old}(s_t = i|v_{1:T}) \right] \cdot \left[\frac{1}{\sum_{t=2}^T p^{old}(s_t = i|v_{1:T})} \right]. \quad (8.29)$$

The measurement matrix update is given by (Dabrowski et al., 2016)

$$B_i^{(n)} = \frac{\sum_{t=1}^T v_t^{(n)} \mathbb{E}_{p^{old}(h_t^{(n)}|V^{(n)})} [h_t^{T(n)}] p^{old}(s_t = i|v_{1:T}^{(n)})}{\sum_{t=1}^T \mathbb{E}_{p^{old}(h_t^{(n)}|V^{(n)})} [h_t^{(n)} h_t^{T(n)}] p^{old}(s_t = i|v_{1:T}^{(n)})}. \quad (8.30)$$

The measurement noise update covariance is given by (Dabrowski et al., 2016)

$$\Sigma_{V_i}^{(n)} = \left[\sum_{t=1}^T (v_t^{(n)} v_t^{T(n)}) p^{old}(s_t = i|v_{1:T}^{(n)}) - \sum_{t=1}^T B_i^{(n)} \mathbb{E}_{p^{old}(h_t^{(n)}|V^{(n)})} [h_t^{(n)}] v_t^{T(n)} p^{old}(s_t = i|v_{1:T}^{(n)}) \right] \cdot \left[\frac{1}{\sum_{t=1}^T p^{old}(s_t = i|v_{1:T}^{(n)})} \right]. \quad (8.31)$$

The initial switching state is given by (Dabrowski et al., 2016)

$$p(s_{t=0} = i) = \frac{p^{old}(s_1 = i|v_{1:T}^{(1:N)})}{\sum_{s_1=1}^S p^{old}(s_1 = i|v_{1:T}^{(1:N)})}. \quad (8.32)$$

The state transition probability is given by (Dabrowski et al., 2016)

$$p(s_t = i|s_{t-1} = j) = \frac{\sum_{t=2}^T p^{old}(s_t = i, s_{t-1} = j|v_{1:T}^{(1:N)})}{\sum_{t=2}^T p^{old}(s_{t-1} = j|v_{1:T}^{(1:N)})}. \quad (8.33)$$

The state transition matrix Φ is a square $S \times S$ matrix. It consists of $p(s_t = i|s_{t-1} = j)$ for each i along the rows and each j along the columns.

The likelihood used for evaluating terminating conditions in the EM algorithm is given by (Dabrowski et al., 2016)

$$p(v_{1:T}^{(1:N)}) = \prod_{n=1}^N \prod_{t=0}^{T-1} \sum_{s_t, s_{t-1}} p(v_t^{(n)}|s_t, s_{t-1}, v_{1:t-1}^{(n)}) \cdot p(s_t|s_{t-1}, v_{1:t-1}^{(n)}) \cdot p(s_{t-1}|v_{1:t-1}^{(n)}). \quad (8.34)$$

9. Results

Each of the methods discussed is applied to operate as an EWS. The methods indicate the probability of a crisis (or a signal in the case of the signal extraction method). Conceptually, the results are measured according to two criteria. Firstly, does the method detect the crises or regimes? Secondly, is the method able to detect the crisis before it occurs? That is, is the method able to predict a crisis. By definition, this second criterion is a requirement for an EWS. A unique approach is presented for testing this second criterion.

9.1. Dataset

The methods presented in this study are tested with the dataset used by Lainà et al. (2015). This dataset was preferred due to a level of the similarity between the economies of the included countries. That is, these are all developed countries with similar financial regulatory systems. Similar countries are preferred in order to include indicators that can be considered systematic for all the countries in the data set. By including developing and transition economies, the possible idiosyncrasy of systemic crises in these economies may convolute the modelling and results. The purpose of this study is to investigate if, and to what extent, DBN's could be an improvement to current modelling approaches.

The dataset focuses on developed European economies with long time series. Economies are chosen according to the data availability. Furthermore, transition economies are intentionally excluded. Eleven countries are included namely, Austria, Belgium, Denmark, Finland, France, Germany, Italy, the Netherlands, Spain, Sweden and United Kingdom. The data consists of quarterly samples spanning the period 1980 to 2013. The dataset consists of crisis events and a set of vulnerability indicators.

A total of 19 systemic banking crises are included in the dataset. For this dataset, a crisis is defined as “the occurrence of simultaneous failures in the banking sector that significantly impairs the capital of the banking system as a whole, which mostly results in large economic effects and government intervention” (Lainà et al., 2015). The crisis events are determined as a consensus in the literature of crises, crisis periods, and crisis dates. The primary source of crisis events is a database constructed by the European System of Central Banks (ESCB) Heads of Research Group. This database contains data on crisis events from several influential studies. The crisis periods relevant to each country are documented in Table 1. Lainà et al. (2015) deviated from their own crisis definition by including the 2008 crises in the dataset. These crises were included as they exhibit periods of elevated stress in the financial sector (Lainà et al., 2015).

The vulnerability indicators are macro-financial factors pertaining to a range of credit, asset, and macro variables. These indicators were selected based on the literature. The indicators or features include house prices, mortgages, mortgages to GDP, household loans, household loans to GDP, private loans, private loans to GDP, consumer price index, GDP, current account surplus to GDP, and loans to deposits. For each of these indicators,

Table 1: Crisis periods between 1980 and 2013 for each country (Lainà et al., 2015). The periods are represented by the last to digits of the year followed by the quarter. For example 78Q1 indicates 1978, quarter 1.

Country	1980s	1990s	2000s
Austria			08Q3-08Q4
Belgium			08Q3-08Q4
Germany			08Q3-08Q4
Spain	78Q1-85Q4		08Q3-08Q4
Finland		91Q3-95Q4	08Q3-08Q4
France		94Q1-95Q4	08Q3-09Q4
Italy		90Q1-95Q4	08Q3-08Q4
Netherlands			08Q3-08Q4
Denmark	87Q1-92Q4		08Q3-10Q4
Great Britain	84Q1-84Q4	90Q3-95Q4	07Q3-08Q4
Sweden		91Q3-95Q4	08Q3-08Q4

the growth, trend, absolute trend deviation (ATD), and relative trend deviation (RTD) are additionally provided. The growth rate is defined as the annual growth. The trend is extracted by using the one-sided Hodrick-Prescott filter. The ATD and RTD are the deviations between the Hodrick-Prescott filter results and the real values.

9.2. Performance Measures

To measure the performance of each method, the precision, recall, F-score, and accuracy are used. These are determined from the confusion matrix Λ described in Section 4.

The recall for class j is calculated as follows (Theodoridis and Koutroumbas, 2009; Murphy, 2012):

$$R_j = \frac{\Lambda(j, j)}{\sum_{k=1}^S \Lambda(j, k)}. \quad (9.1)$$

Recall can also be interpreted as a sensitivity measure. It is a measure of the fraction crises detected.

The precision for class j is calculated as follows (Theodoridis and Koutroumbas, 2009; Murphy, 2012):

$$P_j = \frac{\Lambda(j, j)}{\sum_{k=1}^S \Lambda(k, j)}. \quad (9.2)$$

Precision can also be interpreted as a confidence measure. It is a measure of the fraction of crisis detections that are actually crises.

The F-score is given as follows (Murphy, 2012):

$$F_j = \frac{2R_jP_j}{R_j + P_j}. \quad (9.3)$$

The F-score is the harmonic mean between the precision and recall.

The accuracy is computed as follows (Theodoridis and Koutroumbas, 2009):

$$Acc = \frac{\sum_{j=1}^S \Lambda(j, j)}{\sum_{j=1}^S \sum_{k=1}^S \Lambda(j, k)}. \quad (9.4)$$

This describes the probability of correctly classifying a sample.

The methods presented in this study provide a measure of the probability of a crisis. The range of a probability is $[0,1]$. A probability of 1 indicates the highest level of certainty of a crisis state. A probability of 0 indicates the highest level of certainty of a tranquil state.

9.3. Evaluation Methodology

The leave-one-out cross validation approach is used in evaluation. This means that, the data of one country are removed from the dataset to be used as a test set. The remaining dataset is used as a training set. The parameters of the particular EWS model are determined by using the training set. The EWS model is subsequently tested by using the test set. This process is performed for each country in the dataset. Cross validation reduces overfitting and provides an indication on how well the model will generalise over an independent set. That is, it provides an indication on how well the model will perform with newly presented data.

In this study, a crisis is detected according to the behaviour of the various indicators. The behaviour is described by temporal dynamics, that is, how the indicator changes over time. It is argued that the dynamics will change as the system transitions to a crisis. Transform operations on the data, such as the growth, and trend deviations provide descriptors of the temporal dynamics. The levels of the indicators may not be useful. For example, consider the consumer price index, as plotted in Figure 4a. The consumer price index increases at a near linear rate. A method such as the signal extraction method would determine a threshold value to define a crisis. Values above the threshold would indicate a crisis. Once the consumer price index has exceeded the threshold, the crisis would not end. That is, as long as the trend follows the linear increasing function. Therefore, the consumer price index provides a poor indicator variable. The ATD of the consumer price index is illustrated in Figure 4b. At the 2008 crisis, there is a noticeable change in dynamics. Therefore, this transform provides a meaningful indicator. In general, the growth, ATD and RTD transforms are considered in this study. The level and trend datasets for the various variables are ignored.

The purpose of an EWS is to provide a warning of a crisis before the crisis occurs. To compare the methods presented in this study, a fixed horizon before a crisis is used. Lainà et al. (2015) indicated that a 12 quarter (3 year) lead time in the signal extraction and logit models was optimal. For all methods other than the logit model, the crisis data consist of the combined crisis period and the 12 quarter leading period. For the logit model, the data from the start of the crisis up to two years after the crisis are removed. The change in behaviour of the indicators during the post-crisis period adversely affects the results of the logit model (Bussiere and Fratzscher, 2006; Demirgüç-Kunt and Detragiache, 1998).

The logit model and the NB-SLDS models are applied as multivariate models. The signal extraction, HMM, and SLDS methods are applied as univariate models. The SLDS and HMM can be used as multivariate models. However, they require samples from all indicators at each time step. These methods cannot deal with missing or unsynchronised data. These

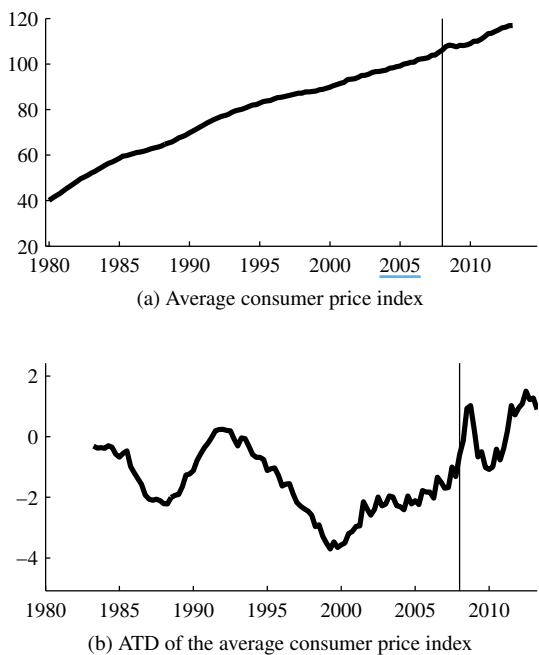


Figure 4: Plot of the consumer price index and the ATD of the consumer price index. The curves indicate the mean value over all the countries. The vertical line is located at the 2008 global crisis.

properties are unfortunately common to banking crisis datasets. The NB-SLDS has specifically been proposed as a method to handle such problems in datasets (Dabrowski et al., 2016).

Lainà et al. (2015) applied the signal extraction and logit model methods to the given dataset. These same methods are applied in this study. However, the results in this study could differ from those presented by Lainà et al. (2015). This is due to the differences in how the dataset is utilised.

In sections 9.4 to 9.8, probability plots are presented and discussed for each method. In section 9.9, the performance results are discussed. Detailed performance results are tabulated in Appendix A.

9.4. Signal Extraction Method Results

The signal extraction method is implemented as described in Algorithm 1. The indicator providing the best performance is the RTD of the GDP. A plot of the probability for a crisis is presented in Figure 5. Figure 5a and Figure 5b present the results for the best and worst performing countries respectively. The best performance of this method is on the dataset of the Netherlands. The worst performance is on the dataset of Belgium. In both cases, the crises are correctly detected. The numerous false detections in the Belgium dataset result in lower accuracy. Both crisis in the Netherlands dataset are detected correctly. The first crisis is detected in the period leading to the crisis. A successful warning is thus issued. However, in the second crisis, detection coincided with the crisis. Note that the signal extraction method produces binary results. It does not provide a measure of the severity of the crisis.

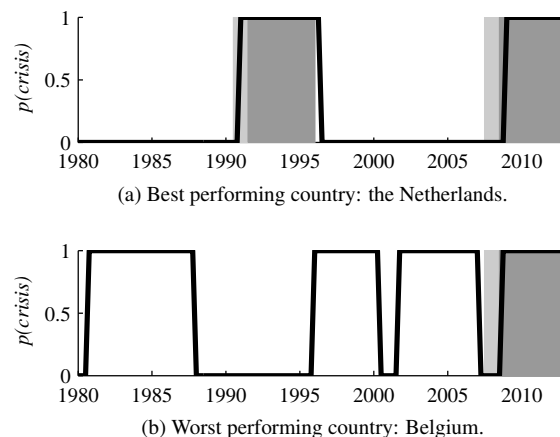


Figure 5: The signal extraction results. The black line indicates the probability of a crisis. The light-grey region indicates the three year period leading to a crisis. The dark-grey region indicates the period of the true crisis.

9.5. Logit Model Results

The logit model is applied as a multivariate model. A set of optimal indicators are selected by using a filter method (Guyon and Elisseeff, 2003). The indicators are selected sequentially. The first indicator provides the optimal result in a univariate logit model. The second indicator is selected as the indicator that provides the best results when added to the univariate model. This process is continued until there is little or no change in performance when adding further indicators. The optimal indicators, in order of significance, are determined the RTD of the GDP, the RTD of current account surplus to GDP, and the RTD of private loans to GDP.

A plot of the probability for a crisis is presented in Figure 6. Figure 6a and Figure 6b present the results for the best and worst performing countries respectively. The best performance is on the dataset of Finland. The worst performance is on the dataset of Italy. Both crises are detected in the period preceding the first crisis. This provides a successful early warning. However, in the second crisis, there is no early warning. In the dataset of Italy, numerous false positives are issued after the first crisis. In the second crisis, the probability of a crisis begins to rise in the pre-crisis period. However, it suddenly drops just before the crisis.

9.6. Hidden Markov Model Results

The HMM parameters are learned using the EM algorithm presented in Section 6.2. The k-means clustering algorithm is used to initialise the HMM parameters (Murphy, 2012) for the EM algorithm. Inference is performed by using the forwards-backwards algorithm presented in Algorithm 2. The indicator that provides the best performance is the ATD of private loans to GDP. The second best performing feature is the growth of real house prices.

A plot of the probability for a crisis is presented in Figure 7. Figure 7a and Figure 7b present the results for the best and worst performing countries respectively. The method performs the best on the dataset of France, and the worst on the dataset of

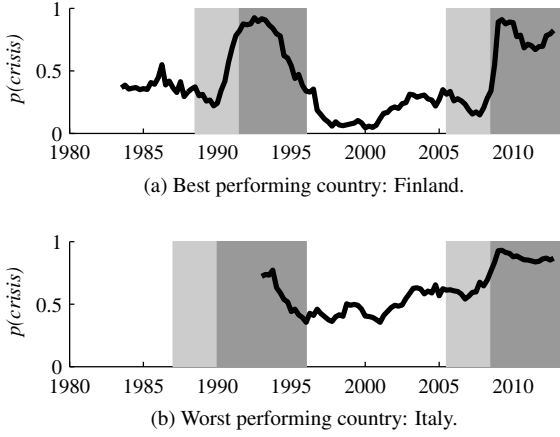


Figure 6: The logit model results. The black line indicates the probability of a crisis. The light-grey region indicates the three year period leading to a crisis. The dark-grey region indicates the period of the true crisis.

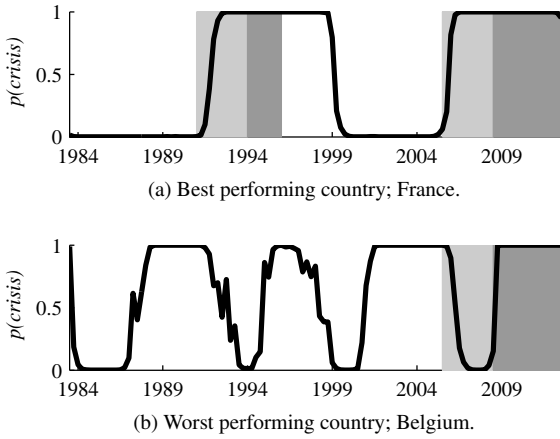


Figure 7: The HMM results. The black line indicates the probability of a crisis. The light-grey region indicates the three year period leading to a crisis. The dark-grey region indicates the period of the true crisis.

Belgium. Both crises in the dataset of France are detected with early warnings. The period of the first crisis is longer than the period of the true crisis. In the dataset of Belgium, the crisis is detected; however, many false positives are given and no early warning is provided for the actual crisis.

9.7. Switching Linear Dynamic System Results

The SLDS parameters are required to be initialised before learning. The initial state matrix $A(s_1)^{(n)}$ is configured for a constant velocity (first derivative) model. The initial noise parameters are set manually, using a brute force approach. The initial switching state transition matrix is set to a uniform distribution. The initial switching state is set with a bias to the

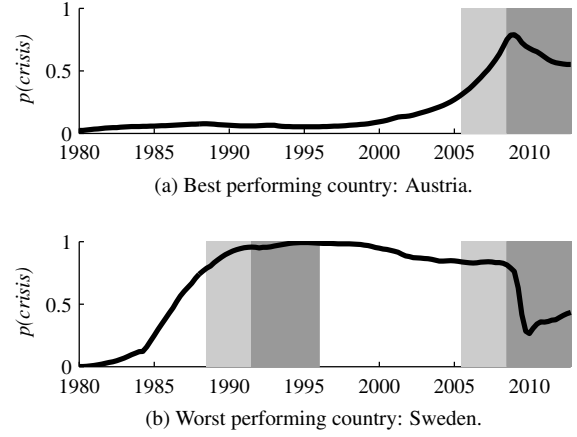


Figure 8: The SLDS results. The black line indicates the probability of a crisis. The light-grey region indicates the three year period leading to a crisis. The dark-grey region indicates the period of the true crisis.

tranquil state. The initial parameters are thus given as follows

$$A(s_1) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \Sigma_h(s_1) = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix},$$

$$B = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}, \quad \Sigma_v = 0.3,$$

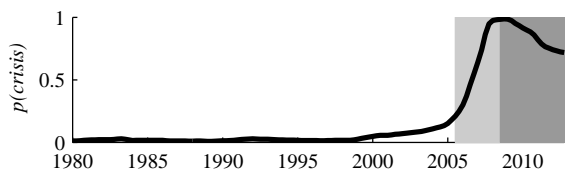
$$\Phi = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}, \quad p(s_1) = [0.8 \quad 0.2]^T.$$

The indicator that provides the optimal performance is the RTD of the GDP. The second most optimal indicator is the ATD of the consumer price index. A plot of the probability for a crisis is presented in Figure 8. Figure 8a and Figure 8b present the results for the best and worst performing countries respectively. The method performs the best on the dataset of Austria and the worst on the dataset of Sweden. For Austria, the crisis is detected with early warning. The probability of a crisis rises during the pre-crisis period and reaches a peak at the time of the crisis. For Sweden's dataset, the first crisis is detected and, according to the model, it continues to the next crisis in 2008. After the 2008 crisis, the probability of a crisis decreases.

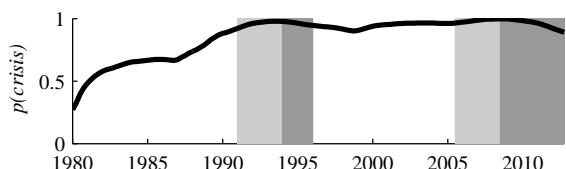
9.8. Naïve Bayes Switching Linear Dynamic System Results

Other than $\Sigma_v^{(n)}$, the NB-SLDS parameters are initialised to the same values as for the SLDS. The covariance, $\Sigma_v^{(n)}$ is initialised to 0.075 as this provides superior performance. (Note that the SLDS did not perform as well with this value). To perform feature selection, the 11 indicators providing an average accuracy above 60% are considered. A search is performed on the various combinations of these indicators. The two optimal indicators are the growth and the RTD of the GDP. Additional indicators did not seem to provide any further benefit.

A plot of the probability for a crisis is presented in Figure 9. Figure 9a and Figure 9b present the results for the best and worst performing countries respectively. The method performs the best on the dataset of Austria and the worst on the dataset of France. The 2008 crisis in Austria is detected with early warning. Comparing Figure 9a with Figure 8a, it appears



(a) Best performing country: Austria.



(b) Worst performing country: France.

Figure 9: The NB-SLDS results. The black line indicates the probability of a crisis. The light-grey region indicates the three year period leading to a crisis. The dark-grey region indicates the period of the true crisis.

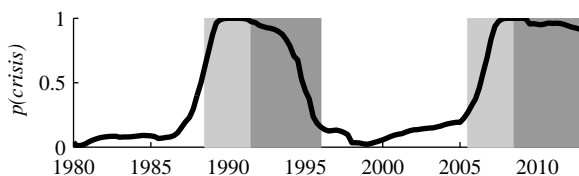


Figure 10: The NB-SLDS results for Finland. Both the 1990-1995 and the 2008 crisis are detected with early warning. The black line indicates the probability of a crisis. The light-grey region indicates the three year period leading to a crisis. The dark-grey region indicates the period of the true crisis.

that NB-SLDS is more confident of a crisis than the SLDS. For France, the model seems to indicate a high probability of a crisis throughout the dataset.

Figure 10 illustrates the NB-SLDS predicting the two Finland crises. Note that both crises are predicted early.

9.9. Performance Results

Two performance tests are conducted. In the first test, the general ability of a model to detect the *tranquil* and *crisis* regimes is considered. In the second test, the ability of a model to *predict* a crisis is considered. The second test is intended to measure the ability of each models as an EWS. Important aspects of the results are summarised in Sections 9.9.1 and 9.9.2. More detailed results are presented in Appendix A. The results and the implications thereof are discussed in Section 9.9.3

9.9.1. Regime Detection Performance

The signal extraction, HMM and SLDS models are tested on each country and each indicator. The logit model and the NB-SLDS are used as multivariate models. These models are tested on each country with a set of optimal indicators. A test is performed with the performance measures being computed over the entire sequence of each test set. This provides a general measure of how well a model performs over all the regimes. In table 2, the results for each method are summarised. For the signal extraction, HMM and SLDS methods, the results are averaged over all countries for the best performing indicator.

Table 2: Average regime detection results over all countries. These results are computed over the complete data sequence of the best performing features. They provide an indication of the regime detection performance.

	Accuracy	Recall	Precision	F-score
Signal Extraction	0.74	0.68	0.67	0.62
Logit Model	0.60	0.45	0.37	0.38
HMM	0.68	0.56	0.63	0.56
SLDS	0.66	0.55	0.65	0.58
NB-SLDS	0.74	0.72	0.71	0.68

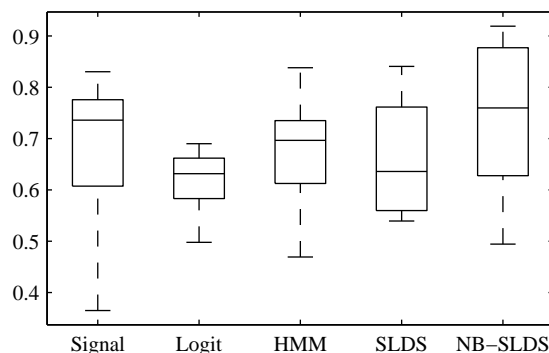


Figure 11: Box plot for the regime detection evaluation. The central mark is the median. The top and bottom edges of the box are the 25th and 75th percentile, respectively. The whiskers extend to the most extreme data points.

For the logit model and the NB-SLDS, the results for the best combination of indicators are averaged over all the countries. Country-specific results for the signal extraction, logit model, HMM, SLDS and NB-SLDS are presented in Tables A.4, A.5, A.6, A.7, and A.8 in Appendix A respectively.

Both the NB-SLDS and the signal extraction methods provide accuracies of 74%. The precision and recall of the NB-SLDS are superior to that of the signal extraction method. The accuracy of the HMM is slightly superior compared with that of the SLDS. However, the SLDS is slightly more precise compared with the HMM. The logit provides the worst performance overall, especially in terms of precision.

In Figure 11, the box plots for the regime detection evaluation for each method are illustrated. The box plot illustrates the variation in the accuracy of the results relevant to the various countries. A preferred method will have little variation and values close to unity. Although the NB-SLDS method provides the superior results, the variation in the results is the greatest. The signal extraction method provides the next-best set of results, with less variation than those of the NB-SLDS. The signal extraction method however, has the lowest valued whisker. The logit model provides the least variation in results. The values of the results are however poor.

9.9.2. Crisis Prediction Performance

To test the predictive capability of a method, the performance measures are computed only over the period prior to a crisis. This is the 12 quarter horizon. The results of this test provide a measure of how well a model performs at predicting a cri-

Table 3: Average crisis predictability results over all countries. These results are computed over the 12 quarters (3 years) leading to a crisis for the best performing features. They provide an indication of the predictability of a crisis.

	Accuracy	Recall	Precision	F-score
Signal Extraction	0.33	1.00	0.33	0.55
Logit Model	0.31	1.00	0.31	0.47
HMM	0.47	1.00	0.47	0.53
SLDS	0.65	1.00	0.65	0.77
NB-SLDS	0.67	1.00	0.67	0.79

sis. This is a relatively unique approach to evaluation. In most other studies, the methods were tested using the regime detection approach discussed in Section 9.9.1. The evaluation of the predictive performance is corresponds more closely with the definition of an EWS. The results for this evaluation are summarised in Table 3. For the signal extraction, HMM, and SLDS methods, the results are averaged over all countries for the best performing indicator. For the logit model and the NB-SLDS, the results for the best combination of indicators are averaged over all countries. The country specific results for the signal extraction, logit model, HMM, SLDS, and NB-SLDS are presented in Tables A.9, A.10, A.11, A.12, and A.13 in Appendix A, respectively.

The precision of the DBN models are significantly higher than those of the logit model and the signal extraction method. Of the DBN methods, the NB-SLDS and the SLDS provide the optimal results.

In Figure 12, the box plots for the crisis prediction evaluation for each method are illustrated. The box plot illustrates the variation in the accuracy of the results relevant to the various countries. The NB-SLDS method provides superior results in terms of the variation and range. The median is however not optimal. The SLDS provides the optimal median and upper percentile. The variation is however greater than that of the NB-SLDS. The signal extraction model has the lowest median value and high variation. The variation in the HMM is significant. As for the regime detection results, the logit model provides the least variation in results. The values of the results are however poor.

9.9.3. Discussion

As indicated in the literature, the signal extraction logit models appear to detect crises effectively. However, this study indicates that they do not provide sufficient early warning. A crisis could be detected too late for a sensible action to be taken. The DBN models provide superior performance in providing early warning. These models may thus be more useful in practice. In the worst case, a model that is better at providing early warning is expected to be at least as good as the regime detecting methods.

The accuracy, precision, and computational complexity of a method are important factors when selecting an EWS. The signal extraction and logit models are less complex than the DBN based models. The regime detection test appears to indicate that the performance of the NB-SLDS is not good enough, in comparison with the signal extraction method, to warrant its

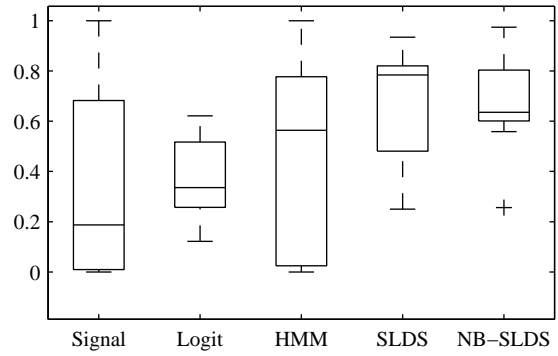


Figure 12: Box whisker plot for the crisis prediction evaluation. The central mark is the median. The top and bottom edges of the box are the 25th and 75th percentiles respectively. The whiskers extend to the most extreme data points. Outliers are plotted individually with the + symbol.

use. However, the crisis prediction test results clearly show that, as an EWS, the NB-SLDS method is superior to the signal extraction method. The signal extraction results indicate that the method provides correct prediction only 33% of the time. The NB-SLDS provides correct predictability 67% of the time. This is a significant improvement. Furthermore, the NB-SLDS provides an indication of the severity of a crisis, whereas the signal extraction method does not. Furthermore, the crisis prediction box plots indicate that the results of the signal extraction method show extremely large variation and a low median value. The NB-SLDS provides superior results with low variance.

The performance of the NB-SLDS is superior to that of the SLDS in the regime detection results. In the crisis prediction results, the models provide similar performances in terms of average accuracy. The RTD of the GDP indicator is a highly informative feature which induces the SLDS to perform well. In general, if a single indicator variable (feature) provides sufficient results, it may not be worth adding variables to form the NB-SLDS. However, when considering the crisis prediction box plots, adding more variables appears to decrease the variation in the results. With its lower variation, the NB-SLDS is clearly the optimal method.

Computationally, the HMM is the simplest of the DBN models. It performed well in the regime detection results. However, it did not perform as well as the SLDS and NB-SLDS in the crisis prediction results. Although, the results outperform those of the signal extraction and logit models. The benefit of the HMM is that it is more widely used than the SLDS and the NB-SLDS. It is thus more likely to be available in various software packages. In practice, a small performance increase could lead to a substantial financial implication. Consequently, an increase in computational complexity, such as that in the NB-SLDS, may be justified.

9.10. Exclusion of the 2008 Crisis Study

Lainà et al. (2015) deviated from their crisis definition when they included the 2008 crises in their dataset. The 2008 crisis coincided with GDP movements. Indicators relating to the real GDP were prominent in this study. It could be questioned

whether these methods are being trained with bias towards the properties of the 2008 financial crisis. Therefore, a study is conducted where all 2008 crises were removed from the dataset. Feature (indicator) selection is performed using the SLDS, signal extraction and logit model methods. The results indicate that for the signal extraction and logit model, indicators relating to mortgages and private loans become prominent. GDP related indicators become less prominent. This finding could possibly indicate that the selection of indicators in these methods is biased towards the 2008 crisis. However, the results for the SLDS method indicate that the GDP and the consumer price index related indicators remain prominent when the 2008 crises data are excluded. Furthermore, it has been demonstrated that the DBN models are clearly able to detect crises not related to the 2008 crisis by using the GDP indicators. See Figure 10 for example.

Indicators relating to the GDP have not been identified in other literature as prominent EWS indicators. Commonly used EWS approaches in literature, such as the logit model and the signal extraction methods, do not consider the dynamics of indicators. These methods consider the magnitudes of the indicator values. The dynamics of indicators are a primary consideration in DBN models. It is expected that the DBN methods will prefer indicators containing relevant dynamics. These preferred indicators may thus differ from those that traditional models prefer. However, it is not the purpose of this study to investigate indicators. Such an investigation may be reserved for future work. The purpose of this study is to demonstrate the importance of evaluating the dynamics of indicators in an EWS.

10. Future Work

In future work, this study could be extended to include larger datasets. Including emerging economies and other transitional economies can and should be considered. In addition, limiting the dataset to specific economies or banks could also be a promising endeavour. The study may additionally be expanded to consider general financial EWS applications. This study provides a novel approach in the context of financial risk in general. Future work could also include an investigation to determine which indicators would provide superior performance in DBN models.

The effect of the definition of a banking crisis on a model should be investigated in future research. The definition essentially defines the ground truth on which the models are trained upon. It is expected that variance in the definition could therefore influence the model.

The financial implication of computational complexity and method performance should be investigated. Furthermore, methods such as Bayesian model selection and Bayesian model averaging should be considered for improving model performance.

11. Summary and Conclusion

In this study, three different dynamic Bayesian network models are compared with the logit model and the signal extraction

method. A recent dataset containing data on various European countries and indicators is utilised. Two different tests are applied to measure the performance of the methods. In the first test, the general ability to detect tranquil and crisis periods over a long period is measured. In the second test, the ability to detect a crisis before it occurs is measured. This second test is unique in the literature. The results indicate that the SLDS and NB-SLDS provide superior performance to the other methods in both tests. Relating to the second test, improved early warning capability is of significant practical importance.

In a validation test, the 2008 crises are excluded from the dataset. The DBN models maintain the preference for the GDP and consumer price index related indicators. This result indicates that the DBN models are not favouring indicators relating to the 2008 crises.

The DBN models provide a measure of the severity of a crisis as a probability. The results indicate that the probability gradually increases before the crisis occurs. This is especially true for the SLDS and NB-SLDS. This information could be of use to policy makers and other stakeholders for developing and implementing policies to reduce the probability of a crisis.

Decision-makers in reserve banks and other regulatory bodies are often compelled to make decisions on early intervention. Such decisions are often based on conflicting model results and other relevant information. An improved EWS, such as the NB-SLDS, could lead to decision-makers placing more emphasis on model results. At best, an improved EWS could help to prevent or limit future crisis events. At worst it could serve as a sifting tool for more thorough investigation into impending crises.

A disadvantage of the SLDS and NB-SLDS is that they are computationally more complex in comparison with the traditional methods. Furthermore, they are comparably not as simple to implement. However, systemic banking crises are generally complex events. The proposed methods, although more complex, are demonstrated to be far more effective. A more effective model could have significant financial implications.

Appendix A. Tables

Detailed results are tabulated in this section. The tables consist of the results for the optimal indicators. A 'NaN' denotes a 'not-a-number' that occurs when dividing by zero.

Table A.4: Signal extraction method *crisis* state results for the best performing indicator. These results are computed over the complete data sequence. They provide an indication of the regime detection performance.

	Accuracy	Recall	Precision	F-score
Austria	0.89	0.83	0.45	0.59
Belgium	0.90	0.68	0.77	0.72
Germany	0.43	0.20	0.78	0.32
Spain	0.84	0.96	0.55	0.70
Finland	0.92	0.93	0.82	0.87
France	0.66	0.43	0.89	0.58
Italy	0.43	0.38	0.73	0.50
Netherlands	0.85	0.58	0.75	0.65
Denmark	0.70	0.67	0.47	0.55
United Kingdom	0.72	0.77	0.56	0.65
Sweden	0.85	1.00	0.56	0.71
Average	0.74	0.68	0.67	0.62

Table A.5: Logit model method *crisis* state results for the best performing indicator. These results are computed over the complete data sequence. They provide an indication of the regime detection performance.

	Accuracy	Recall	Precision	F-score
Austria	0.65	0.19	0.31	0.24
Belgium	0.69	0.23	0.38	0.29
Germany	0.58	0.19	0.45	0.26
Spain	0.64	0.32	0.27	0.29
Finland	0.65	0.65	0.43	0.52
France	0.56	0.58	0.25	0.35
Italy	0.47	0.54	0.36	0.43
Netherlands	0.64	0.23	0.47	0.30
Denmark	0.57	0.62	0.38	0.47
United Kingdom	0.55	0.85	0.39	0.53
Sweden	0.61	0.60	0.35	0.44
Average	0.60	0.45	0.37	0.38

Table A.6: HMM method *crisis* state results for the best performing indicator. These results are computed over the complete data sequence. They provide an indication of the regime detection performance.

	Accuracy	Recall	Precision	F-score
Austria	0.73	0.01	0.00	0.00
Belgium	0.47	0.27	0.69	0.39
Germany	0.81	0.57	1.00	0.73
Spain	0.70	0.64	0.21	0.31
Finland	0.60	0.58	0.71	0.64
France	0.84	0.77	0.88	0.82
Italy	0.73	0.95	0.54	0.69
Netherlands	0.68	0.40	0.56	0.47
Denmark	0.66	0.64	0.87	0.74
United Kingdom	0.54	0.69	0.47	0.56
Sweden	0.74	0.66	0.99	0.79
Average	0.68	0.56	0.63	0.56

Table A.7: SLDS method *crisis* state results for the best performing indicator. These results are computed over the complete data sequence. They provide an indication of the regime detection performance.

	Accuracy	Recall	Precision	F-score
Austria	0.84	0.67	0.58	0.62
Belgium	0.78	0.51	0.32	0.39
Germany	0.72	0.44	0.83	0.57
Spain	0.64	0.61	0.33	0.43
Finland	0.63	0.56	0.84	0.68
France	0.55	0.45	0.82	0.58
Italy	0.54	0.53	0.77	0.62
Netherlands	0.80	0.58	0.50	0.54
Denmark	0.56	0.57	0.46	0.51
United Kingdom	0.64	0.64	0.96	0.77
Sweden	0.57	0.52	0.77	0.62
Average	0.66	0.55	0.65	0.58

Table A.8: NB-SLDS method *crisis* state results for the best performing indicator. These results are computed over the complete data sequence. They provide an indication of the regime detection performance.

	Accuracy	Recall	Precision	F-score
Austria	0.92	0.87	0.76	0.81
Belgium	0.88	0.74	0.74	0.74
Germany	0.76	0.48	0.89	0.63
Spain	0.71	0.80	0.40	0.53
Finland	0.86	0.86	0.83	0.85
France	0.49	0.43	0.97	0.59
Italy	0.60	0.58	0.75	0.65
Netherlands	0.90	0.79	0.73	0.76
Denmark	0.64	0.84	0.35	0.49
United Kingdom	0.62	0.67	0.80	0.73
Sweden	0.78	0.85	0.62	0.72
Average	0.74	0.72	0.71	0.68

Table A.9: Signal extraction method *crisis* state results for the best performing indicator. These results are computed over the 12 quarters (3 years) leading to a crisis. They provide an indication of the predictability of a crisis.

	Accuracy	Recall	Precision	F-score
Austria	0.00	NaN	0.00	NaN
Belgium	0.00	NaN	0.00	NaN
Germany	1.00	1.00	1.00	1.00
Spain	0.00	NaN	0.00	NaN
Finland	0.12	1.00	0.12	0.21
France	0.27	1.00	0.27	0.42
Italy	0.73	1.00	0.73	0.84
Netherlands	0.77	1.00	0.77	0.87
Denmark	0.54	1.00	0.54	0.70
United Kingdom	0.19	1.00	0.19	0.32
Sweden	0.04	1.00	0.04	0.07
Average	0.33	1.00	0.33	0.55

Table A.10: Logit model method *crisis* state results for the best performing indicator. These results are computed over the 12 quarters (3 years) leading to a crisis. They provide an indication of the predictability of a crisis.

	Accuracy	Recall	Precision	F-score
Austria	0.29	1.00	0.29	0.45
Belgium	0.38	1.00	0.38	0.55
Germany	0.39	1.00	0.39	0.56
Spain	0.24	1.00	0.24	0.38
Finland	0.31	1.00	0.31	0.47
France	0.26	1.00	0.26	0.41
Italy	0.32	1.00	0.32	0.48
Netherlands	0.42	1.00	0.42	0.59
Denmark	0.35	1.00	0.35	0.52
United Kingdom	0.13	1.00	0.13	0.22
Sweden	0.38	1.00	0.38	0.55
Average	0.31	1.00	0.31	0.47

Table A.11: HMM method *crisis* state results for the best performing indicator. These results are computed over the 12 quarters (3 years) leading to a crisis. They provide an indication of the predictability of a crisis.

	Accuracy	Recall	Precision	F-score
Austria	0.00	1.00	0.00	0.00
Belgium	0.30	1.00	0.30	0.46
Germany	1.00	1.00	1.00	1.00
Spain	0.00	1.00	0.00	0.00
Finland	0.71	1.00	0.71	0.83
France	0.78	1.00	0.78	0.88
Italy	0.56	1.00	0.56	0.72
Netherlands	0.02	1.00	0.02	0.04
Denmark	0.77	1.00	0.77	0.87
United Kingdom	0.05	1.00	0.05	0.09
Sweden	0.98	1.00	0.98	0.99
Average	0.47	1.00	0.47	0.53

Table A.12: SLDS method *crisis* state results for the best performing indicator. These results are computed over the 12 quarters (3 years) leading to a crisis. They provide an indication of the predictability of a crisis.

	Accuracy	Recall	Precision	F-score
Austria	0.50	1.00	0.50	0.66
Belgium	0.25	1.00	0.25	0.40
Germany	0.78	1.00	0.78	0.88
Spain	0.42	1.00	0.42	0.59
Finland	0.82	1.00	0.82	0.90
France	0.82	1.00	0.82	0.90
Italy	0.80	1.00	0.80	0.89
Netherlands	0.48	1.00	0.48	0.64
Denmark	0.51	1.00	0.51	0.68
United Kingdom	0.93	1.00	0.93	0.97
Sweden	0.86	1.00	0.86	0.93
Average	0.65	1.00	0.65	0.77

Table A.13: NB-SLDS method *crisis* state results for the best performing indicator. These results are computed over the 12 quarters (3 years) leading to a crisis. They provide an indication of the predictability of a crisis.

	Accuracy	Recall	Precision	F-score
Austria	0.64	1.00	0.64	0.78
Belgium	0.60	1.00	0.60	0.75
Germany	0.90	1.00	0.90	0.95
Spain	0.56	1.00	0.56	0.72
Finland	0.83	1.00	0.83	0.91
France	0.97	1.00	0.97	0.99
Italy	0.62	1.00	0.62	0.77
Netherlands	0.61	1.00	0.61	0.75
Denmark	0.26	1.00	0.26	0.41
United Kingdom	0.66	1.00	0.66	0.79
Sweden	0.72	1.00	0.72	0.84
Average	0.67	1.00	0.67	0.79

Acknowledgement

This work is supported by the Barclays Africa Chair in Actuarial Science in the Department of Insurance and Actuarial Science, University of Pretoria and by the US Office of Naval Research (ONR) Global.

The data used in this study were kindly provided by Dr Peter Sarlin, the corresponding author of Lainà et al. (2015)

Abiad, A., Feb. 2003. Early Warning Systems: A Survey and a Regime-Switching Approach. IMF Working Papers 03/32, International Monetary Fund.

URL <https://ideas.repec.org/p/imf/imfwpa/03-32.html>

Ahn, J. J., Oh, K. J., Kim, T. Y., Kim, D. H., 2011. Usefulness of support vector machine to develop an early warning system for financial crisis. Expert Systems with Applications 38 (4), 2966 – 2973.

URL <http://www.sciencedirect.com/science/article/pii/S095741741000881X>

Alessi, L., Antunes, A., Babecky, J., Baltussen, S., Behn, M., Bonfim, D., Bush, O., Detken, C., Frost, J., Guimaraes, R., Havranek, T., Joy, M., Kauko, K., Mateju, J., Monteiro, N., Neudorfer, B., Peltonen, T., Rodrigues, P., Rusnak, M., Schudel, W., Sigmund, M., Stremmel, H., Smidkova, K., van Tilburg, R., Vasicek, B., Zigraiova, D., 2015. Comparing different early warning systems: Results from a horse race competition among members of the macroprudential research network. Mpra paper, University Library of Munich, Germany.

Alessi, L., Detken, C., Aug. 2014. Identifying excessive credit growth and leverage. Working Paper Series 1723, European Central Bank.

URL <https://ideas.repec.org/p/ecb/ecbwps/20141723.html>

Bar-Shalom, Y., Li, X., 1993. Estimation and Tracking: Principles, Techniques, and Software. The Artech House radar library. Artech House, Incorporated.

Barber, D., 2006. Expectation correction for smoothed inference in switching linear dynamical systems. The Journal of Machine Learning Research 7, 2515–2540.

Barber, D., 2012. Bayesian Reasoning and Machine Learning. Cambridge University Press.

Borio, C. E., Drehmann, M., 2009. Assessing the risk of banking crises—revisited. BIS Quarterly Review, March.

Boyacioglu, M. A., Kara, Y., mer Kaan Baykan, 2009. Predicting bank financial failures using neural networks, support vector machines and multivariate statistical methods: A comparative analysis in the sample of savings deposit insurance fund (sdf) transferred banks in turkey. Expert Systems with Applications 36 (2, Part 2), 3355 – 3366.

URL <http://www.sciencedirect.com/science/article/pii/S095741740800078X>

Bussiere, M., Fratzscher, M., 2006. Towards a new early warning system of financial crises. Journal of International Money and Finance 25 (6), 953 – 973.

- URL <http://www.sciencedirect.com/science/article/pii/S0261560606000532>
- Celik, A. E., Karatepe, Y., 2007. Evaluating and forecasting banking crises through neural network models: An application for turkish banking sector. *Expert Systems with Applications* 33 (4), 809 – 815.
URL <http://www.sciencedirect.com/science/article/pii/S0957417406002132>
- Cerchiello, P., Giudici, P., 2016. Conditional graphical models for systemic risk estimation. *Expert Systems with Applications* 43, 165 – 174.
URL <http://www.sciencedirect.com/science/article/pii/S0957417415006041>
- Dabrowski, J. J., de Villiers, J. P., Beyers, C., 2016. Naive bayes switching linear dynamical system: A model for dynamic system modelling, classification and data fusion. *Information Fusion-Under review*.
- Davis, E. P., Karim, D., 2008. Comparing early warning systems for banking crises. *Journal of Financial Stability* 4 (2), 89 – 120.
URL <http://www.sciencedirect.com/science/article/pii/S1572308908000144>
- Demirgüç-Kunt, A., Detragiache, E., 1998. The determinants of banking crises in developing and developed countries. *Staff Papers (International Monetary Fund)* 45 (1), 81–109.
URL <http://www.jstor.org/stable/3867330>
- Demirgüç-Kunt, A., Detragiache, E., 2005. Cross-country empirical studies of systemic bank distress: A survey. *National Institute Economic Review* 192 (1), 68–83.
- Duttgupta, R., Cashin, P., Apr. 2008. The Anatomy of Banking Crises. IMF Working Papers 08/93, International Monetary Fund.
URL <https://ideas.repec.org/p/imf/imfwpa/08-93.html>
- Faranda, D., Pons, F. M. E., Giachino, E., Vaienti, S., Dubrulle, B., 2015. Early warnings indicators of financial crises via auto regressive moving average models. *Communications in Nonlinear Science and Numerical Simulation* 29 (13), 233 – 239.
URL <http://www.sciencedirect.com/science/article/pii/S1007570415001653>
- Feki, A., Ishak, A. B., Feki, S., 2012. Feature selection using bayesian and multiclass support vector machines approaches: Application to bank risk prediction. *Expert Systems with Applications* 39 (3), 3087 – 3099.
URL <http://www.sciencedirect.com/science/article/pii/S0957417411013030>
- Gaytán, A., Johnson, C. A., 2002. A review of the literature on early warning systems for banking crises.
- Graciela Kaminsky, Saul Lizondo, C. M. R., 1998. Leading indicators of currency crises. *Staff Papers (International Monetary Fund)* 45 (1), 1–48.
URL <http://www.jstor.org/stable/3867328>
- Guyon, I., Elisseeff, A., Mar. 2003. An introduction to variable and feature selection. *J. Mach. Learn. Res.* 3, 1157–1182.
URL <http://dl.acm.org/citation.cfm?id=944919.944968>
- Honohan, P., Honohan, P., 1997. Banking system failures in developing and transition countries: Diagnosis and prediction. In: *Diagnosis and Prediction*, BIS Working Paper, No. 39 (Basle: Bank for International Settlements. pp. 29–1.
- Iturriaga, F. J. L., Sanz, I. P., 2015. Bankruptcy visualization and prediction using neural networks: A study of u.s. commercial banks. *Expert Systems with Applications* 42 (6), 2857 – 2869.
URL <http://www.sciencedirect.com/science/article/pii/S0957417414007118>
- Jagic, T., Bojnec, S., Jagic, V., 2015. Optimized spiral spherical self-organizing map approach to sector analysis the case of banking. *Expert Systems with Applications* 42 (13), 5531 – 5540.
URL <http://www.sciencedirect.com/science/article/pii/S0957417415001700>
- Kaminsky, G. L., Reinhart, C. M., 1999. The twin crises: The causes of banking and balance-of-payments problems. *American Economic Review* 89 (3), 473–500.
URL <http://www.aeaweb.org/articles.php?doi=10.1257/aer.89.3.473>
- Kim, C.-J., 1994. Dynamic linear models with markov-switching. *Journal of Econometrics* 60 (1-2), 1 – 22.
- Kirchgässner, G., Wolters, J., Hassler, U., 2012. *Introduction to Modern Time Series Analysis*. Springer Texts in Business and Economics. Springer Berlin Heidelberg.
- Lainà, P., Nyholm, J., Sarlin, P., 2015. Leading indicators of systemic banking crises: Finland in a panel of {EU} countries. *Review of Financial Economics* 24, 18 – 35.
URL <http://www.sciencedirect.com/science/article/pii/S1058330014000512>
- Lin, C.-S., Khan, H. A., Chang, R.-Y., Wang, Y.-C., 2008. A new approach to modeling early warning systems for currency crises: Can a machine-learning fuzzy expert system predict the currency crises effectively? *Journal of International Money and Finance* 27 (7), 1098 – 1121.
URL <http://www.sciencedirect.com/science/article/pii/S0261560608000818>
- Lin, S. L., 2009. A new two-stage hybrid approach of credit risk in banking industry. *Expert Systems with Applications* 36 (4), 8333 – 8341.
URL <http://www.sciencedirect.com/science/article/pii/S0957417408007094>
- Lin, S.-L., Wu, S.-J., 2011. Is grey relational analysis superior to the conventional techniques in predicting financial crisis? *Expert Systems with Applications* 38 (5), 5119 – 5124.
URL <http://www.sciencedirect.com/science/article/pii/S0957417410010961>
- Manasse, P., Roubini, N., 2009. rules of thumb for sovereign debt crises. *Journal of International Economics* 78 (2), 192 – 205.
URL <http://www.sciencedirect.com/science/article/pii/S0022199609000440>
- Martinez Peria, M., 2002. A regime-switching approach to the study of speculative attacks: A focus on ems crises. In: Hamilton, J., Raj, B. (Eds.), *Advances in Markov-Switching Models*. Studies in Empirical Economics. Physica-Verlag HD, pp. 159–194.
URL http://dx.doi.org/10.1007/978-3-642-51182-0_8
- Mesot, B., Barber, D., 2007. Switching linear dynamical systems for noise robust speech recognition. *Audio, Speech, and Language Processing, IEEE Transactions on* 15 (6), 1850–1858.
- Murphy, K., 1998. Learning switching kalman filter models. *Compaq Cambridge Research Lab Tech Report* 98-10.
- Murphy, K. P., July 2002. *Dynamic bayesian networks: Representation, inference and learning*. Ph.D. thesis, University of California, Berkeley.
- Murphy, K. P., 2012. *Machine Learning: A Probabilistic Perspective*. Adaptive computation and machine learning series. MIT Press.
- Nag, A., Mitra, A., 1999. Neural networks and early warning indicators of currency crisis. *Reserve Bank of India Occasional Papers* 20 (2), 183–222.
- Pavlovic, V., Rehg, J., Cham, T.-J., Murphy, K., 1999. A dynamic bayesian network approach to figure tracking using learned dynamic models. In: *Computer Vision, 1999. The Proceedings of the Seventh IEEE International Conference on*. Vol. 1. pp. 94–101 vol.1.
- Rose, A. K., Spiegel, M. M., 2012. Cross-country causes and consequences of the 2008 crisis: Early warning. *Japan and the World Economy* 24 (1), 1 – 16.
URL <http://www.sciencedirect.com/science/article/pii/S0922142511000417>
- Sevim, C., Oztekin, A., Bali, O., Gumus, S., Guresen, E., 2014. Developing an early warning system to predict currency crises. *European Journal of Operational Research* 237 (3), 1095 – 1104.
URL <http://www.sciencedirect.com/science/article/pii/S0377221714001829>
- Theodoridis, S., Koutroumbas, K., 2009. *Pattern Recognition*, 4th Edition. Elsevier.
- Turaga, P., Chellappa, R., Subrahmanian, V. S., Udrea, O., Nov 2008. Machine recognition of human activities: A survey. *Circuits and Systems for Video Technology, IEEE Transactions on* 18 (11), 1473–1488.