A pragmatic approach to including complex natural modes of vibration in aeroelastic analysis

International Aerospace Symposium of South Africa

14 to 16 September, 2015

Stellenbosch, South Africa

Louw van Zyl

CSIR

our future through science

Problem statement

- Aeroelasticity is often described as the study of the interaction of inertial, elastic and aerodynamic forces that occur when an elastic body is exposed to a fluid flow (Wikipedia).
- The aim of a flutter analysis is to determine the speed above which structural vibrations will grow exponentially and potentially cause structural failure.
- On the one hand it is necessary to model how the structure would respond to forces applied to it, and on the other hand it is necessary to model what aerodynamic forces would be generated due to the movement of the structure.
- This presentation concerns mainly the structural dynamic component of the aeroelastic problem, and specifically the structural damping forces (which is usually not mentioned in the definition of aeroelasticity).

Structural Dynamics

- The general structural dynamic equation of motion is

$$[M]\{\ddot{x}\}+[C]\{\dot{x}\}+[K]\{x\}=\{f\}$$

- Where the x_i represent physical displacements, the f_i physical forces and the matrices can be finite element model matrices or something more abstract. These mass, damping and stiffness matrices are generally full matrices.

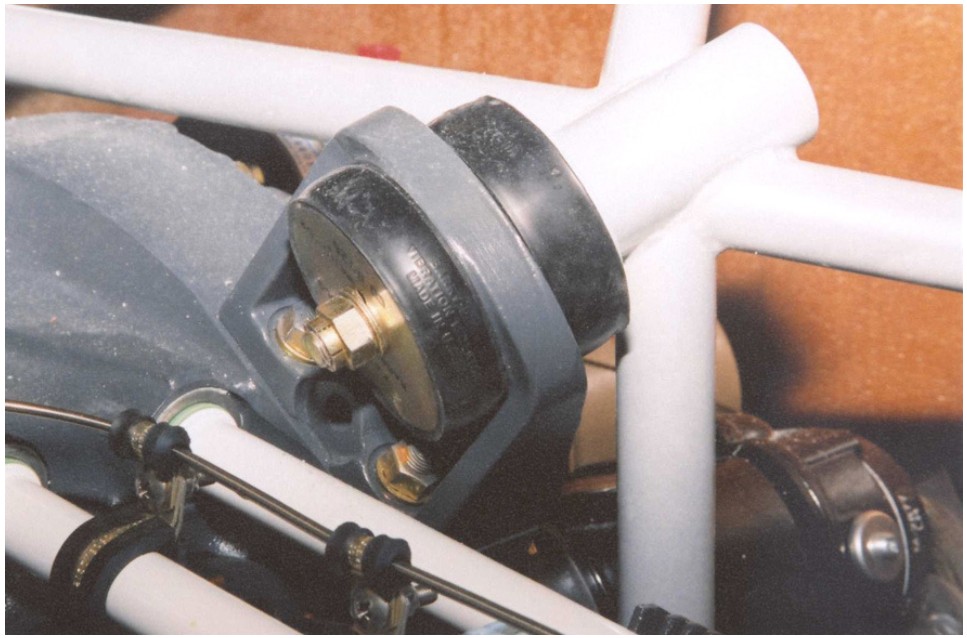
- The eigenvalues of the corresponding un-damped equation are the natural frequencies of the structure (actually, the square of the angular frequencies in radians per second)

$$s^2[M]\{x\}+[K]\{x\}=\mathbf{0}$$

Structural Dynamics (continued)

- The corresponding eigenvectors are real-valued and are the natural mode shapes of the structure
- By pre- and post-multiplying the structural dynamic equation of motion by a subset of these eigenvectors, the problem is transformed from a physical basis to a modal basis, i.e. the degrees of freedom become modal deflections rather than physical deflections.
- We are usually only interested in a small number of natural modes of a structure, defined by a frequency range of interest. The resulting modal basis structural dynamic model is orders of magnitude smaller than the physical model.
- Pre- and post-multiplying the physical mass and stiffness matrices by the eigenvectors diagonalises them. Under the condition of proportional damping, the damping matrix is also diagonalised and the eigenvectors of the un-damped equation of motion are also eigenvectors of the damped equation of motion.

- It is not always justified to assume proportional damping: aircraft engines are relatively large masses mounted on various types of mountings. This is a major source of non-proportional damping in aircraft.



Properties of normal modes

- A normal mode of an oscillating system is a pattern of motion in which all parts of the system move sinusoidally with the same frequency and with a fixed phase relation. (Wikipedia)
- In the case of proportionally damped systems, the phase relation between any two points is either in phase or 180 degrees out of phase. The mode shapes can therefore be described by real numbers (positive and negative) whereas complex numbers are required to describe the mode shapes of a non-proportionally damped system.
- When a structure is made to oscillate in one of its natural modes and the excitation is stopped, the structure will continue to oscillate in the same mode shape, even though the amplitude will decrease exponentially. This is true for both proportionally damped and non-proportionally damped structures.

Properties of normal modes

- It is important to note that the un-damped, real-valued, normal modes of the structure together with the full damping matrix in the case of non-proportionally damped structures is a complete structural dynamic model of the structure.
- It is convenient to use real-valued mode shapes in aeroelastic analysis, especially for the calculation of the unsteady aerodynamic forces.
- The unsteady aerodynamic code therefore needs no modification, only the flutter solver needs to read in an use a full damping matrix in stead of a diagonal one.

Obtaining the structural dynamic model

- There are two main options: finite element modelling and ground vibration testing.
- In a finite element model the user has to specify the damping model – it is the user's own fault if he chooses a difficult model
- In ground vibration testing the damping model must be determined experimentally – the user is not to blame if it turns out to be non-proportionally damped
- There are two main ground vibration testing methods: Phase separation ("broadband") and phase resonance ("sine dwell")
- In phase separation testing the test consists of measuring a large number of transfer functions, typically in the order of a hundred responses and in the order of ten excitation points. The structural dynamic model is obtained by post-processing of the measured data.
- In phase separation testing the structure is made to oscillate in each of its natural modes in turn and the mode shape and modal parameters are measured directly

Obtaining the structural dynamic model from phase resonance testing

- In phase separation testing the structure is made to oscillate in its **un-damped** normal modes. Several exciters may be required to achieve the desired phase relationship over the whole structure. The modal parameters are measured for one mode at a time, therefore the interaction between modes (due to the off-diagonal damping matrix terms) appears to be lost.
- The sine-dwell method does however leave a record of the input forces and velocities at the excitation positions when the mode was excited. This record is used to determine the off-diagonal damping matrix terms for a set of modes.

Obtaining the modal damping matrix from phase resonance testing

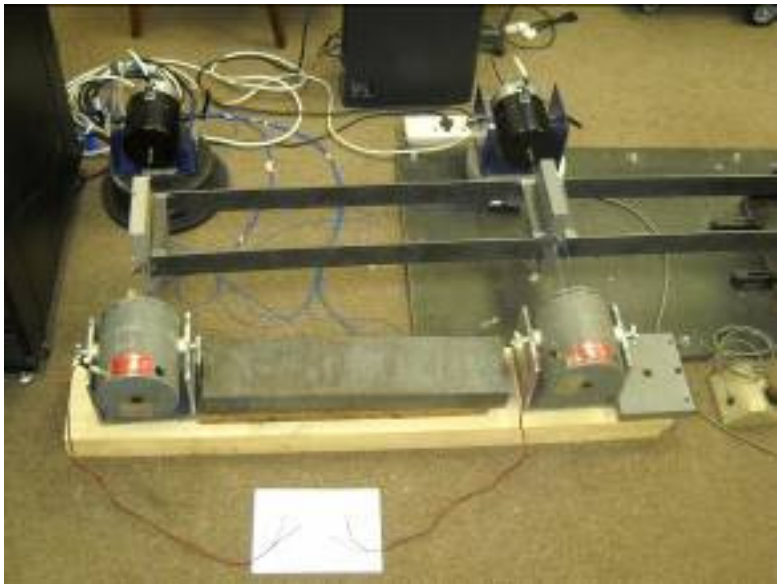
- The excitation forces are expressed in terms of a specific model, viz. that each excitation degree of freedom is connected to ground through a viscous damper and that each pair of excitation degrees of freedom are connected by a damper.

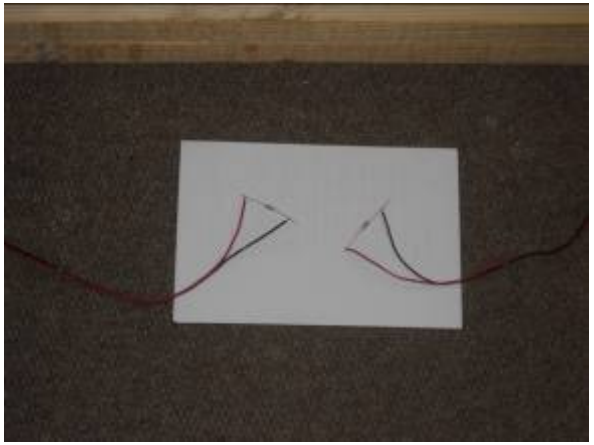
$$F_{ij} = v_{ij} C_{ii} + \sum_{k=1}^{n} \left(v_{ij} - v_{kj} \right) C_{ik}$$

Where *Fij* and *vij* are the force and velocity, respectively, in degree of freedom *i* used in the isolation of mode *j*. C_{ik} is the damping constant of the damper between degrees of freedom *i* and *k*, except that when *i*=*k*, it is the damping constant of the damper between degree of freedom *i* and ground.

- Once the damping values are known, they are used to construct the corresponding physical damping matrix. The final step is to generalize the physical damping matrix using the displacement vectors of the excitation degrees of freedom.

Experimental setup





Experimental setup

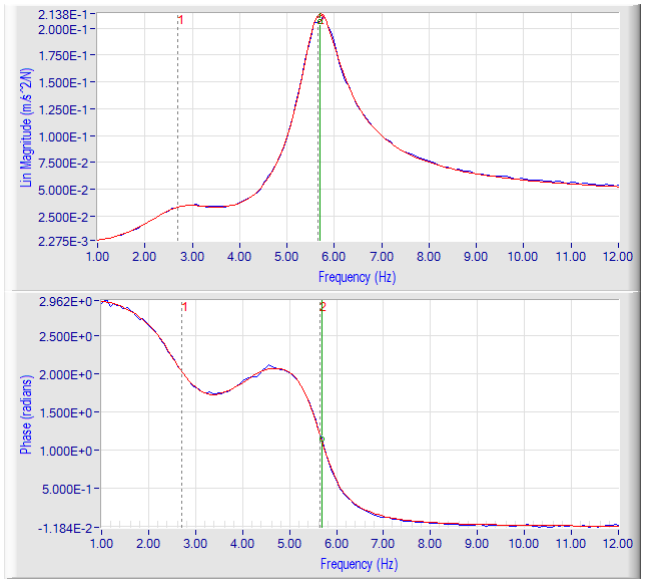
- The system has only two degrees of freedom
- Two electro-mechanical exciters (the grey ones) are used as dampers. The external resistance determines the damping constant.
- The other two exciters are used to excite the structure.
- Impedance heads measure input force and response at the excitation positions.
- A setup in which one damper has minimum damping (open loop) and the other maximum damping (short circuit) produced significantly non-proportional damping.
- The results that follow are for this setup, for both phase separation tests and phase resonance tests.

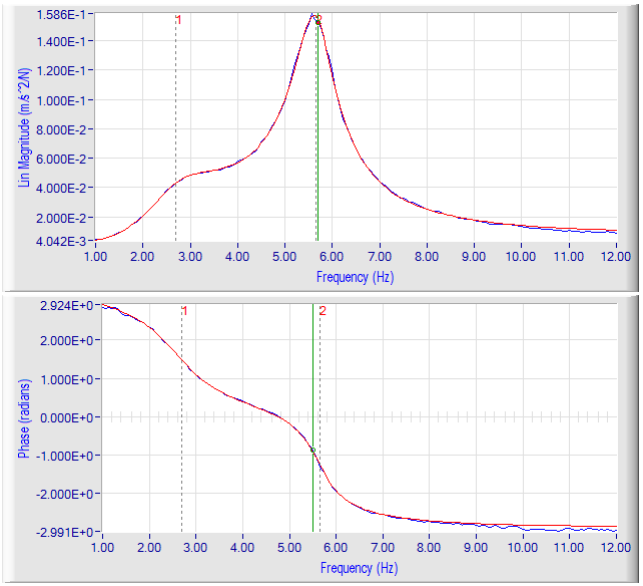
Analysis using MATLAB SDT

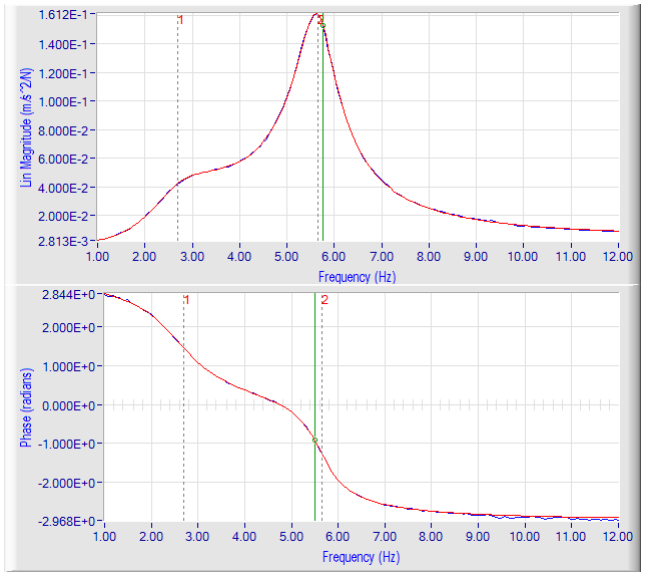
- The first step is identifying “poles” in the responses
- The second step is to calculate “residues” in order to fit the measured transfer functions. At this stage there is one residue per pole per response d.o.f per excitation d.o.f
- The poles and residues are typically iteratively refined to improve the fit .
- The final step is to transform the parameters used to fit the individual transfer functions to a structural dynamic model.
- SDT offers a choice of models: A pole-residue model that is suitable for proportionally damped structures, and a full damping matrix model for non-proportionally damped structures
- In the former case the number of residues is reduced substantially.
- There are pre-requisites for the latter type of model in terms of the number of sensors, actuators and modes that must be kept in mind at the test stage.

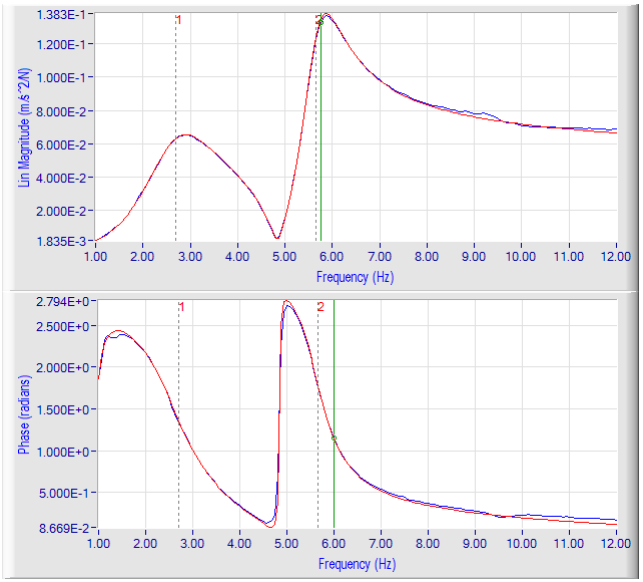
Analysis using MATLAB SDT

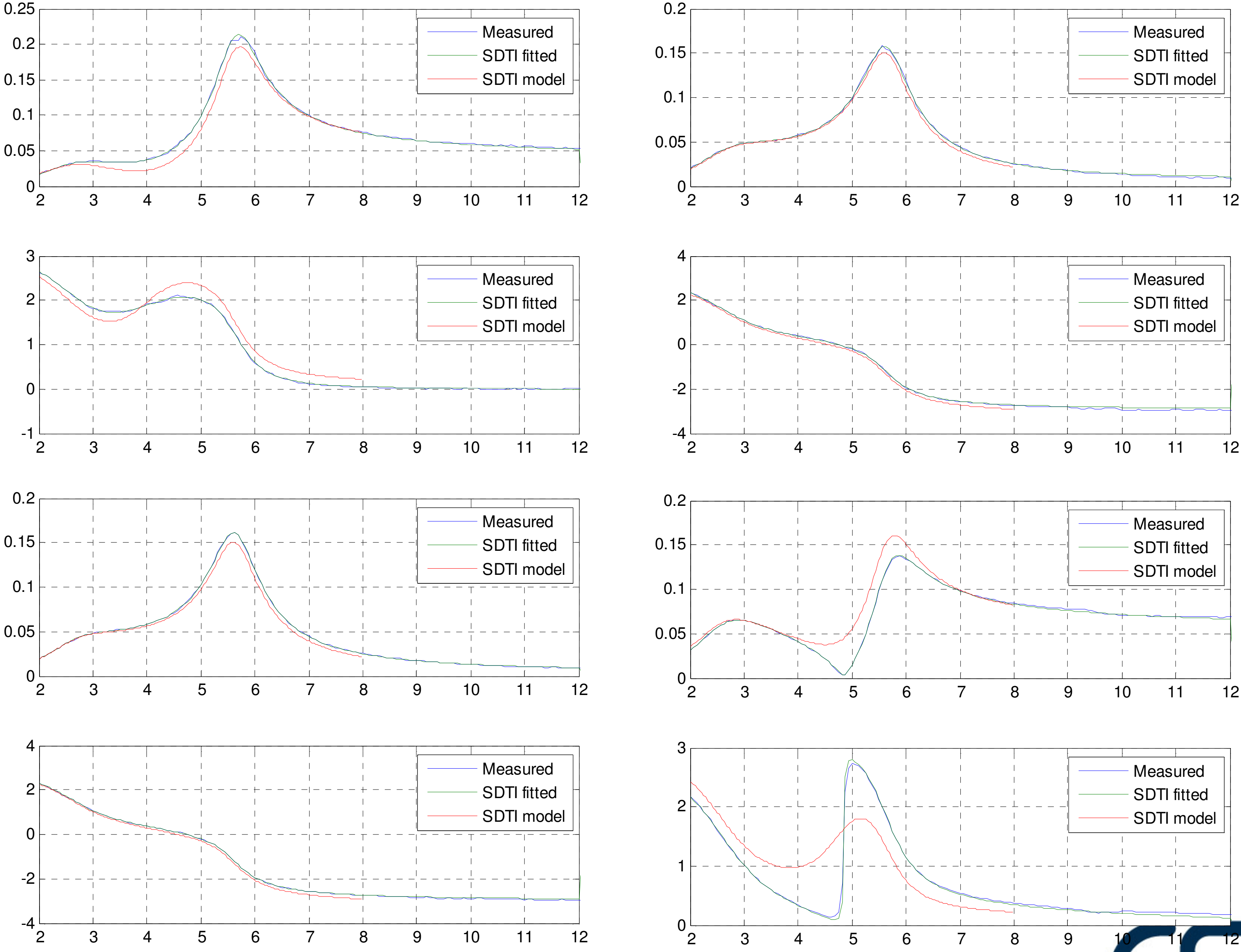
- The first step is identifying “poles” in the responses
- The second step is to calculate “residues” in order to fit the measured transfer functions. At this stage there is one residue per pole per response d.o.f per excitation d.o.f
- The poles and residues are typically iteratively refined to improve the fit .
- The final step is to transform the parameters used to fit the individual transfer functions to a structural dynamic model.
- SDT offers a choice of models: A pole-residue model that is suitable for proportionally damped structures, and a full damping matrix model for non-proportionally damped structures
- In the former case the number of residues is reduced substantially.
- There are pre-requisites for the latter type of model in terms of the number of sensors, actuators and modes that must be kept in mind at the test stage.

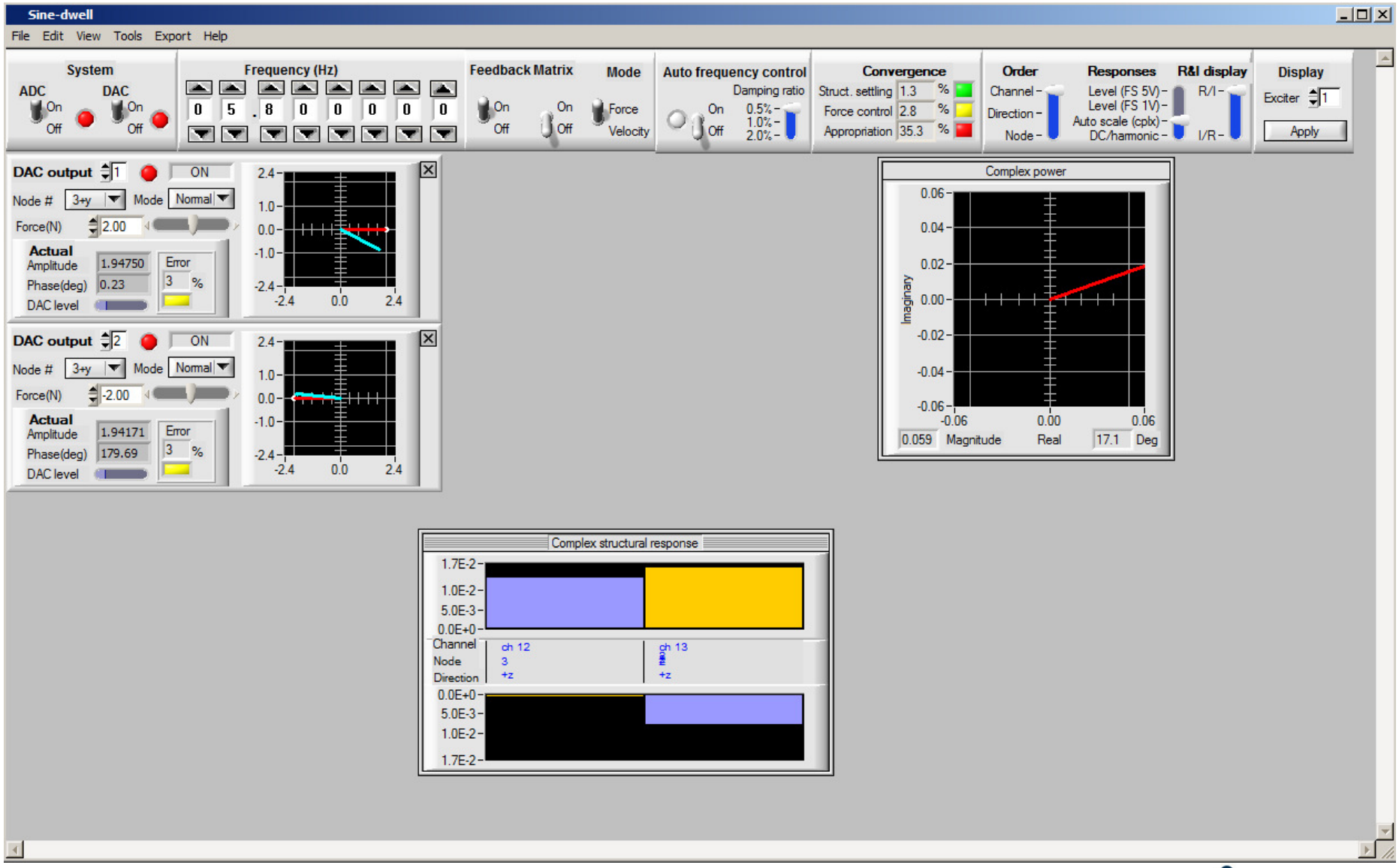






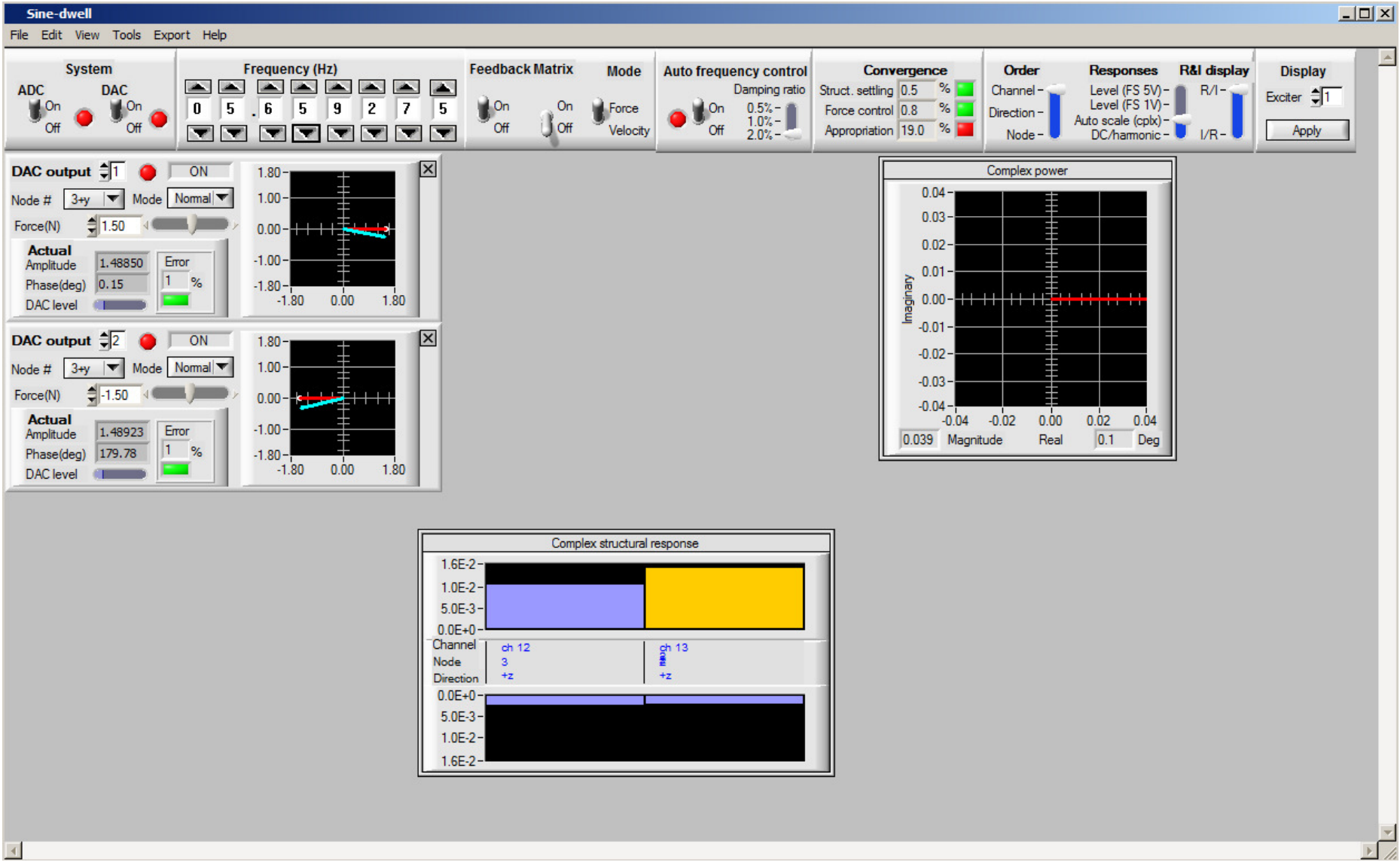






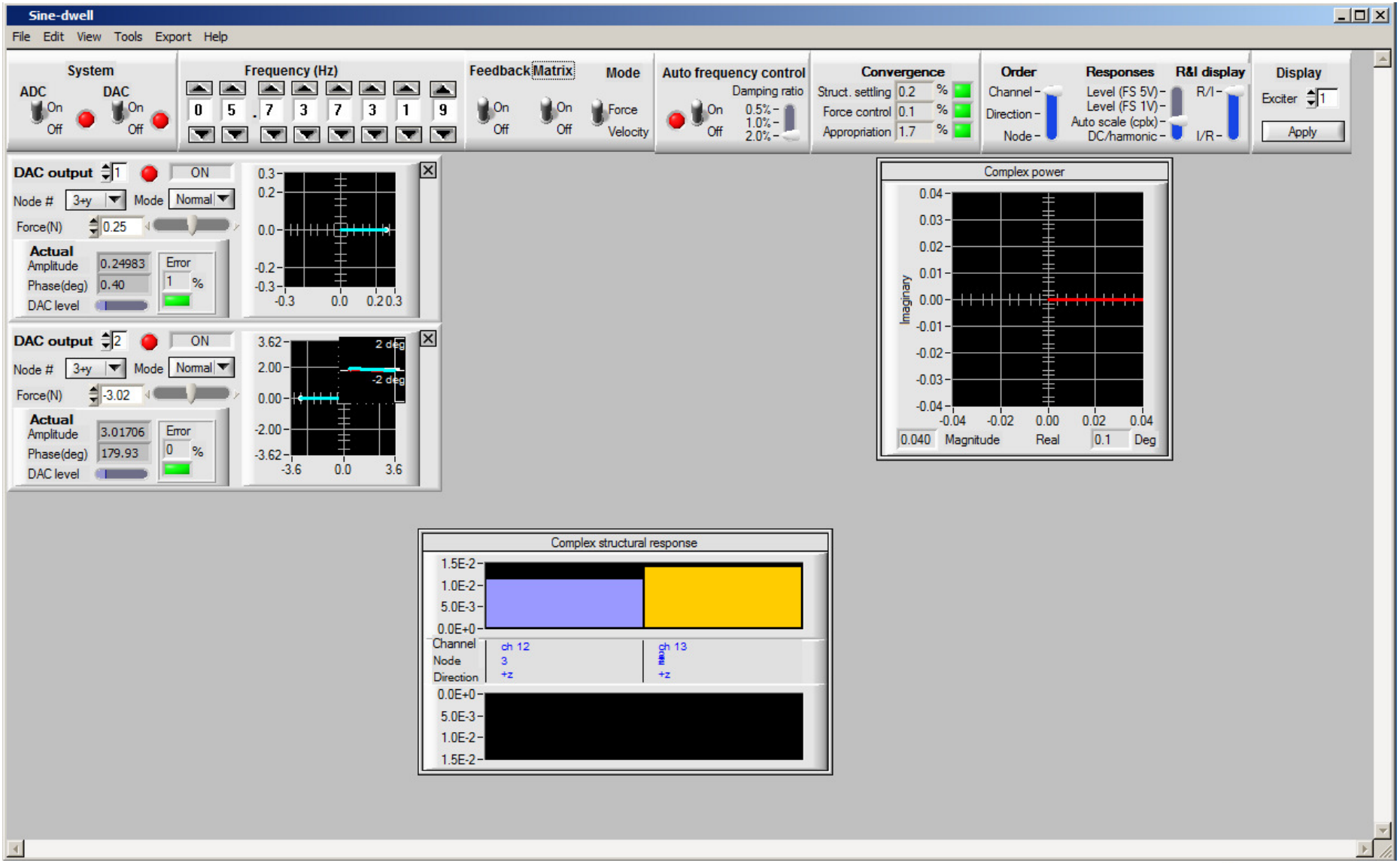
Sine-dwell testing: exciting close to the modal frequency

our future through science



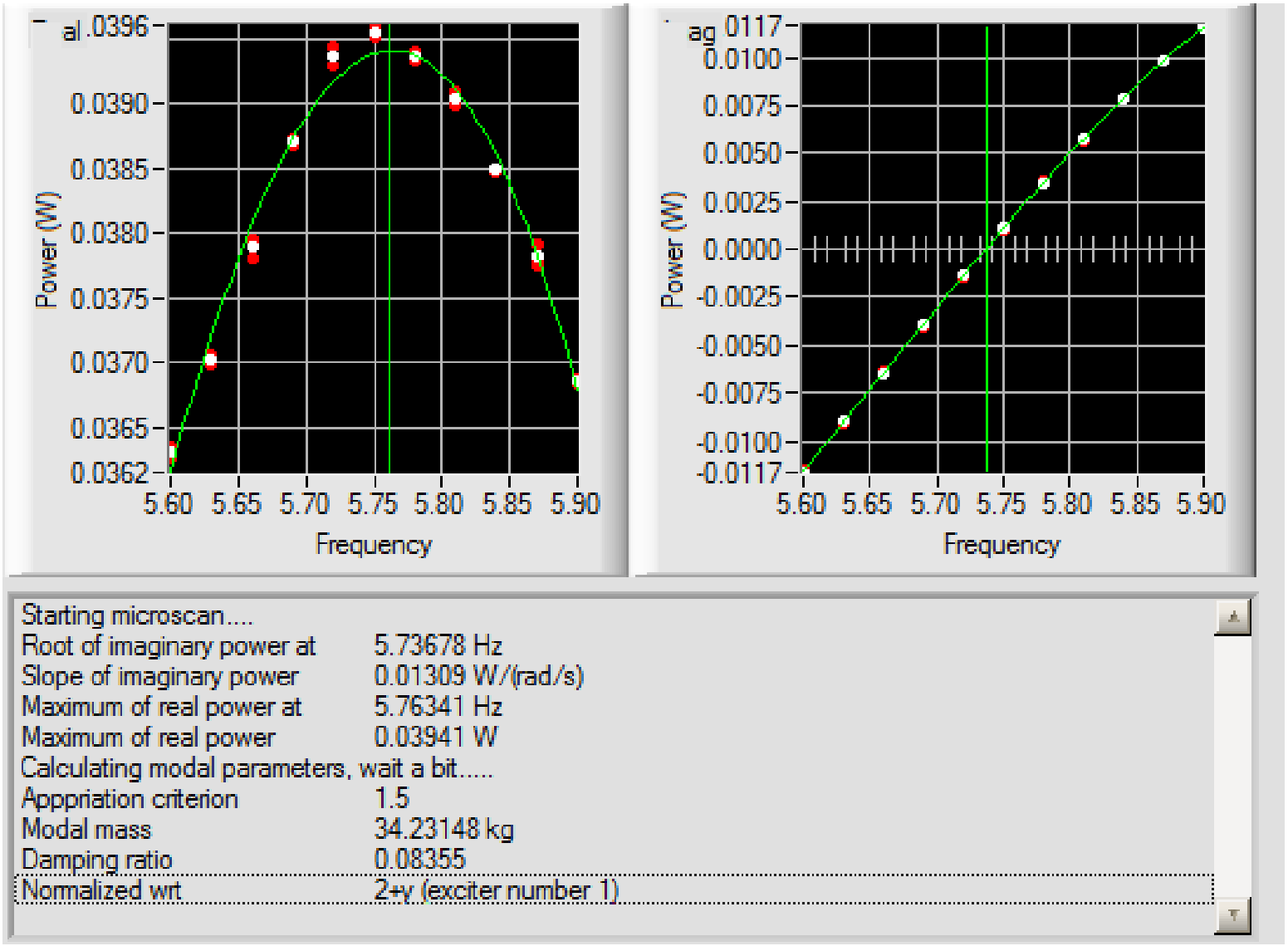
Sine-dwell testing: exciting at the modal frequency but with the wrong force ratio

our future through science



Sine-dwell testing: exciting at the modal frequency and with the correct force ratio

our future through science



Sine-dwell testing: extracting modal parameters

Sine-dwell testing: force and velocity

Mode 1

Exciter number	Degree of freedom	Force	Velocity
1	2+y	-0.001936	0.007463
2	3+y	3.090704	0.011558

Mode 2

Exciter number	Degree of freedom	Force	Velocity
1	2+y	0.248883	0.013825
2	3+y	-3.019044	-0.012175

Sine-dwell testing: damping matrix

Solution (physical dampers)

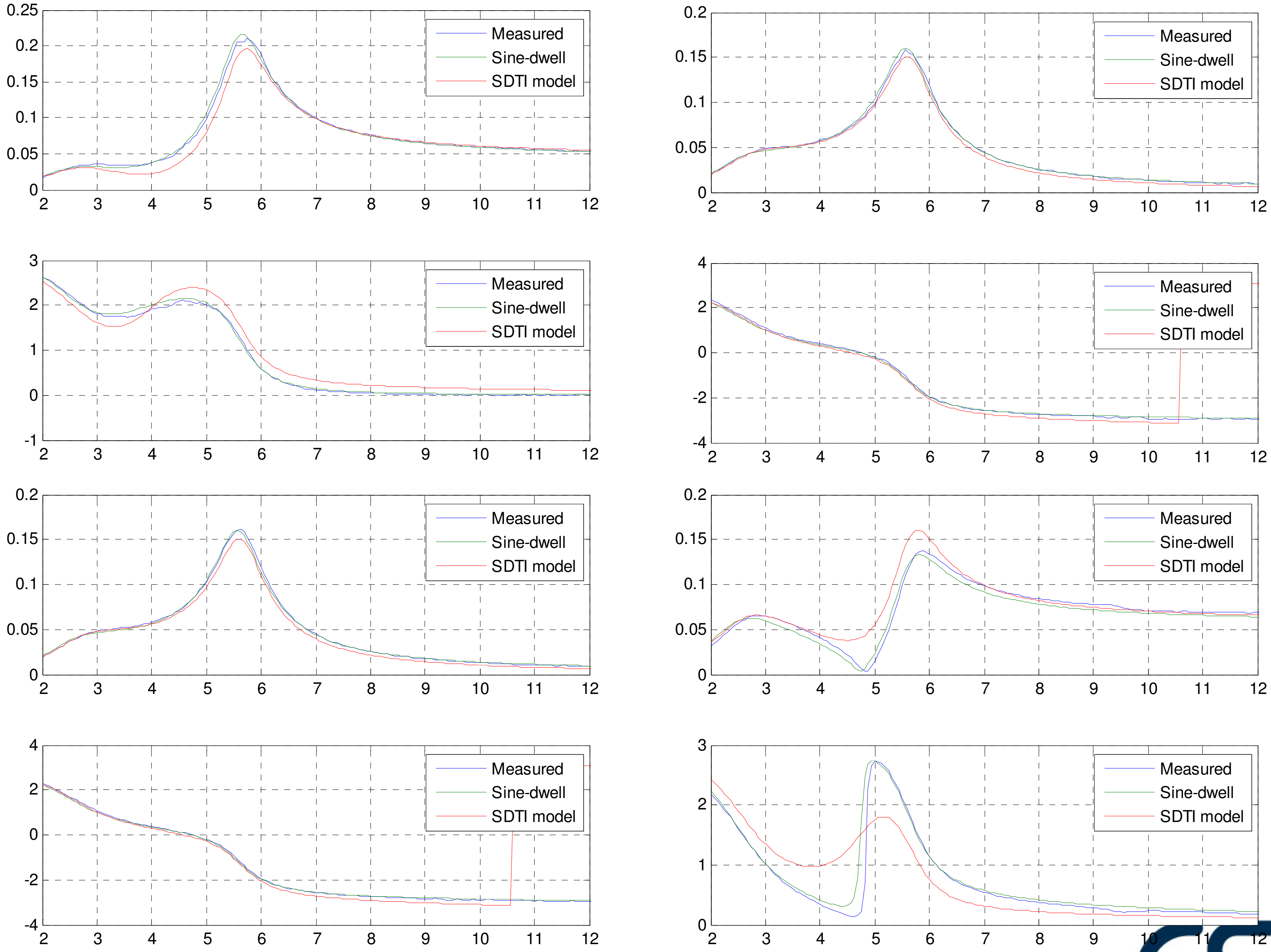
1	1	15.33
1	2	-1.09
2	2	258.59

Physical damping matrix

14.24	1.09
1.09	257.50

Generalized damping matrix

264.8	-217.1
-217.1	212.0



Conclusion

- The analytical tools for modelling structures with non-proportional damping is available in both phase separation and phase resonance testing.
- The only significant difference is that a full (as opposed to diagonal) modal damping matrix needs to be determined and used in the flutter solver.
- Careful planning of phase separation tests is necessary to ensure that it will in fact be possible to extract the full modal damping matrix.
- In phase resonance testing the number of excitation degrees of freedom should be kept to a minimum.
- The significance of non-proportional damping in aeroleastic analysis remains to be seen