What's all this about entanglement?

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CSIR National Laser Centre

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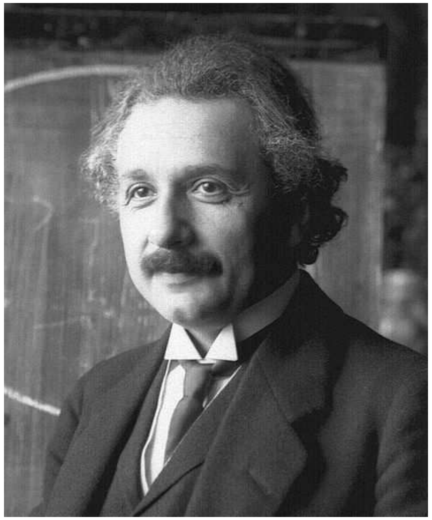
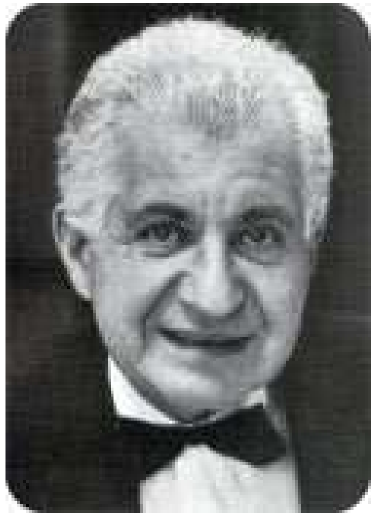
our future through science

Einstein-Podolsky-Rosen

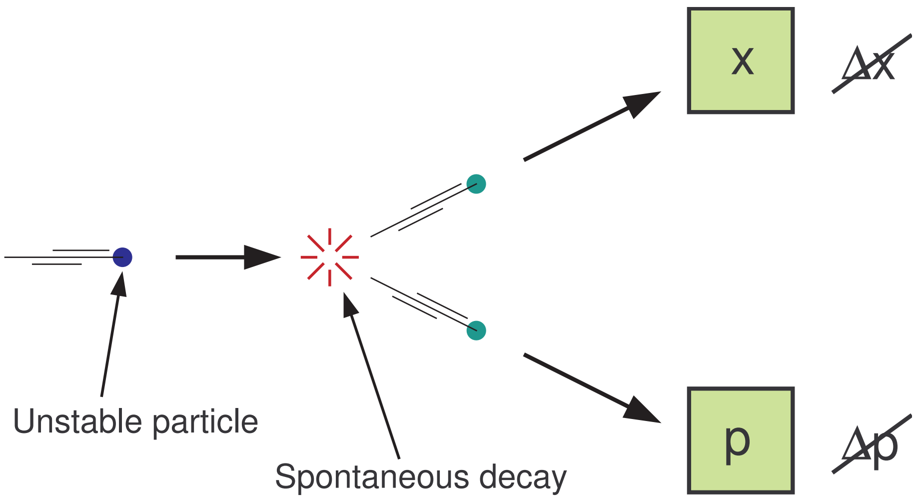


Albert Einstein



Boris Podolsky

Nathan Rosen



Quantum mechanics:

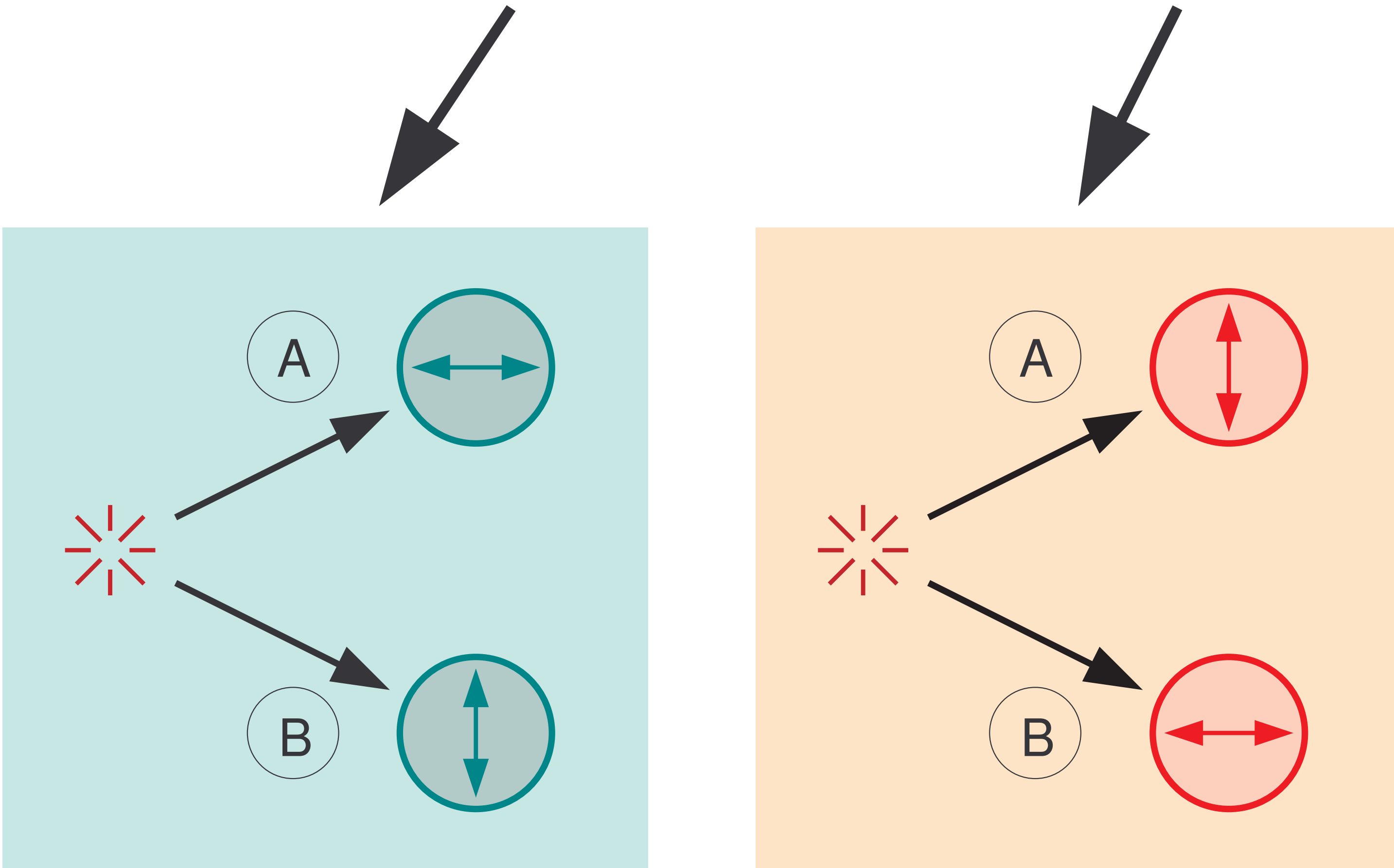
measurements on one particle <u>dictate</u> the state of the other particle.

Multiple reality

$$|\Psi\rangle = \frac{1}{\sqrt{2}} |H\rangle_A |V\rangle_B - \frac{1}{\sqrt{2}} |V\rangle_A |H\rangle_B$$

Reality #1

Reality #2

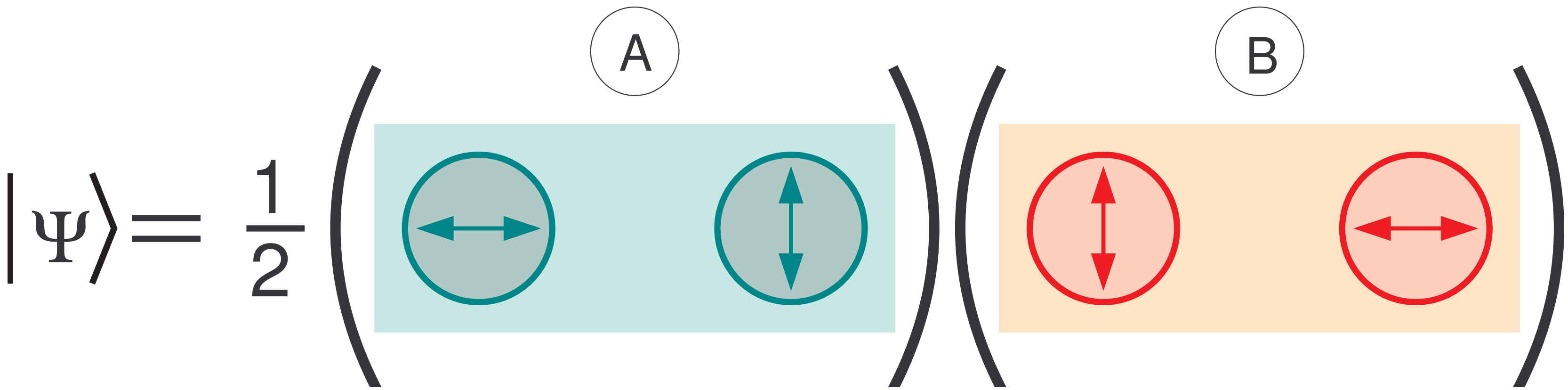


Separability

$$|\Psi\rangle = \frac{1}{2}\,|H\rangle_A\,|V\rangle_B - \frac{1}{2}\,|H\rangle_A\,|H\rangle_B + \frac{1}{2}\,|V\rangle_A\,|V\rangle_B - \frac{1}{2}\,|V\rangle_A\,|H\rangle_B$$

... can be factored (separated)

$$|\Psi\rangle = \frac{1}{2}\left(|H\rangle_A + |V\rangle_A\right)\left(|H\rangle_B - |V\rangle_B\right)$$



Separability $\Rightarrow$ Not entangled

Spontaneous parametric down-conversion

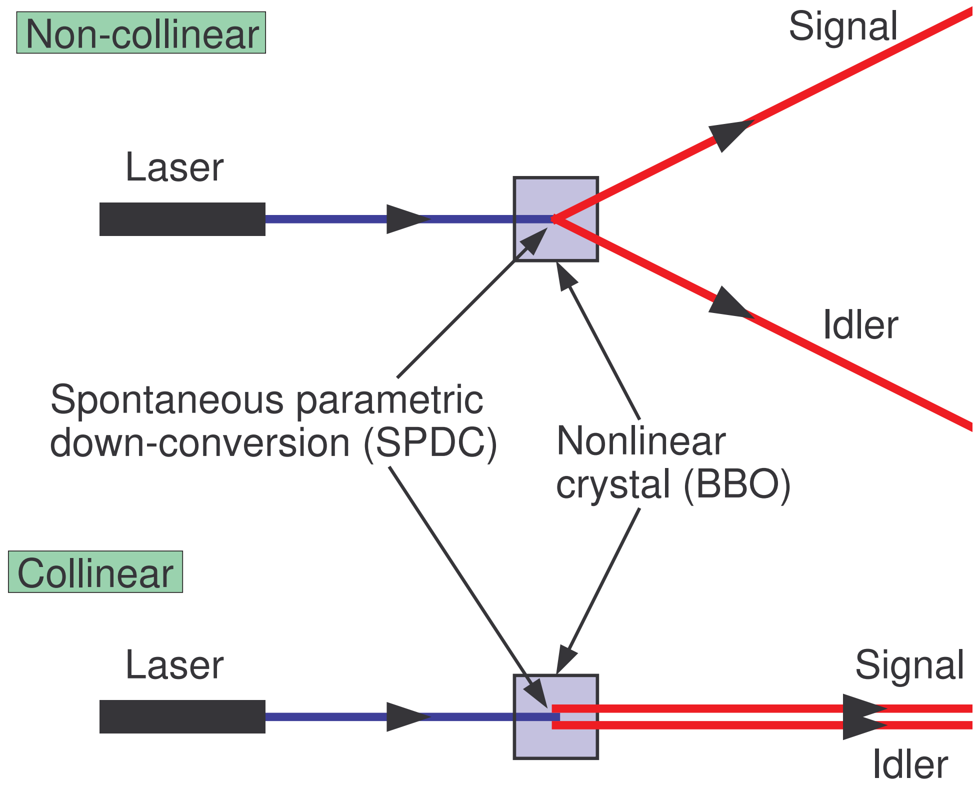
One incoming photon $\rightarrow$ Two outgoing photons

Energy conservation:

$\omega_{\text{pump}} = \omega_{\text{signal}} + \omega_{\text{idler}}$

Momentum conservation:

$\mathbf{k}_{\text{pump}} = \mathbf{k}_{\text{signal}} + \mathbf{k}_{\text{idler}}$



Degenerate phase matching conditions:

$\omega_{\text{signal}} = \omega_{\text{idler}} = \frac{1}{2}\omega_{\text{pump}}$ or $\lambda_{\text{signal}} = \lambda_{\text{idler}} = 2\lambda_{\text{pump}}$

At NLC: $\lambda_{\text{pump}} = 355$ nm and $\lambda_{\text{signal}} = \lambda_{\text{idler}} = 710$ nm

Entanglement in momentum

Due to momentum conservation (remember $\mathbf{p} = \hbar\mathbf{k}$):
(summation $\leftrightarrow$ integration)

$$\begin{aligned} |\Psi\rangle_{\text{SPDC}} &= \sum_n \alpha_n \, |\mathbf{k}_n\rangle_A \, |\mathbf{k}_p - \mathbf{k}_n\rangle_B \\ &= \alpha_1 \, |\mathbf{k}_1\rangle_A \, |\mathbf{k}_p - \mathbf{k}_1\rangle_B + \alpha_2 \, |\mathbf{k}_2\rangle_A \, |\mathbf{k}_p - \mathbf{k}_2\rangle_B \\ &\quad + \alpha_3 \, |\mathbf{k}_3\rangle_A \, |\mathbf{k}_p - \mathbf{k}_3\rangle_B + \ldots \end{aligned}$$

where $|\mathbf{k}_p\rangle$ is the pump state and $|\mathbf{k}_n\rangle$ are plane wave states

Each term represent a different ‘reality’
The complete state does not factorize $\Rightarrow$ the state is entangled

Entanglement in spatial modes

Entanglement in momentum basis $\Rightarrow$ entanglement in any modal basis

Why? — 2 reasons:

- ▷ Different modal bases are all related by local unitary transformation $|M_m\rangle = U_{m,n} |\mathbf{k}_n\rangle$ for $U_{m,n} U_{n,p}^{\dagger} = \mathcal{I}_{m,p}$.
- ▷ local unitary transformation does not affect entanglement
  Example: $|x\rangle = C|a\rangle - S|b\rangle$; $|y\rangle = S|a\rangle + C|b\rangle$, where $C^2 + S^2 = 1$.

$$\begin{aligned}
\sqrt{2}\,|\psi\rangle &= |x\rangle_A |x\rangle_B + |y\rangle_A |y\rangle_B \qquad \leftarrow \quad \text{entangled} \\
&= (C|a\rangle_A - S|b\rangle_A)|x\rangle_B + (S|a\rangle_A + C|b\rangle_A)|y\rangle_B \\
&= |a\rangle_A (C|x\rangle_B + S|y\rangle_B) + |b\rangle_A (-S|x\rangle_B + C|y\rangle_B) \\
&= |a\rangle_A |a\rangle_B + |b\rangle_A |b\rangle_B \qquad \leftarrow \quad \text{entangled}
\end{aligned}$$

OAM bases

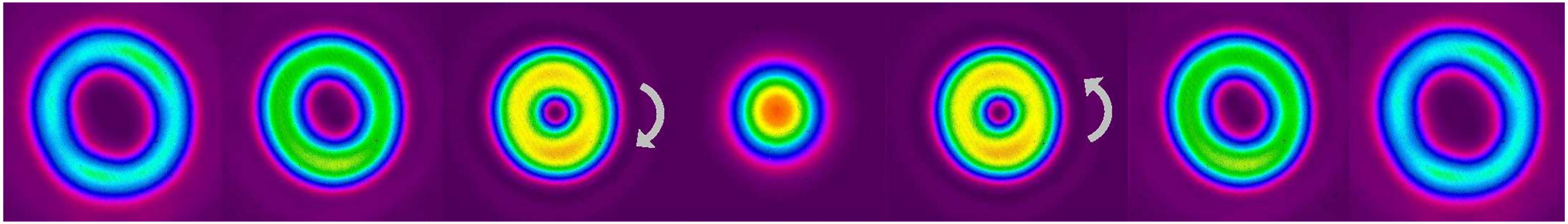
Modes of an orbital angular momentum (OAM) basis:

$$M_{\text{OAM}}(r, \phi) = R_\ell(r) \exp(i\ell\phi)$$

in cylindrical coordinates

ℓ — azimuthal index (integer)
OAM is proportional to ℓ

$R_\ell(r)$ — mode profile function
(examples: Laguerre-Gauss or Bessel-Gauss)



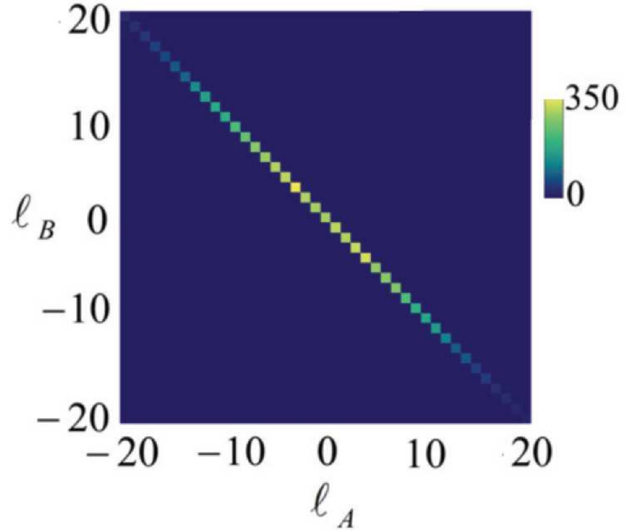
OAM entanglement

In terms of OAM modes: $$|\Psi\rangle_{\text{SPDC}} = \sum_{\ell} \alpha_\ell \, |\ell\rangle_A \, |-\ell\rangle_B$$

Why? In thin-crystal limit (crystal length $\ll$ pump Rayleigh range):

Three-way overlap:

$$\alpha_\ell = \int M_p(\mathbf{x}) M_s^*(\mathbf{x}) M_i^*(\mathbf{x}) \; d^2x$$



Assume pump is without OAM

$\Rightarrow$ the azimuthal integration:

$$\int_0^{2\pi} \exp(-i\ell_s\phi) \exp(-i\ell_i\phi) \; d\phi = \begin{cases} 2\pi \quad \text{for} \quad \ell_s = -\ell_i \\ 0 \quad \text{otherwise} \end{cases}$$

$\Rightarrow$ the azimuthal indices ℓ_s and ℓ_i are anti-correlated

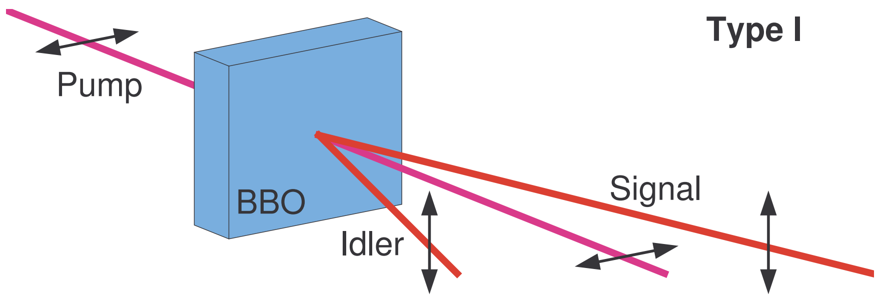
Phase matching conditions

Depends on:

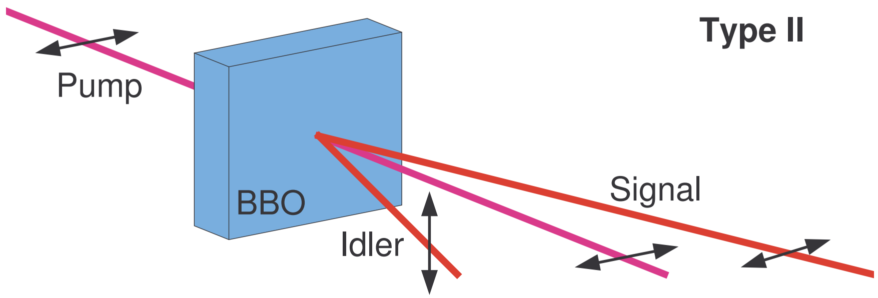
- ▷ Down-converted wavelength (usually degenerate)
- ▷ Dispersion properties (refractive index depends on wavelength)
- ▷ Birefringent medium (ordinary/extra-ordinary index)
- ▷ Polarization

Two types:

- ▷ Type I phase matching
  ⇒ down-converted photons have the same polarization (both are ordinary)

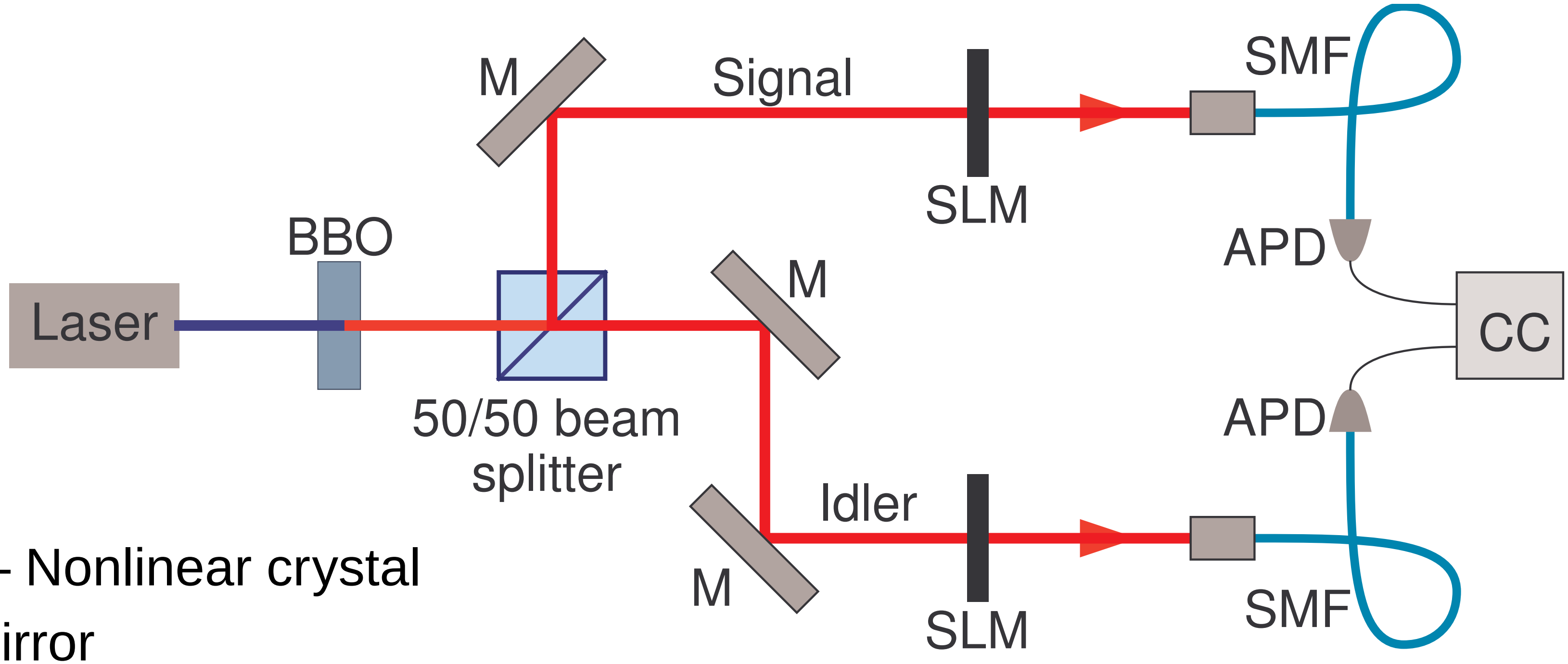


- ▷ Type II phase matching
  ⇒ down-converted photons have perpendicular polarizations (ordinary + extra-ordinary)



NLC experimental setup

Conditions: Type I, collinear and degenerate



BBO — Nonlinear crystal

M — Mirror

SLM — Spatial light modulator (reflective!)

SMF — Single mode fibre

APD — Avalance photo diode

CC — Coincidence counter

Wavelength filters — not shown

4f - imaging (2 lenses each) — not shown

Spatial light modulator

A spatial light modulator (SLM) is a kind of 'mirror' that introduces an arbitrary programmable phase function in the reflection of an incident optical beam

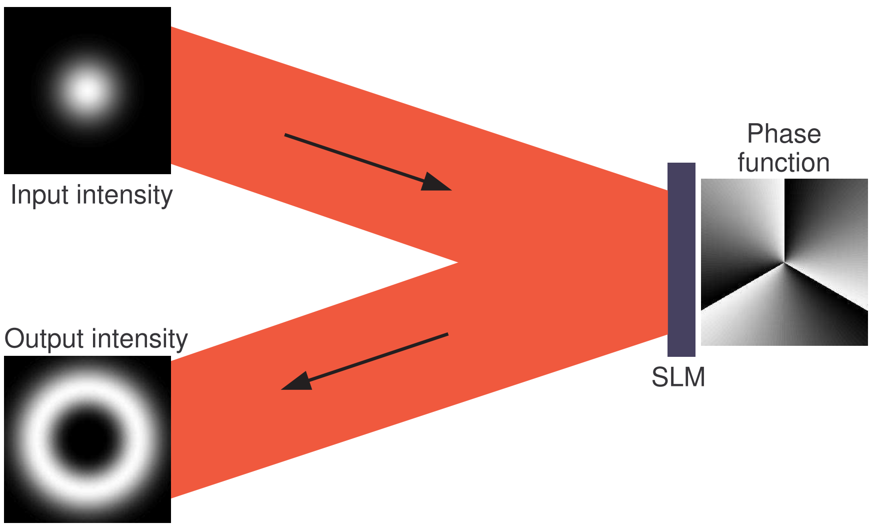
$$f(\mathbf{x}) \to f(\mathbf{x}) \exp[i\theta_{\mathrm{SLM}}(\mathbf{x})]$$

$f(\mathbf{x})$ — complex amplitude of input beam
θ_{SLM} — programmable phase function on the SLM

Example: helical phase with $\ell = 3$.

SLMs are the main controlling devices in our experiments.

Their versatility makes them very powerful.



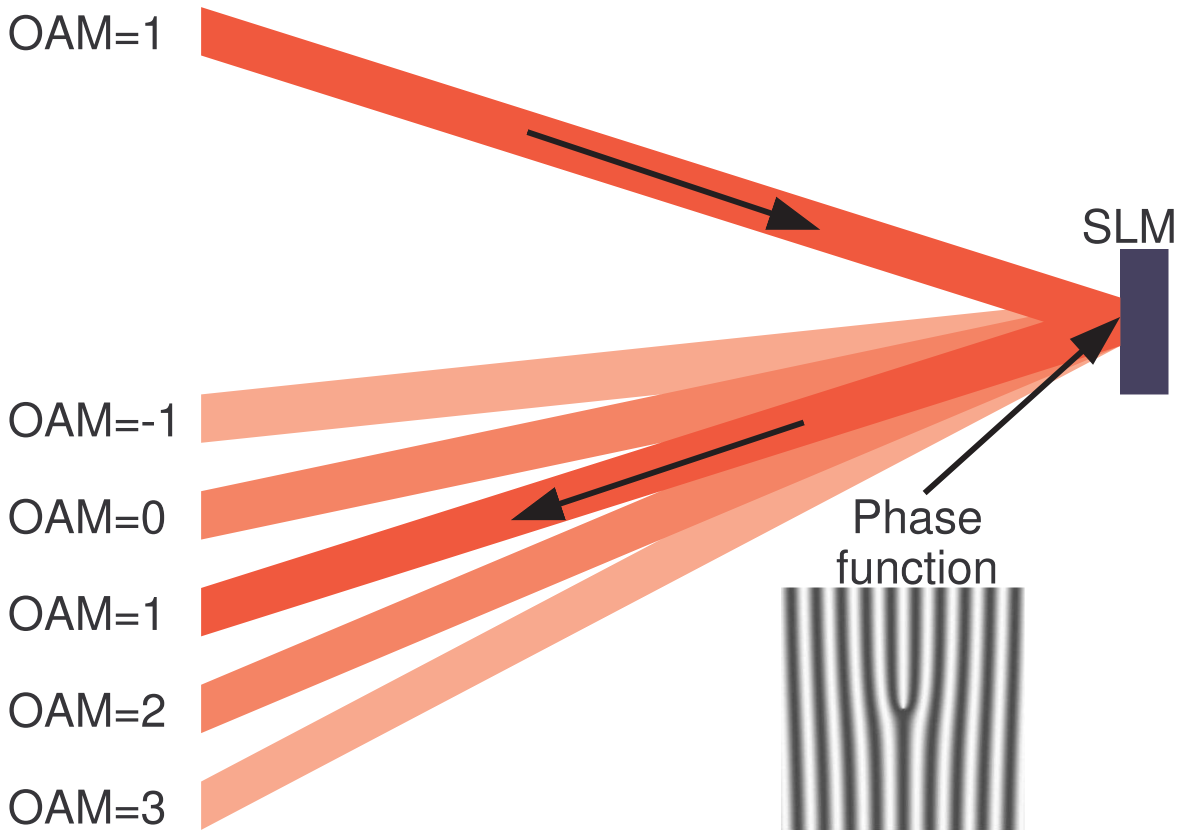
Phase grating

Add a phase grating to the helical phase (OAM$=\ell$) on the SLM

Produce different diffraction orders: $n = ..., -2, -1, 0, 1, 2, ...$

Grating equation: $$\sin(\theta_{\text{out}}) = \sin(\theta_{\text{in}}) + \frac{n\lambda}{d}$$

Each order adds (or subtracts) OAM=$n\ell$



Single mode optical fibre

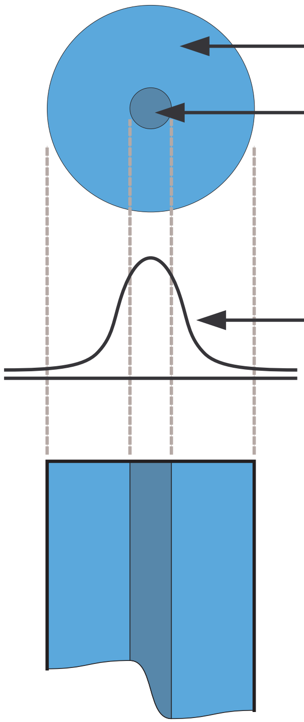
Mode of single mode fibre (SMF) is approximately Gaussian.

Coupling efficiency is computed by overlap integral:

$$\eta_{\mathrm{SMF}} = \int M_{\mathrm{in}}(\mathbf{x}) M^*_{\mathrm{SMF}}(\mathbf{x})\, d^2x$$

Azimuthal integration:

$$\int_0^{2\pi} \exp(i\ell_{\mathrm{in}}\phi)\, d\phi = \begin{cases} 2\pi & \text{for} \quad \ell_{\mathrm{in}} = 0 \\ 0 & \text{otherwise} \end{cases}$$

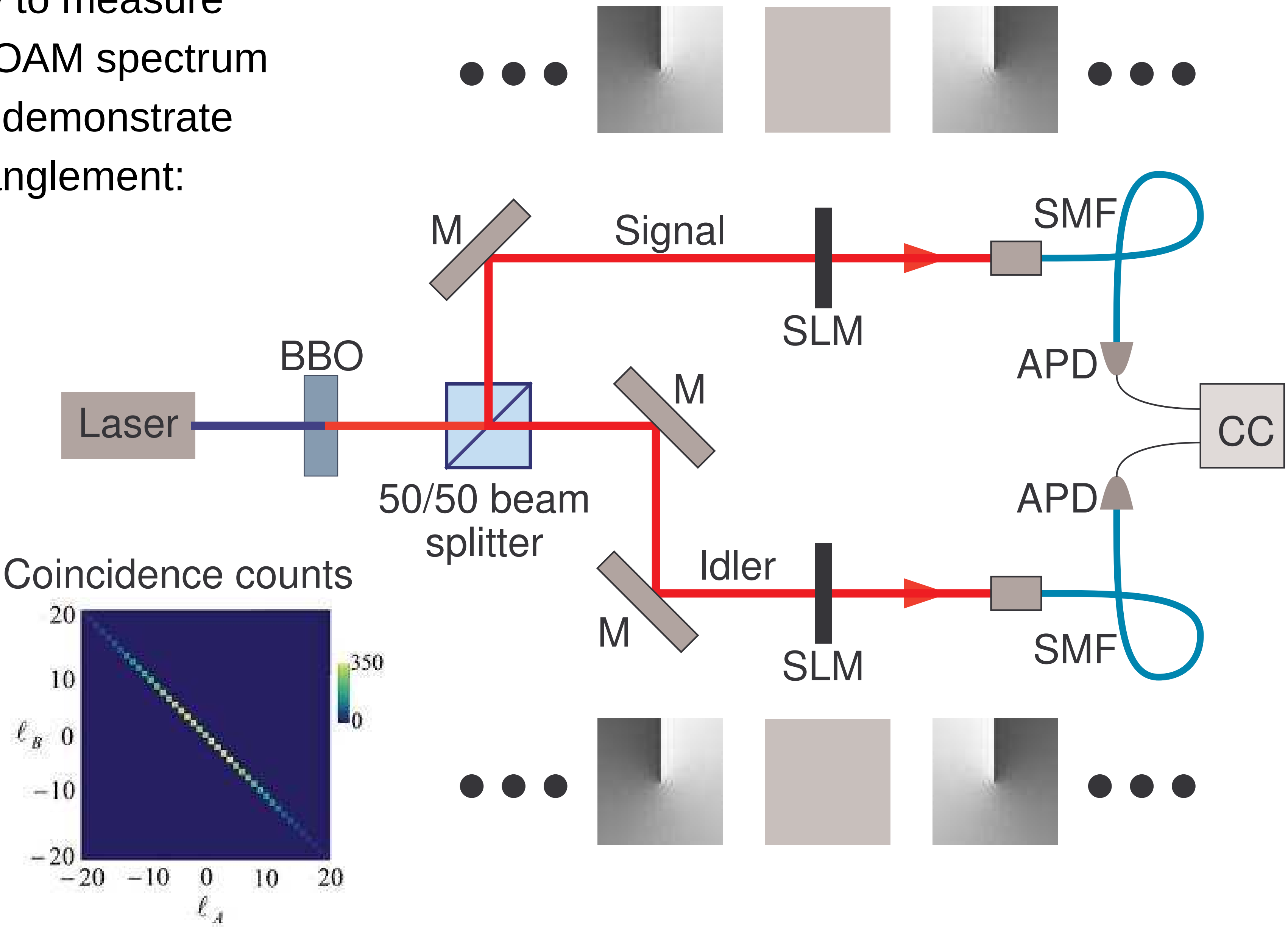


$\Rightarrow$ only if diffraction order has $\ell_{\mathrm{in}} = 0$ will the light couple into the SMF.

Therefore, SLM + SMF gives one the ability to measure the OAM of an input beam.

OAM spectrum

How to measure
the OAM spectrum
and demonstrate
entanglement:



Quantum state

Pure single photon state: $|\psi\rangle = \alpha\,|a\rangle + \beta\,|b\rangle$

where $|\alpha|^2 + |\beta|^2 = 1$ and $\langle a|b\rangle = 0$.

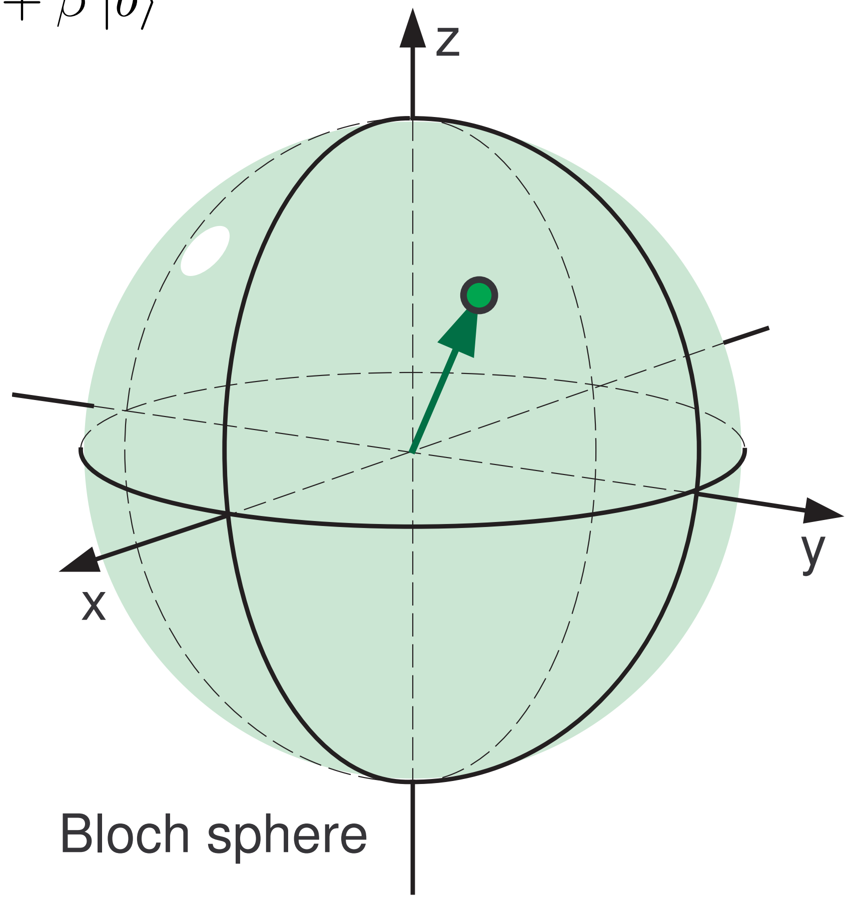
All single photon qubit states lie on the Bloch sphere.

For bi-photon, multi-partite or higher-dimensional states (qudits) the state space is more complex.

Density operator: $\rho = |\psi\rangle\,\langle\psi|$

Mixed state: $$\rho = \sum_n |\psi_n\rangle\, P_n\, \langle\psi_n|$$

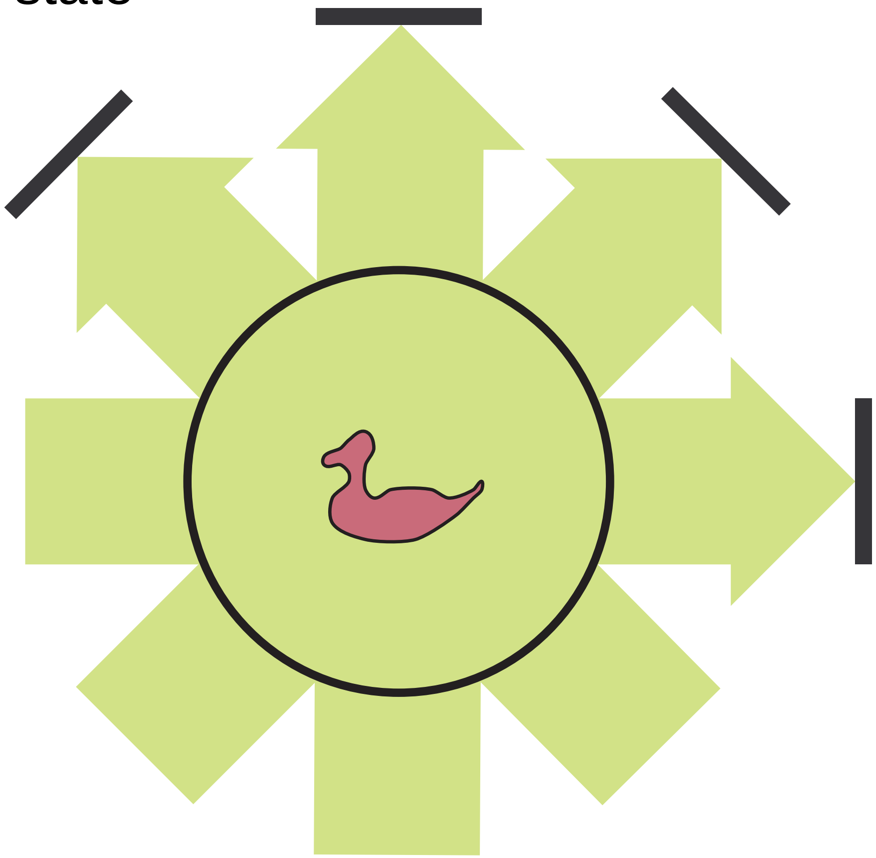
P_n — probabilities



Bloch sphere

Quantum state tomography

To quantify the entanglement of a state,
one needs to know the exact quantum state
$\Rightarrow$ Quantum state tomography

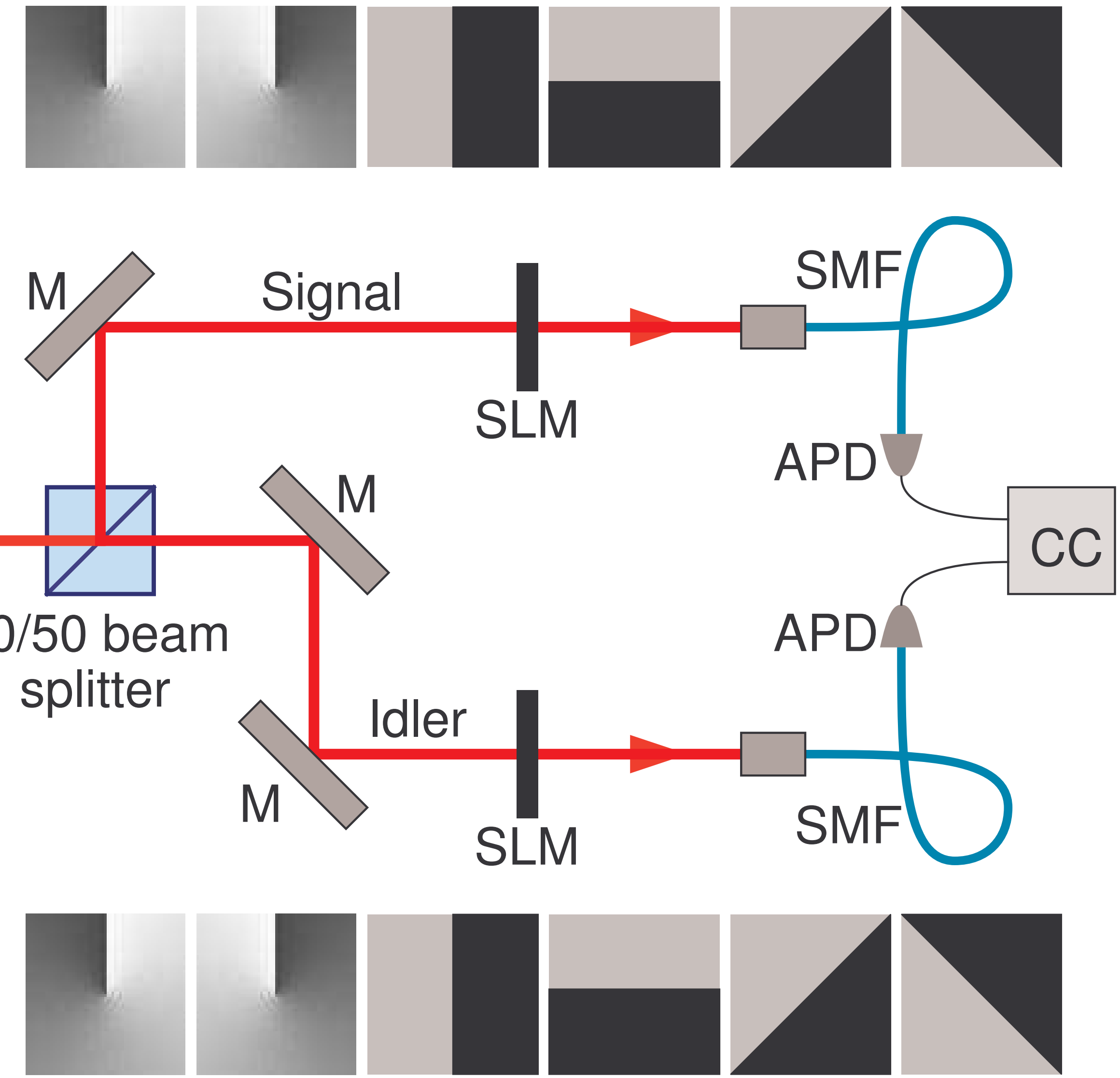


Tomography is a process whereby one reconstructs an object from different observations of the object.

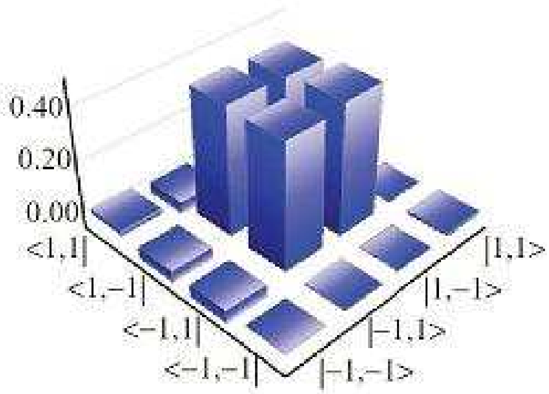
Quantum state tomography:
reconstructs a quantum state ρ from different observations $\mathrm{tr}\{\mathcal{P}_n \rho\}$ where $\mathcal{P}_n$ are different projections.

NLC tomography experiment

Full quantum state tomography
for qubit (or qudit) states
in terms of OAM:

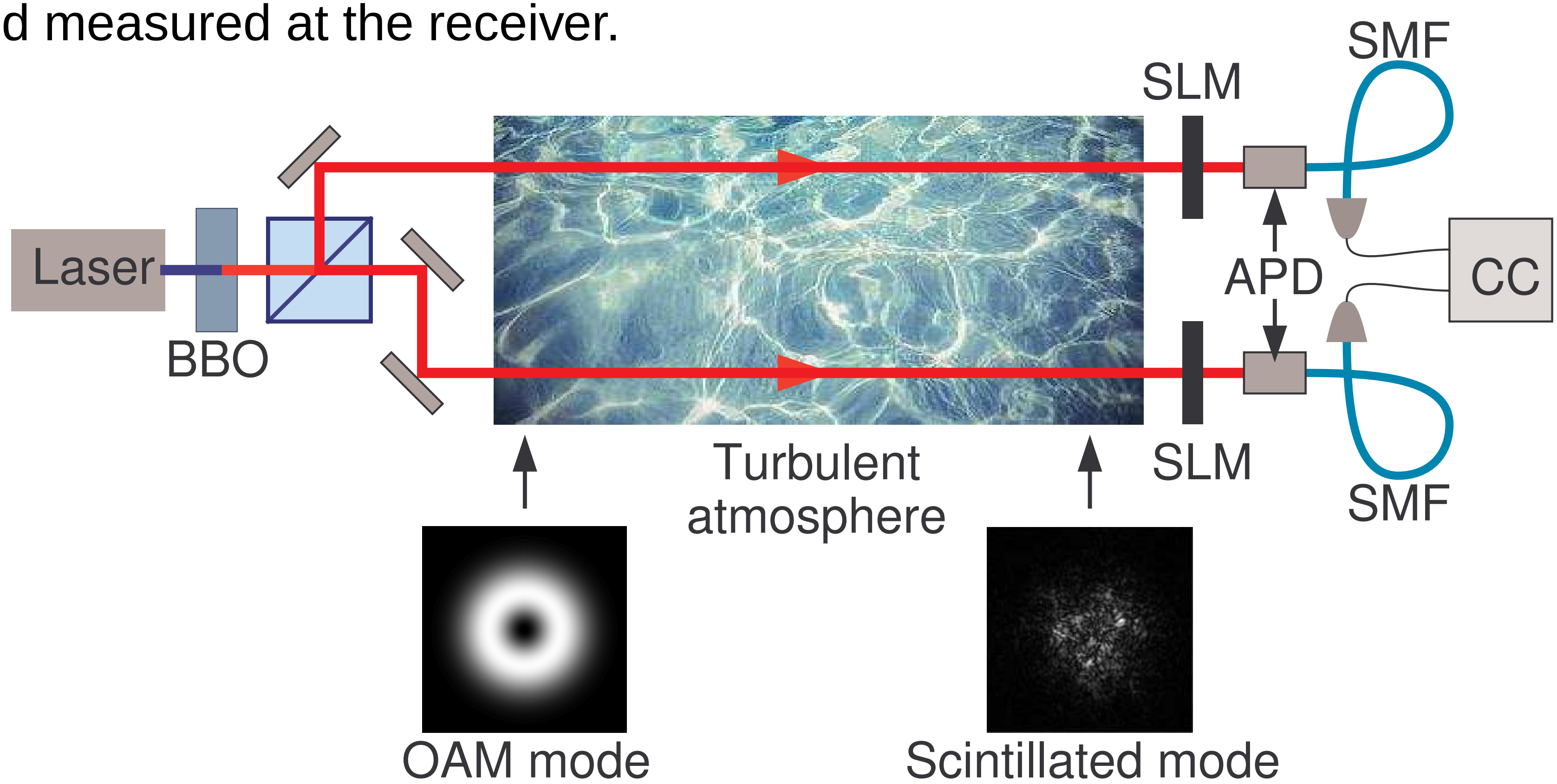


Qubit density matrix



Free-space quantum communication

The entangled photon pair is sent through the atmosphere and measured at the receiver.

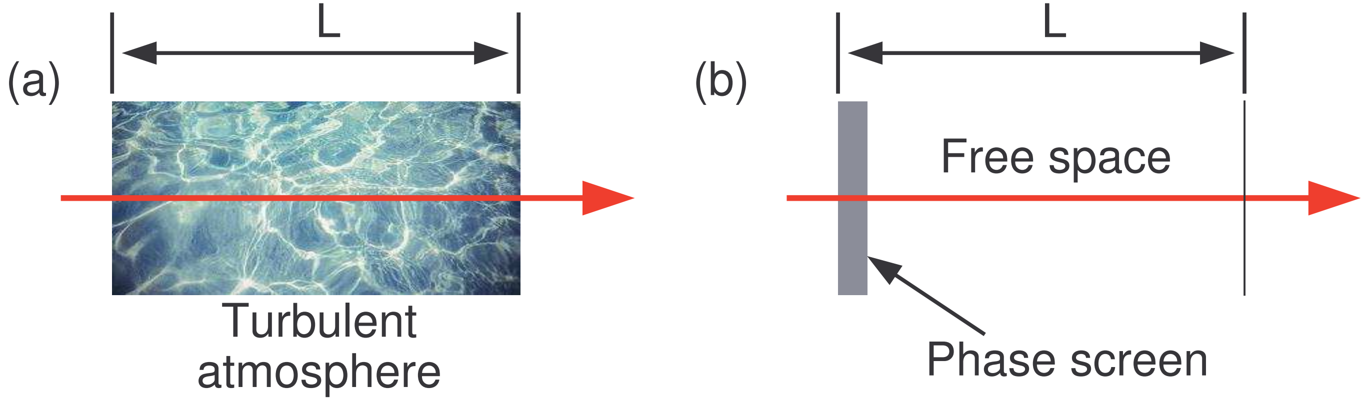


Turbulence distorts the OAM modes

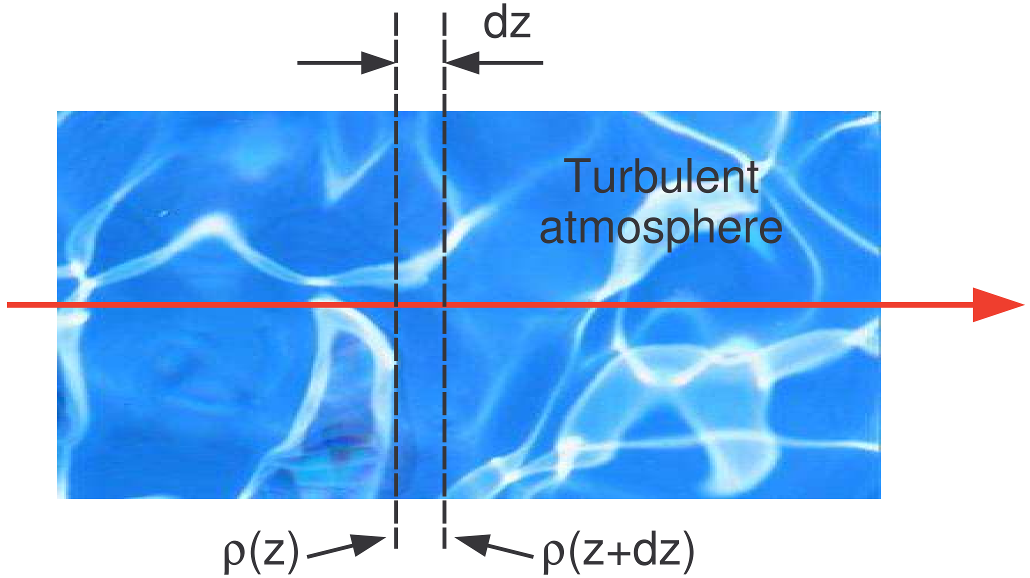
$\Rightarrow$ loss of entanglement

Single or multiple phase screen(s)

Single phase screen approach:

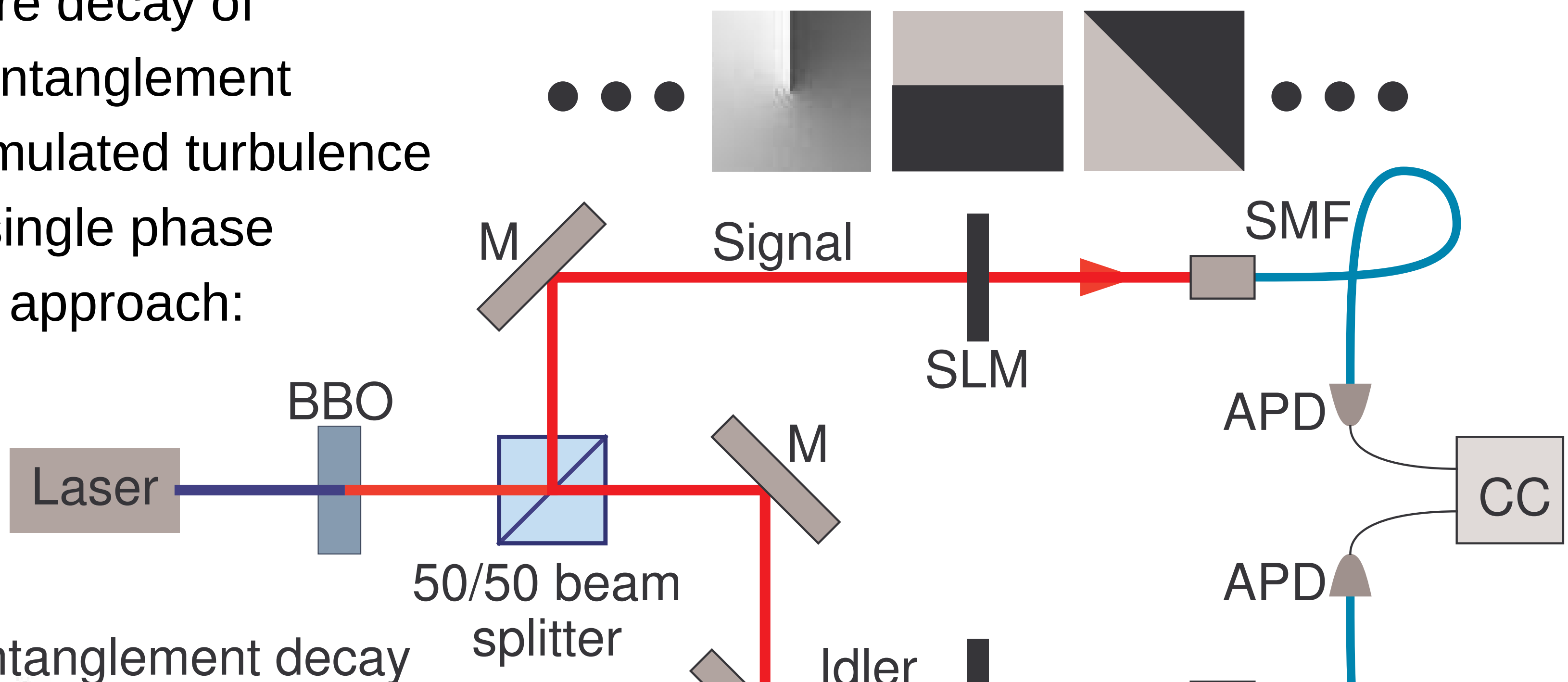


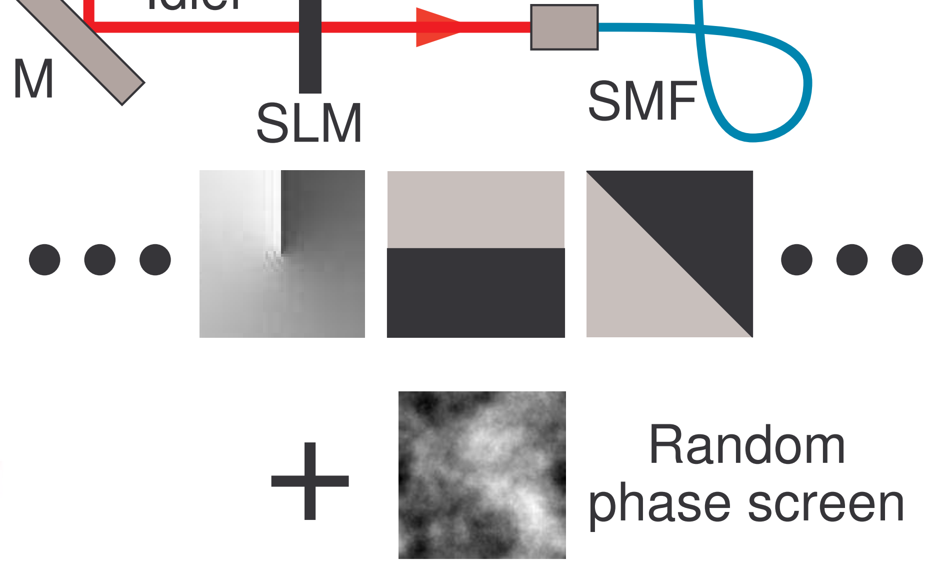
Multiple phase screen approach:



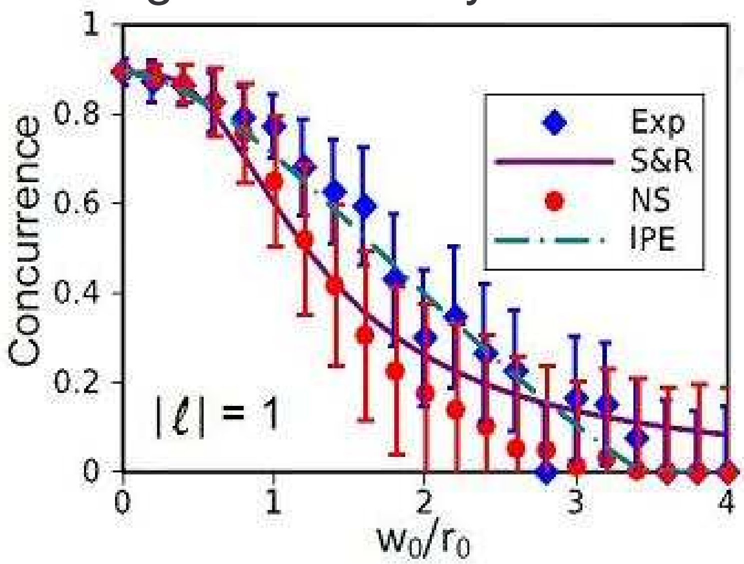
NLC turbulence experiment

Measure decay of OAM entanglement with simulated turbulence using single phase screen approach:



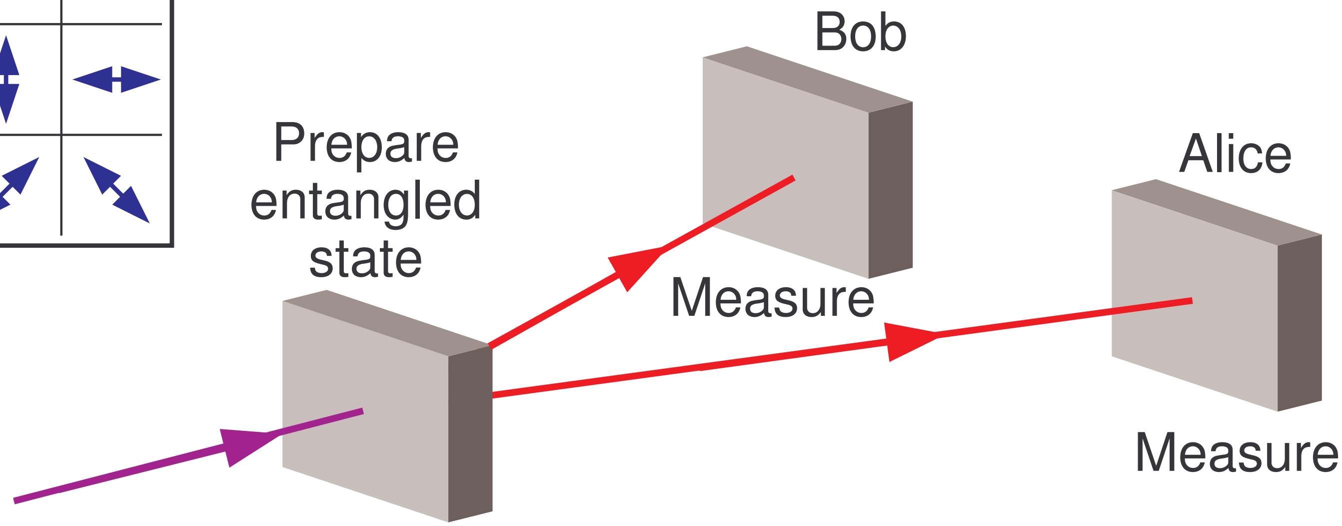


Entanglement decay



Ekert 91 protocol

Basis	0	1
+	↕	↔
×	⤢	⤡



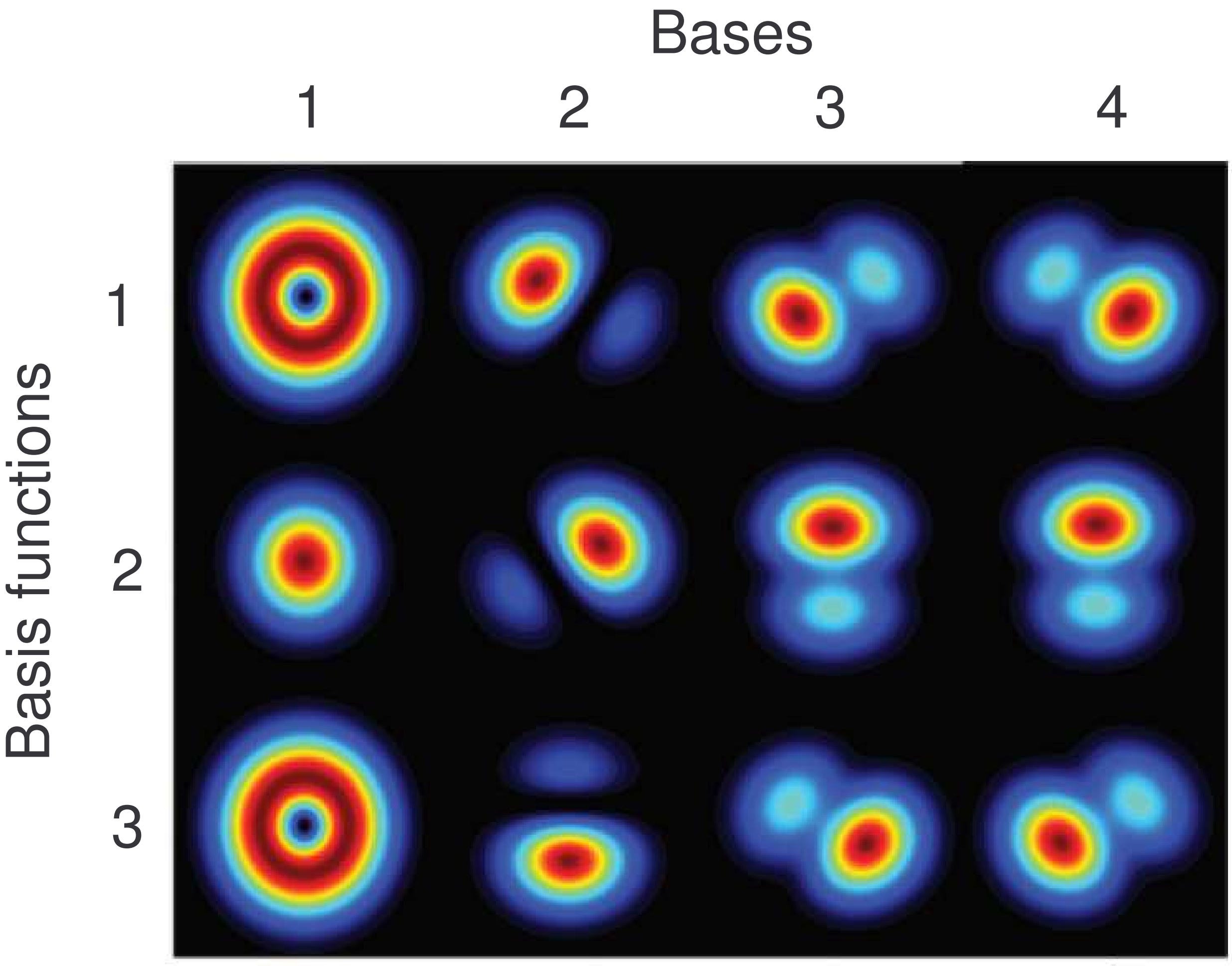
	1	2	3	4	5	6
Alice	+	+	×	+	×	+
	↔	↕	⤢	↕	⤡	↔
Bob	+	×	×	+	×	×
	↔	⤡	⤢	↕	⤡	⤡
Key	1		0	0	1	

	7	8	9	10	11	12
Alice	+	×	×	+	×	+
	↕	⤢	⤡	↔	⤢	↕
Bob	+	×	+	+	×	×
	↕	⤢	↕	↔	⤢	⤡
Key	0	0		1	0	

Mutually unbiased bases

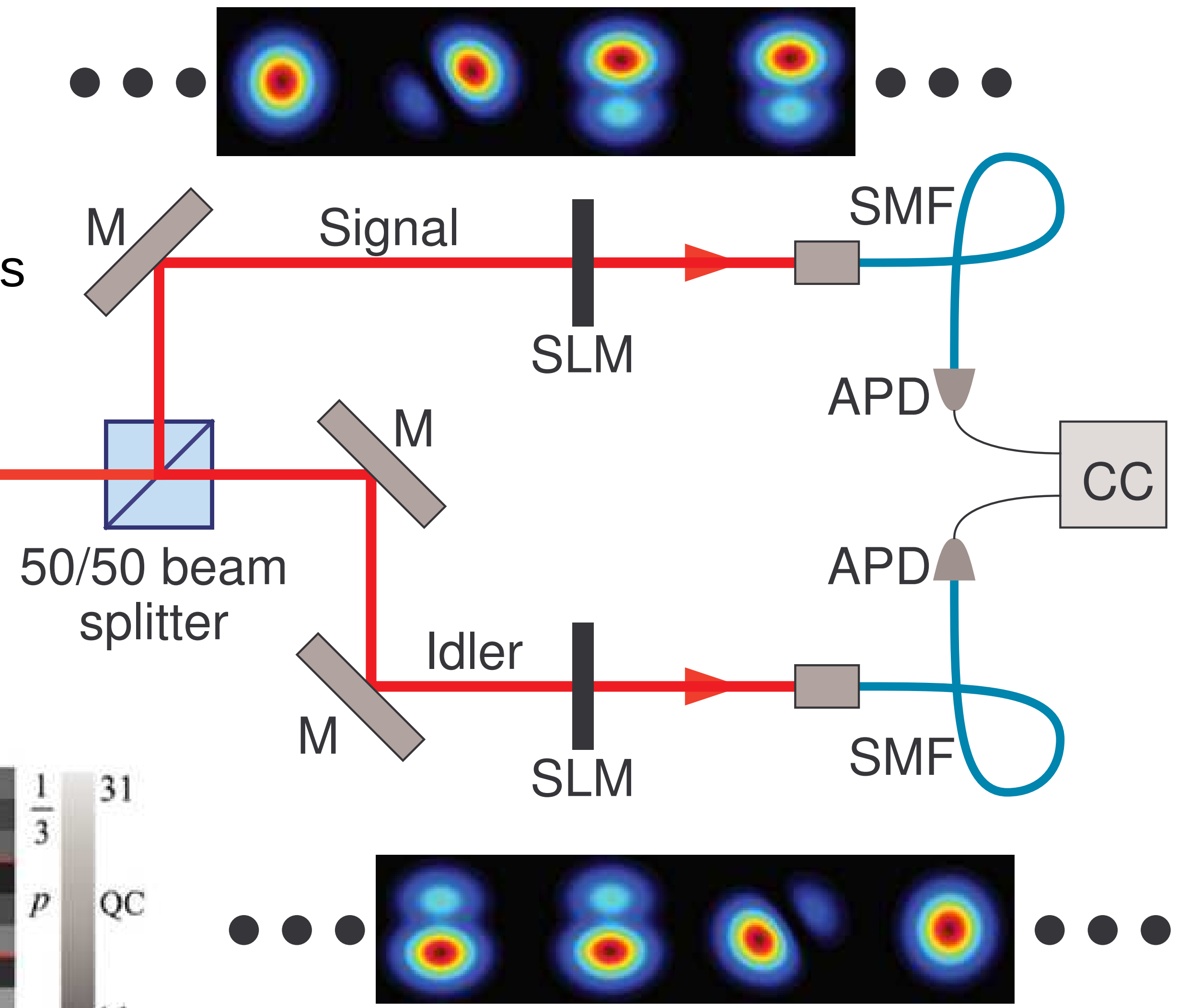
QKD in higher dimensions needs
mutually unbiased bases in higher dimensions

$$|\langle\phi_{a,n}|\phi_{b,m}\rangle|^2 = \frac{1}{d} \quad \text{for} \quad a \neq b$$

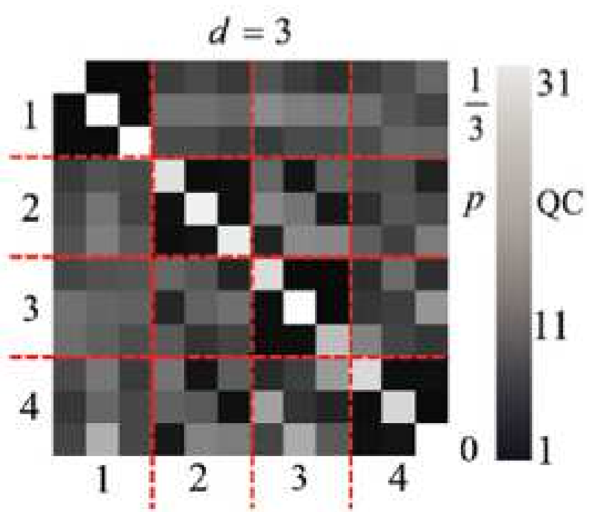


NLC QKD experiment

Perform quantum key distribution
by measuring
the coincidence counts
for different
mutually unbiased bases

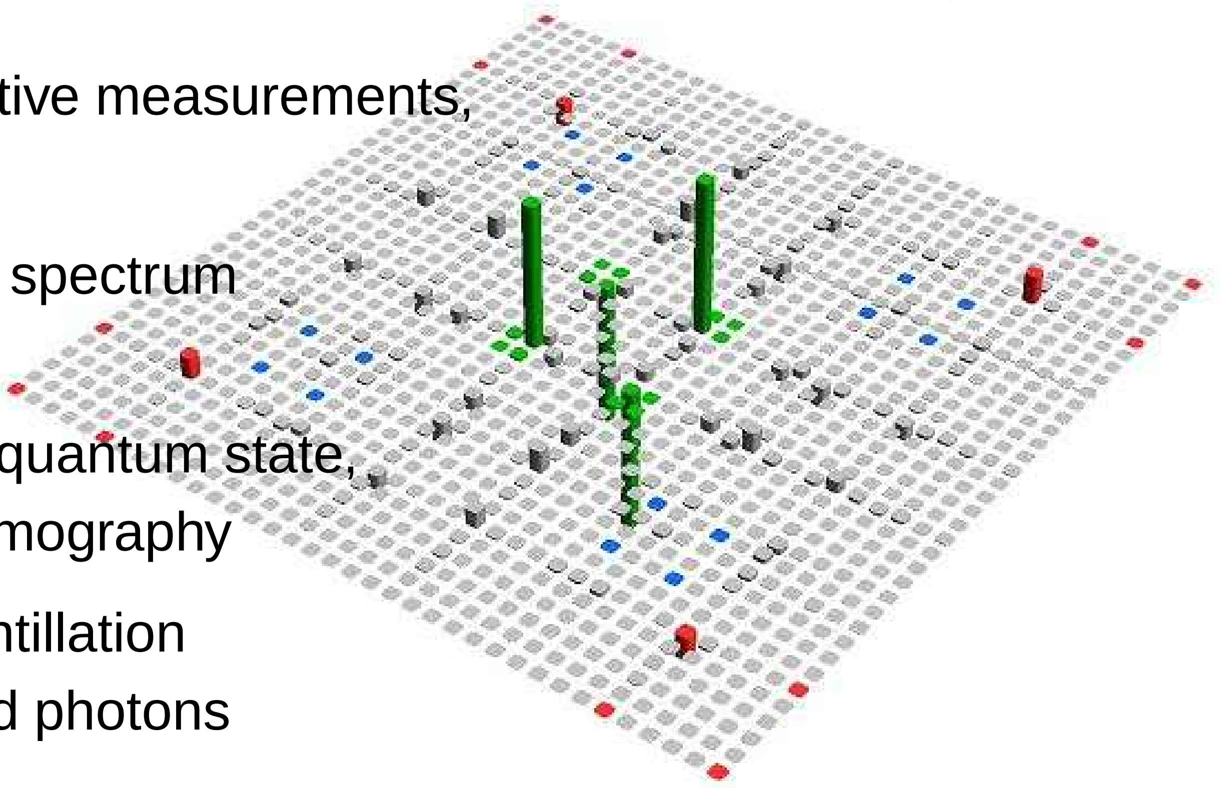
Coincidence counts



Summary

At the NLC, we ...

- ... prepare OAM entangled photons, using SPDC
- ... perform projective measurements, using SLM +SMF
- ... measure OAM spectrum of SPDC output
- ... determine the quantum state, using full state tomography
- ... investigate scintillation of OAM entangled photons
- ... etc.



Stay tuned!