General aspects of optical vortices

F. Stef Roux

CSIR National Laser Centre

PO Box 395, Pretoria 0001, South Africa

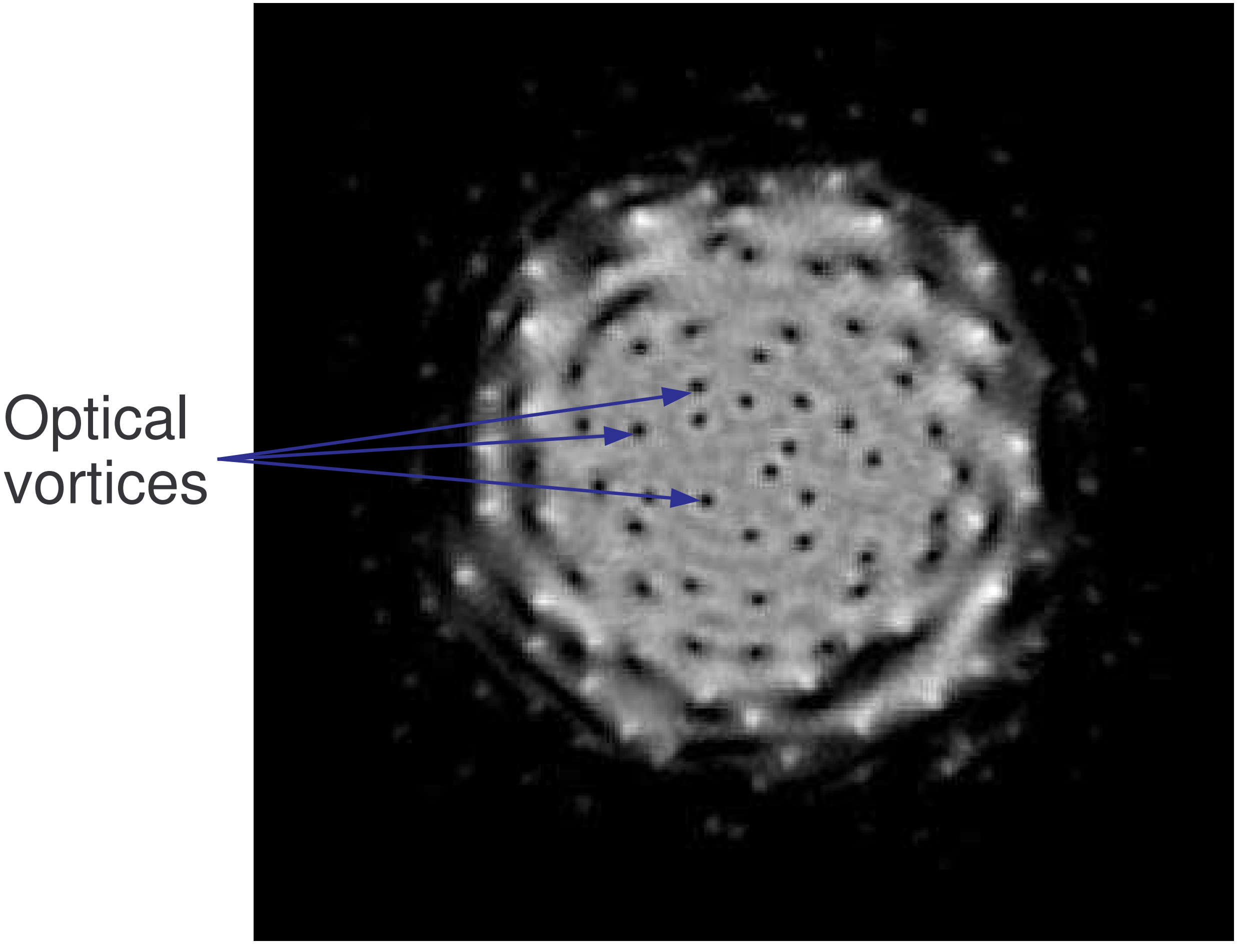
our future through science

Contents

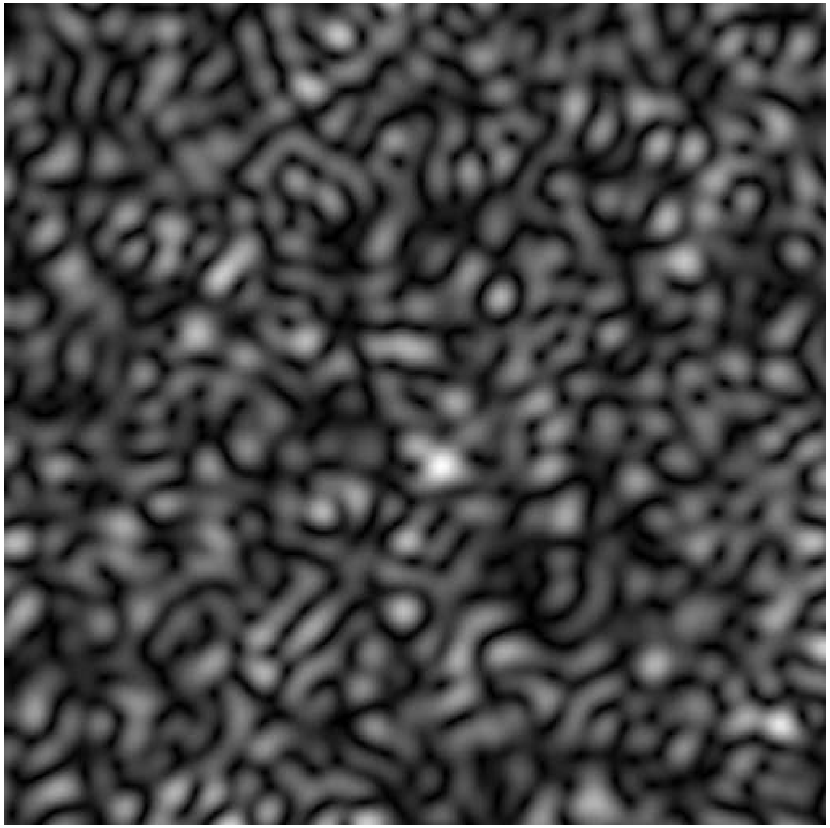
- ▷ Definition of an optical vortex
- ▷ Topological charge and vortex morphology
- ▷ How to detect a vortex — interferometry
- ▷ How to generate optical vortices

our future through science

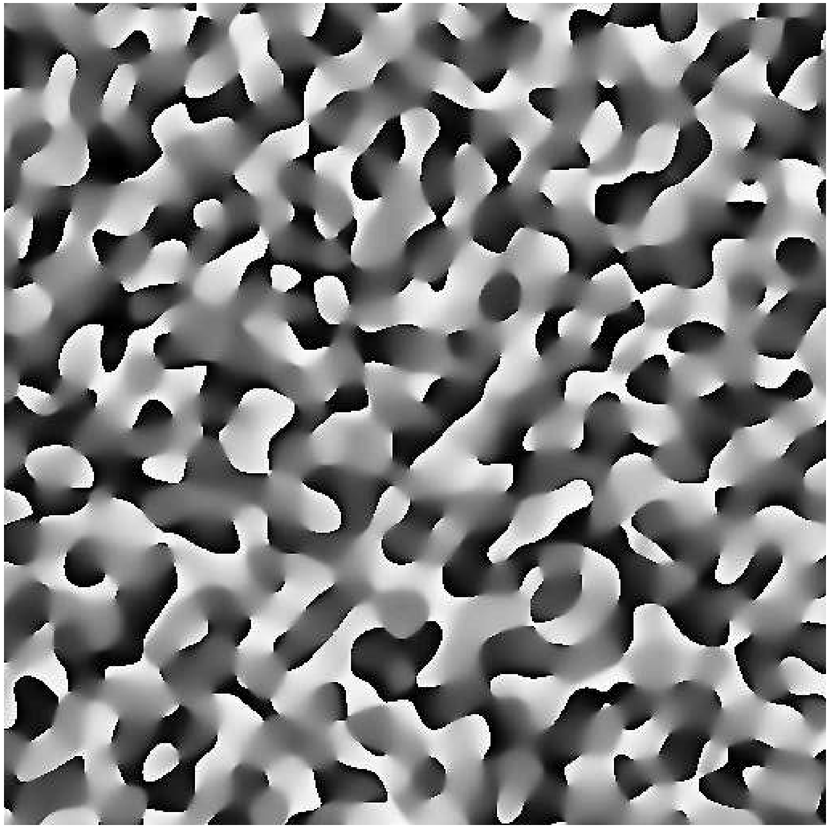
Persistent dark spots



Speckle field

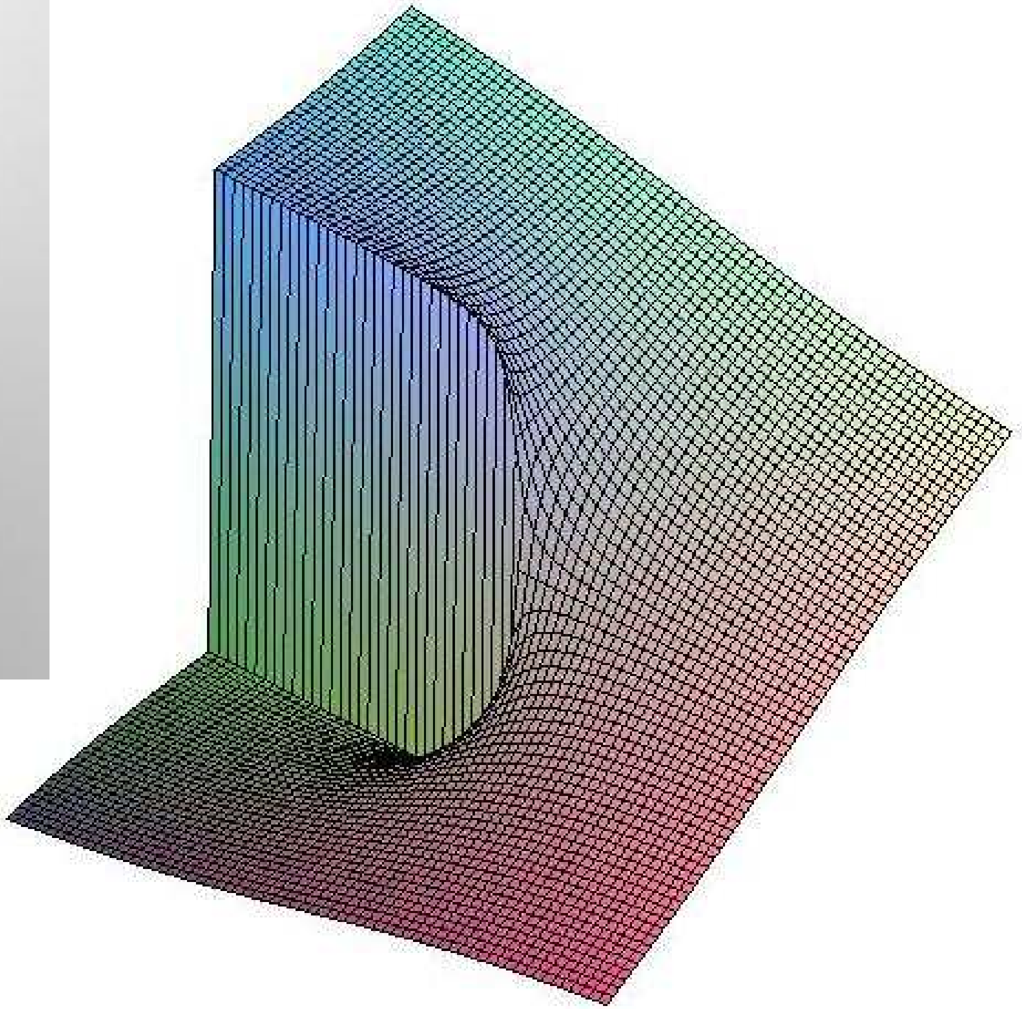


Amplitude



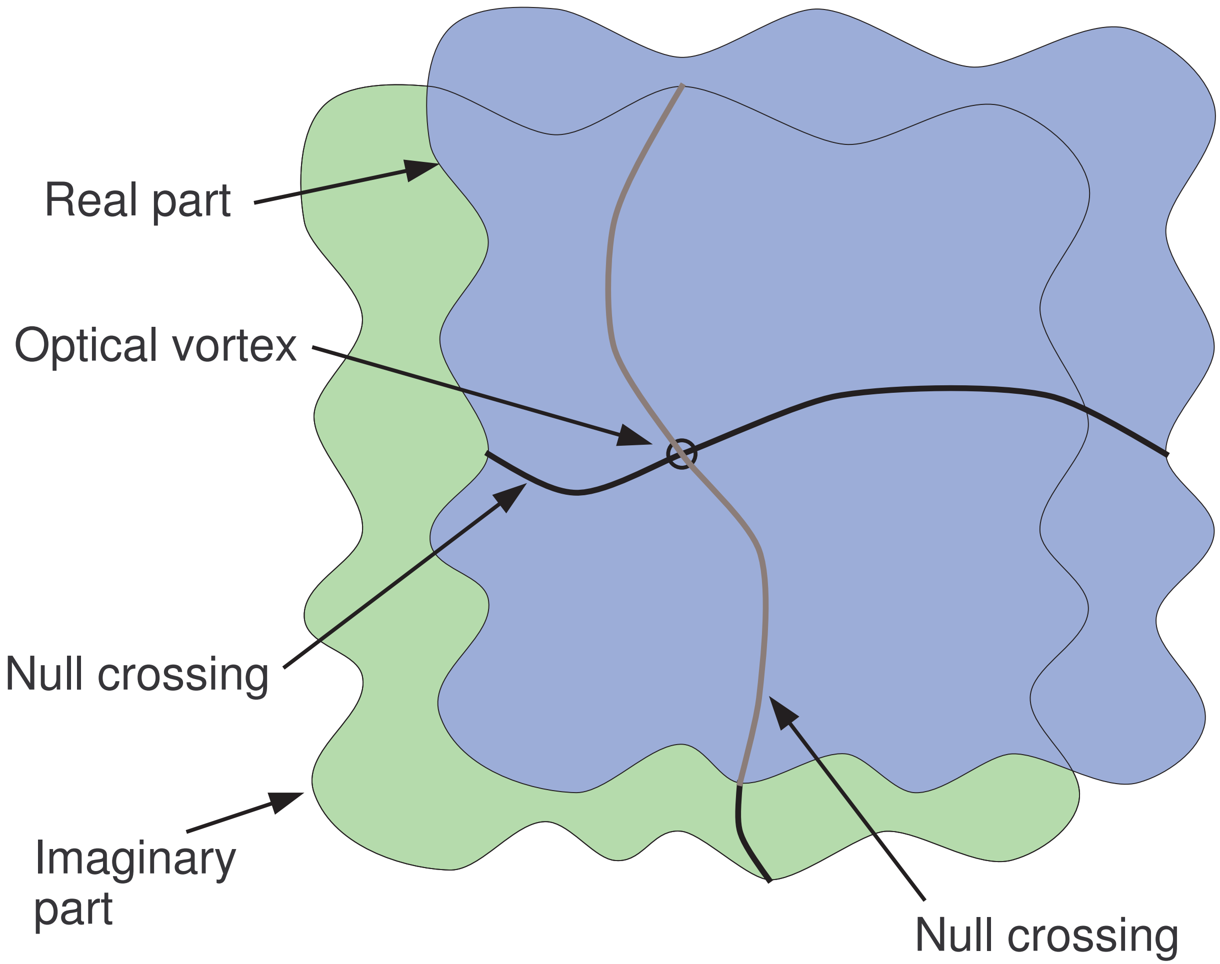
Phase

Singular phase function



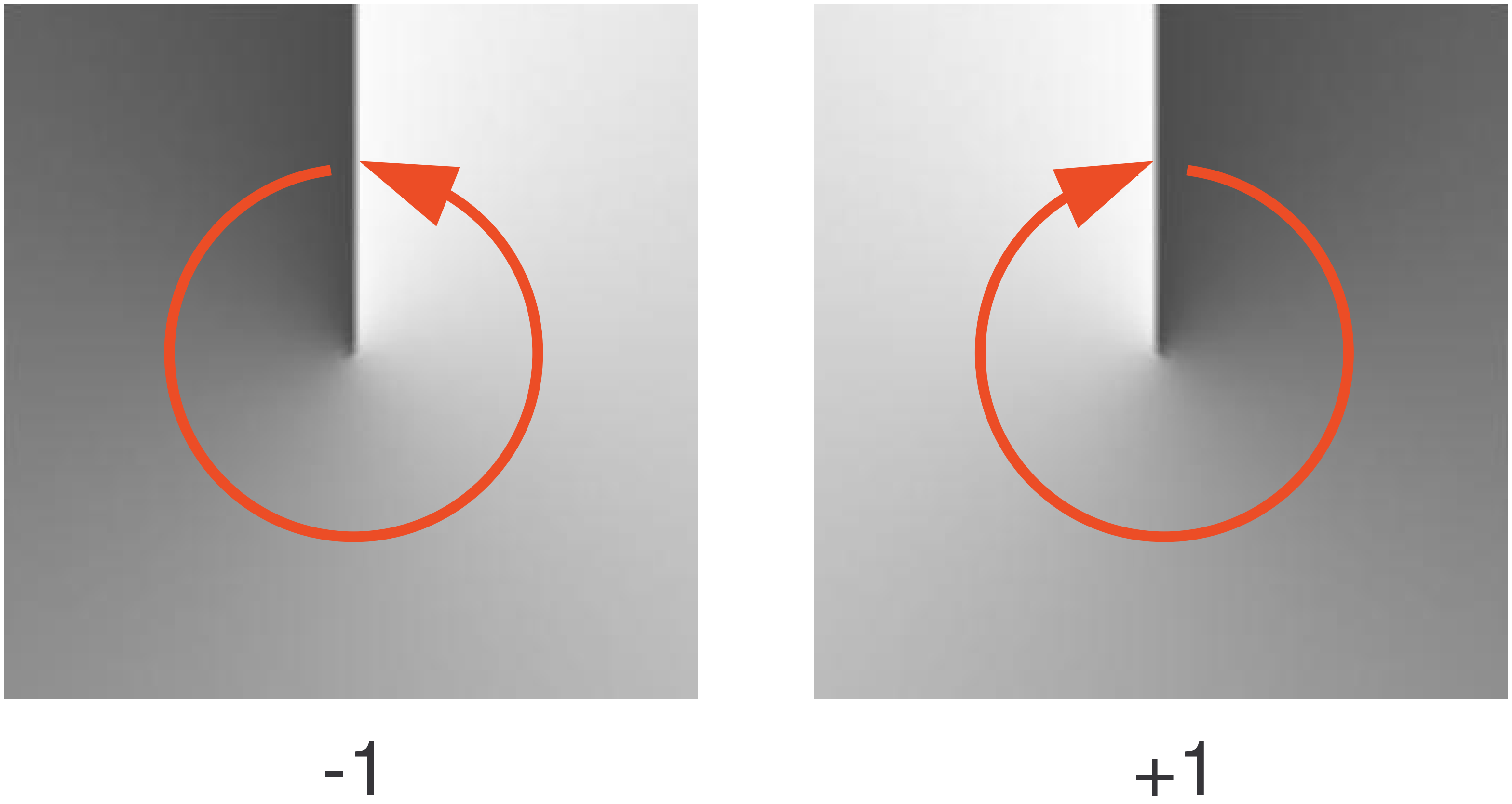
our future through science

Null crossings



our future through science

Topological charge



Expression of a canonical vortex

Basic complex valued function for a vortex (at origin):

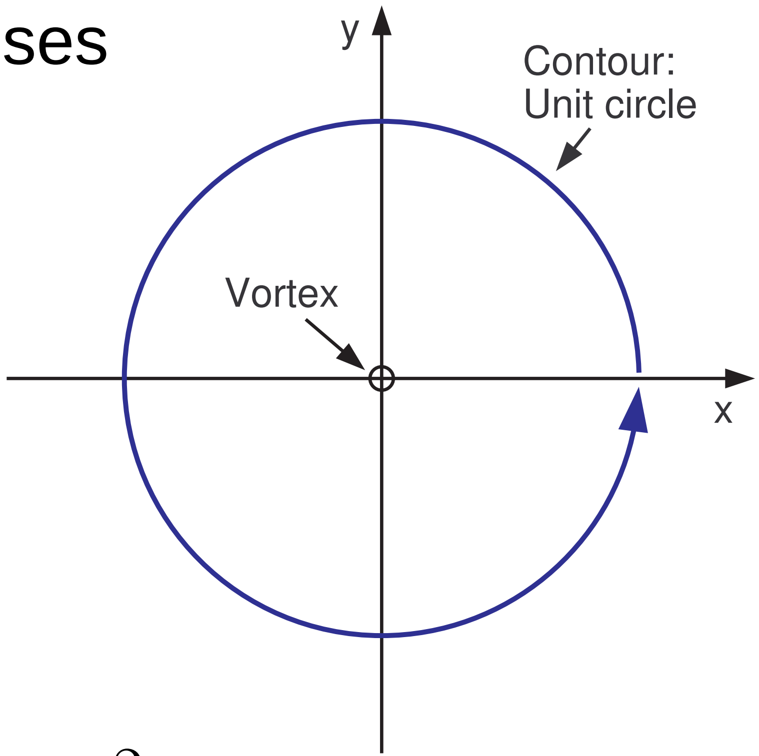
Positive: $V_+(x,y) = x + iy = \rho \exp(i\phi)$

Negative: $V_-(x,y) = x - iy = \rho \exp(-i\phi)$

our future through science

Topological charge integral

Index integral:
Integrate gradient of the phase function around a closed contour that encloses the phase singularity.



$$\oint_C \nabla\theta(x,y)\cdot\hat{\mathrm{d}}s = \nu\ 2\pi$$

Topological charge computation

$$\theta(x,y) = -i\ln\left(\frac{f(x,y)}{|f(x,y)|}\right) \Rightarrow \theta_+(\rho,\phi) = \phi$$

$$\nabla\theta = \nabla\phi = \frac{1}{\rho}\hat{\phi} \quad \text{and} \quad \hat{\mathrm{d}}s = \rho\hat{\phi}\mathrm{d}\phi$$

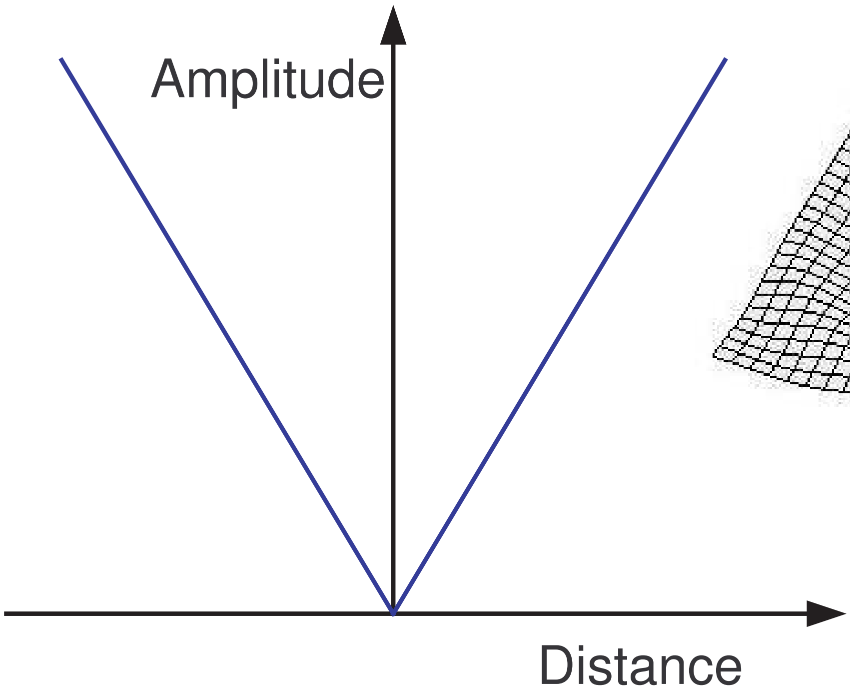
$$\oint \nabla\theta \cdot \hat{\mathrm{d}}s = \int_0^{2\pi} \mathrm{d}\phi = 2\pi$$

$\Rightarrow$ Topological charge: $\nu = +1$

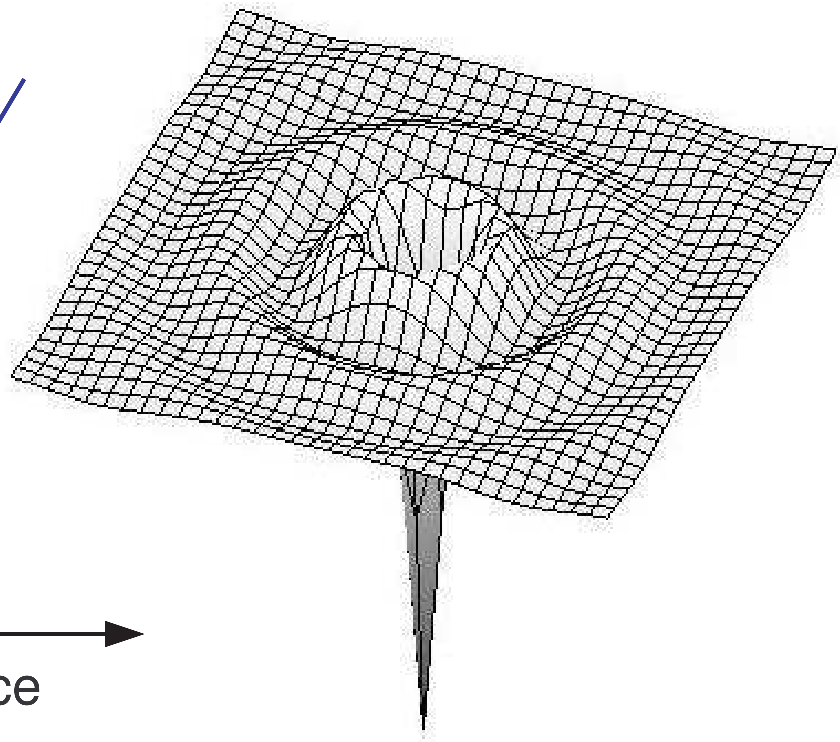
our future through science

Profile function

Linear profile function:



Profile function after CGH:



our future through science

Laguerre-Gaussian beams

$$\mathrm{LG}_{nm}(r,\phi,t) = K \exp(im\phi) L_n^{|m|}\left(\frac{2r^2}{1+t^2}\right) \exp\left(\frac{-r^2}{1-it}\right)$$

Normalised coordinates:

$$(u,v,t) = \left(\frac{x}{\omega_0}, \frac{y}{\omega_0}, \frac{z}{\rho} = \frac{z\lambda}{\pi\omega_0^2}\right)$$

$L_n^{|m|}$ — ass. Laguerre polynomials; n — nonzero integer
m — signed integer (Topological charge)
K — normalisation constant:

$$K = \frac{r^{|m|}(1+it)^n}{(1-it)^{n+|m|+1}} \sqrt{\frac{n!2^{m+1}}{\pi(n+m)!}}$$

our future through science

Topological charge and OAM

For $m \neq 0$ the Laguerre-Gaussian beam has an optical vortex on the axis with topological charge m.

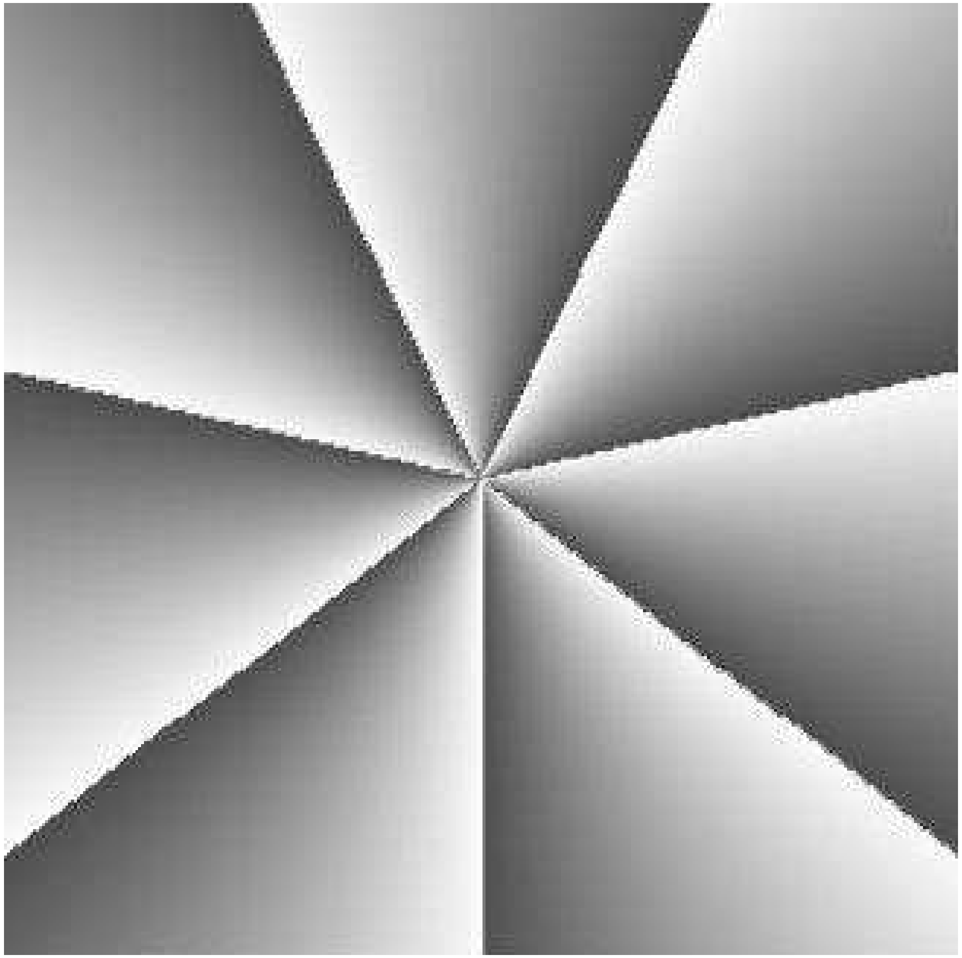
The Laguerre-Gaussian beams carry orbital angular momentum (OAM), which can be transfered to small particles.

The amount of OAM is proportional to the intensity and proportional to m.

our future through science

Higher order topological charge

One can have vortices with arbitrary high topological charge, but they are not stable.



General complex expression

Taylor series expansion of arbitrary complex valued function around a (1st order) vortex at (x_0, y_0):

$$f(x,y) = 0 + a_x(x - x_0) + a_y(y - y_0) + \ldots$$

Complex valued coefficients:

$$a_x = a_{xr} + i a_{xi} \qquad a_y = a_{yr} + i a_{yi}$$

$\Rightarrow$ 6 real valued parameters for each vortex:

$$a_{xr}, a_{xi}, a_{yr}, a_{yi}, x_0, y_0$$

our future through science

Parameterisation

Remove global amplitude and global phase:

$$a_x x + a_y y = A\exp(i\Omega)\left[\xi(x+iy)+\zeta(x-iy)\right]$$

where $|\xi|^2 + |\zeta|^2 = 1$

For canonical vortices:
$\xi = 1$ and $\zeta = 0$ ($\nu = +1$) or $\xi = 0$ and $\zeta = 1$ ($\nu = -1$)

Other cases: noncanonical

our future through science

Vortex morphology

Shape (morphology) of the vortex is given by ξ and ζ. Analogues to Jones vectors (polarisation):

$$\eta = \begin{bmatrix} \xi \\ \zeta \end{bmatrix} = \begin{bmatrix} \cos(\psi/2)\exp(i\beta/2) \\ \sin(\psi/2)\exp(-i\beta/2) \end{bmatrix}$$

Morphology angles:

- $0 \leq \psi \leq \pi$ — helicity ($\cos\psi$)
- $0 \leq \beta < 2\pi$ — orientation

CSIR
our future through science

Noncanonical vortex notation

Shifted noncanonical vortex in helical coordinates:

$$V = \xi \left(w - w_0 \right) + \zeta \left(\overline{w} - \overline{w}_0 \right)$$

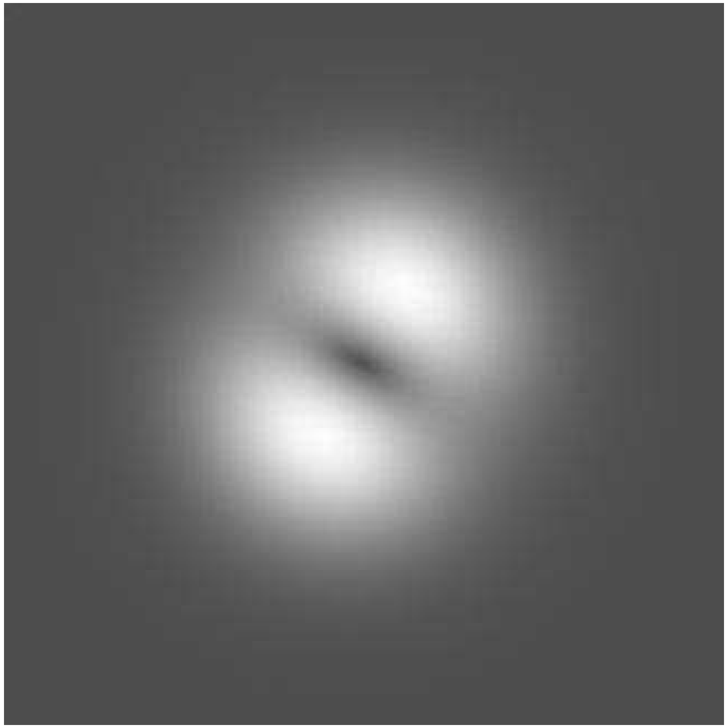
Helical coordinates: $w = u + iv$ and $\overline{w} = u - iv$

Vortex location in terms of helical coordinates:
$w_0 = u_0 + iv_0$ and $\overline{w}_0 = u_0 - iv_0$

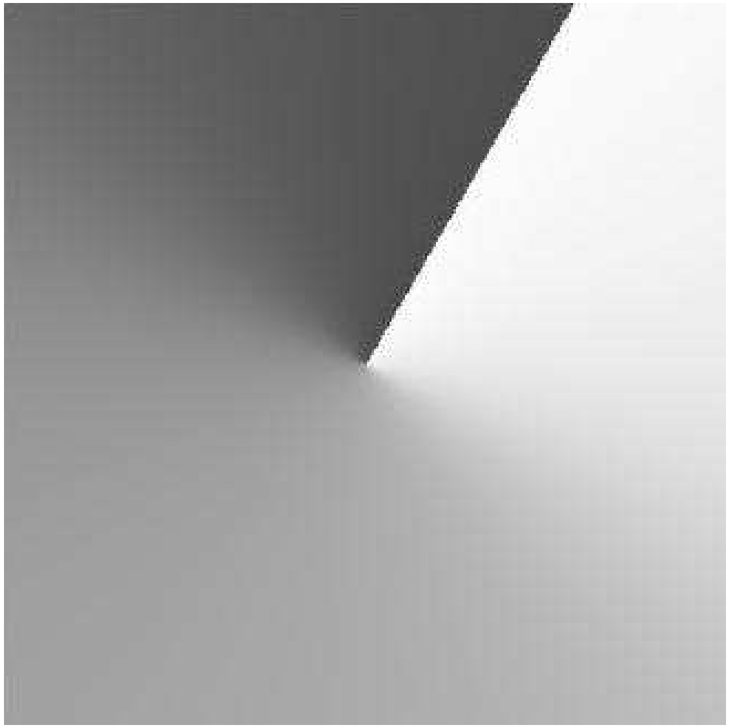
CSIR
our future through science

Noncanonical vortex example

Amplitude



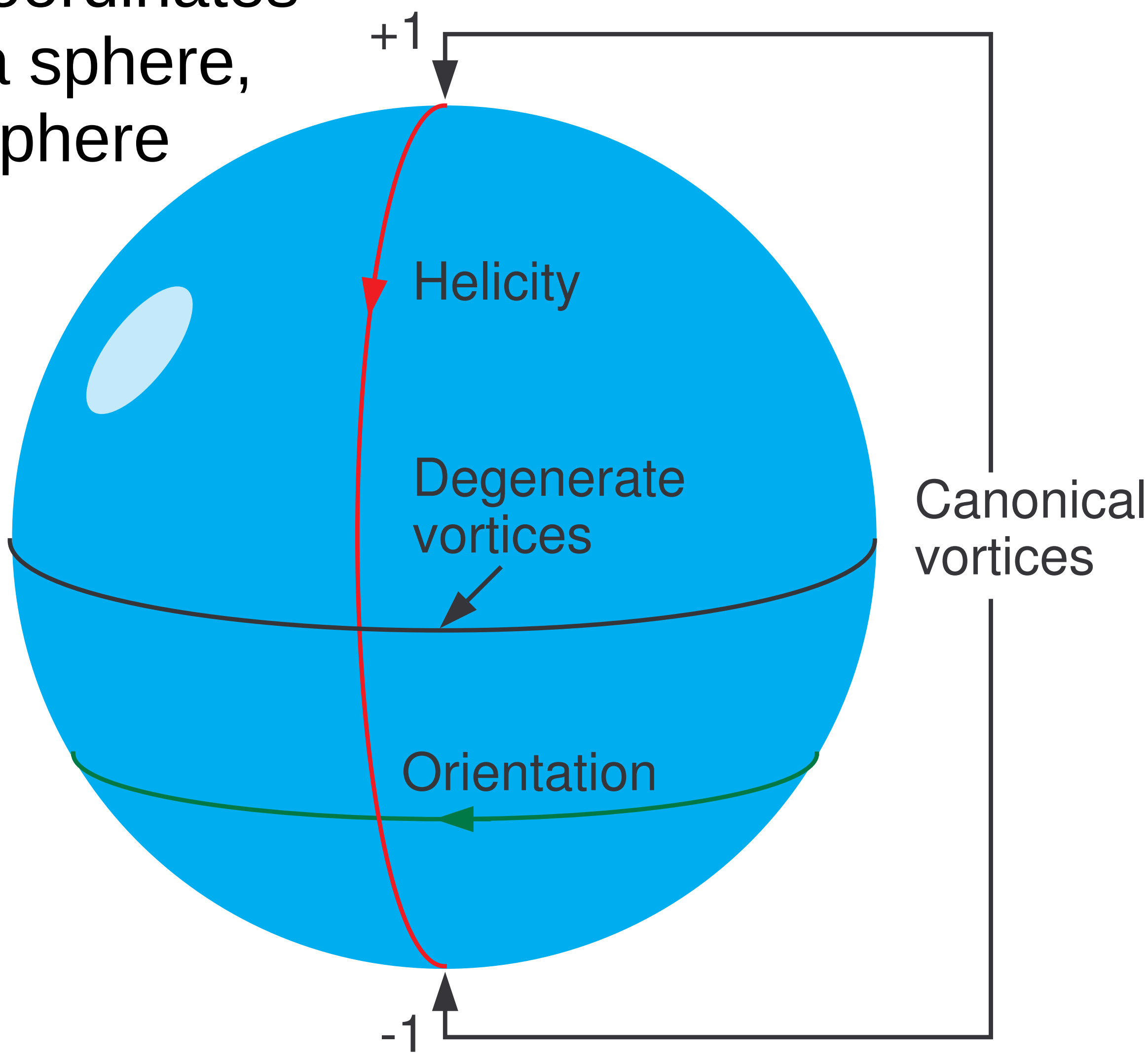
Phase



our future through science

Morphology representation

The morphology angle can be seen as angular coordinates for the surface of a sphere, like the Poincaré sphere for polarisation.



Extracting morphology

Given an arbitrary complex function $f(x, y)$ one can extract the morphology of any vortex.
Define vortex derivatives:

$$\partial_{\pm} f = \frac{1}{2}\left(\frac{\partial f}{\partial x} \mp i \frac{\partial f}{\partial y}\right)$$

$$\partial_+ V_+ = \partial_- V_- = 1 \qquad \partial_+ V_- = \partial_- V_+ = 0$$

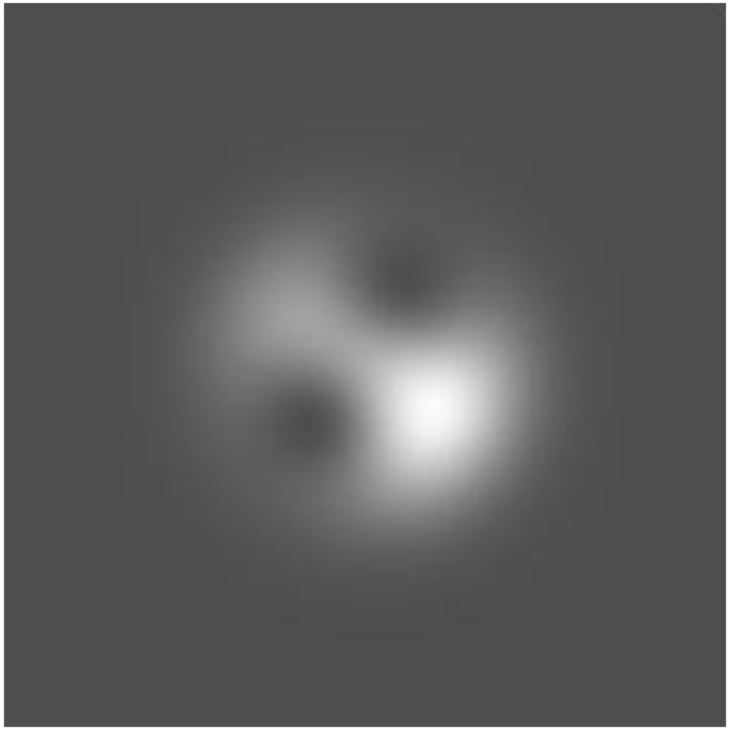
$$\cos\psi = \frac{|\partial_+ f|^2 - |\partial_- f|^2}{|\partial_+ f|^2 + |\partial_- f|^2} \qquad \text{(helicity)}$$

$$\exp(i\beta) = \frac{\partial_+ f \, (\partial_- f)^*}{|\partial_+ f||\partial_- f|} \qquad \text{(orientation)}$$

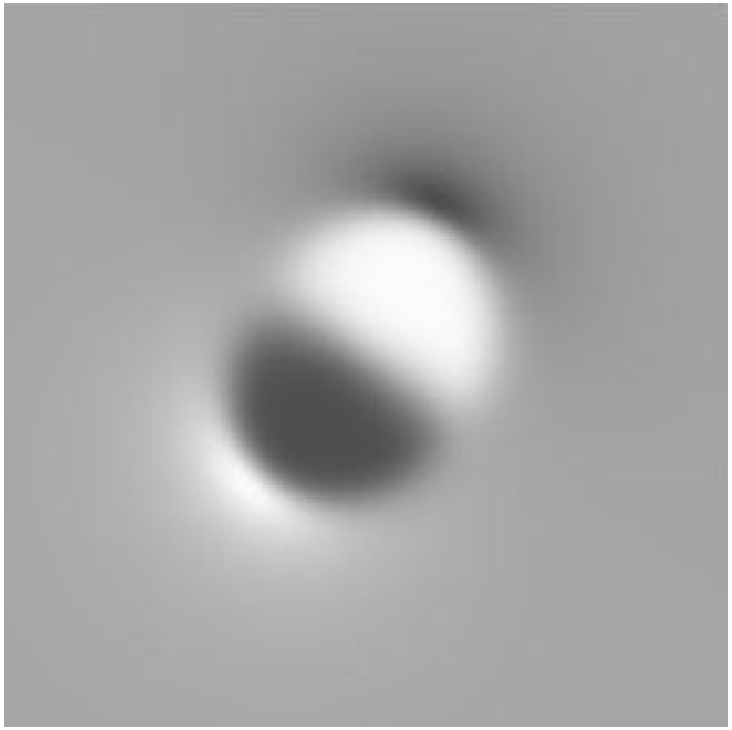
CSIR
our future through science

Morphology distribution

Beam amplitude

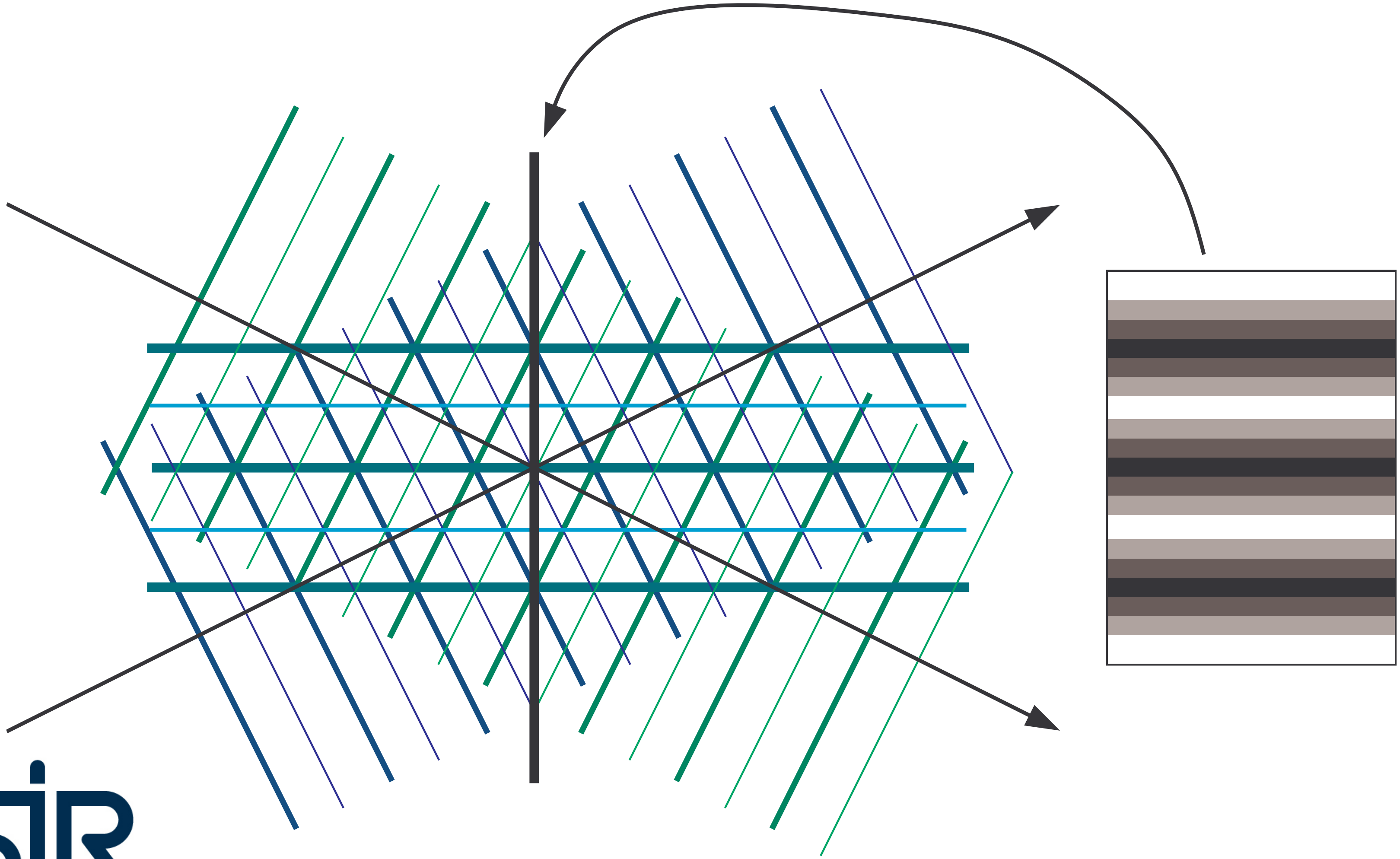


Helicity



Interference

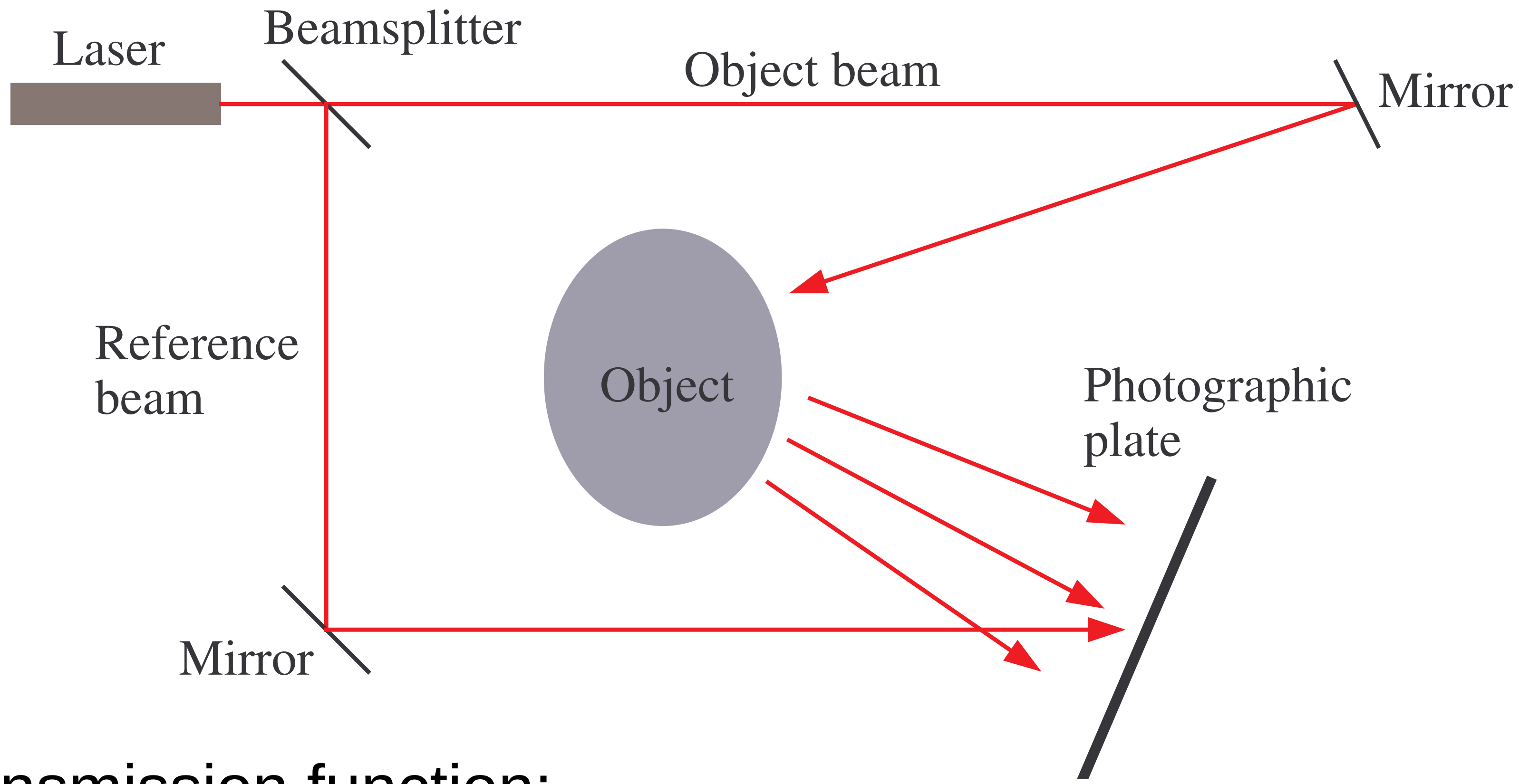
$$|f+g|^2 = (f+g)(f^*+g^*) = |f|^2 + |g|^2 + 2|f||g|\cos\left(\theta_f - \theta_g\right)$$



Plane wave plus vortex wave

Interference between a plane wave and a vortex beam gives a line with a branch point.

Holography

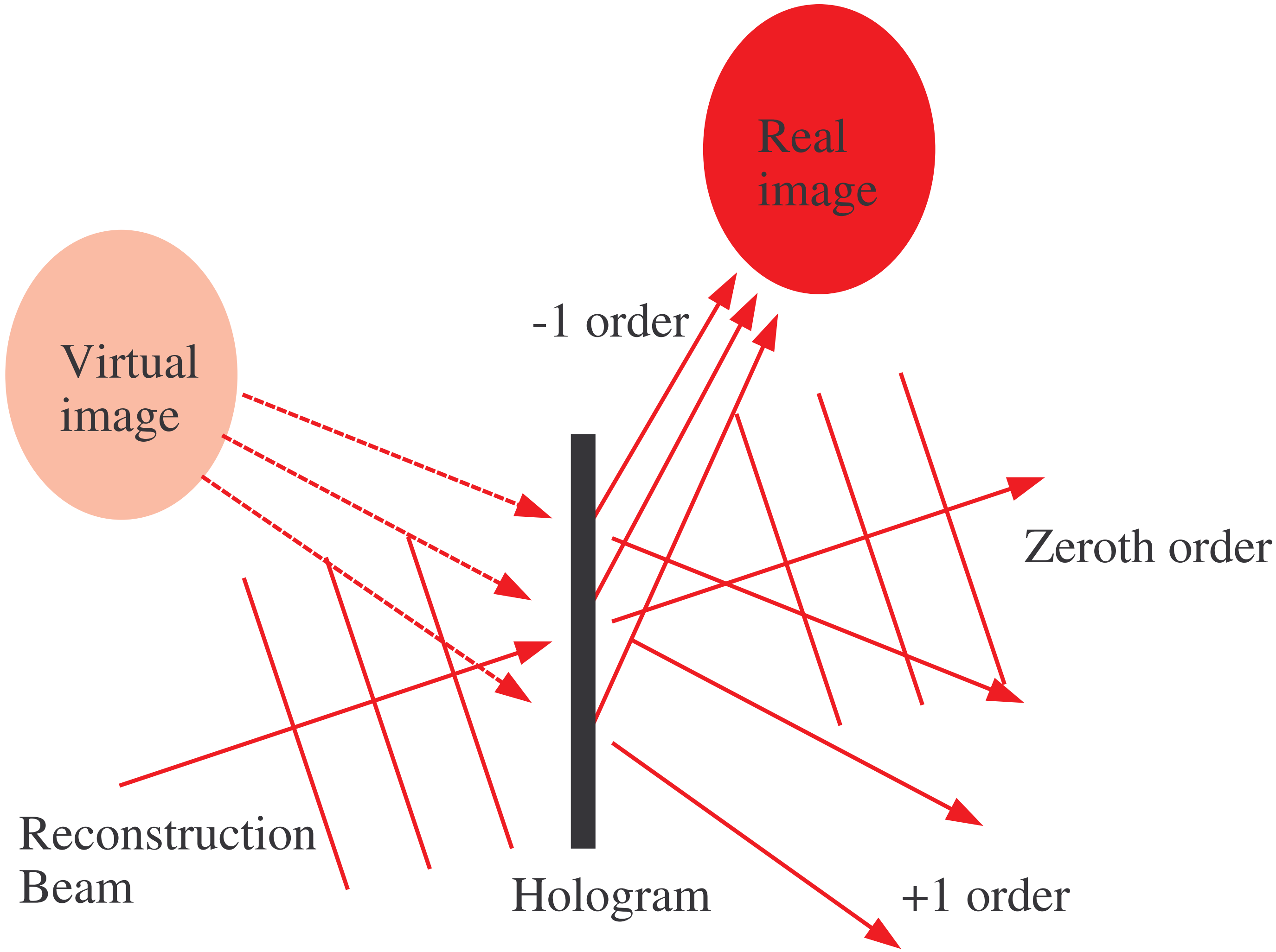


Transmission function:

$$t(x, y) = t_0 + K\left(|f|^2 + |g|^2 + fg^* + f^*g\right)$$

Reconstruction

$$gt(x,y) = g\left(t_0 + K|f|^2 + K|g|^2\right) + Kf|g|^2 + Kf^*g^2$$



our future through science

Computer generated hologram

Compute an artificial amplitude transmission function for an arbitrary phase function:

$$t(x,y) = \frac{1}{2} + \frac{1}{2}\cos\left[\theta(x,y)\right]$$

$\theta(x,y)$ — phase function to be implemented

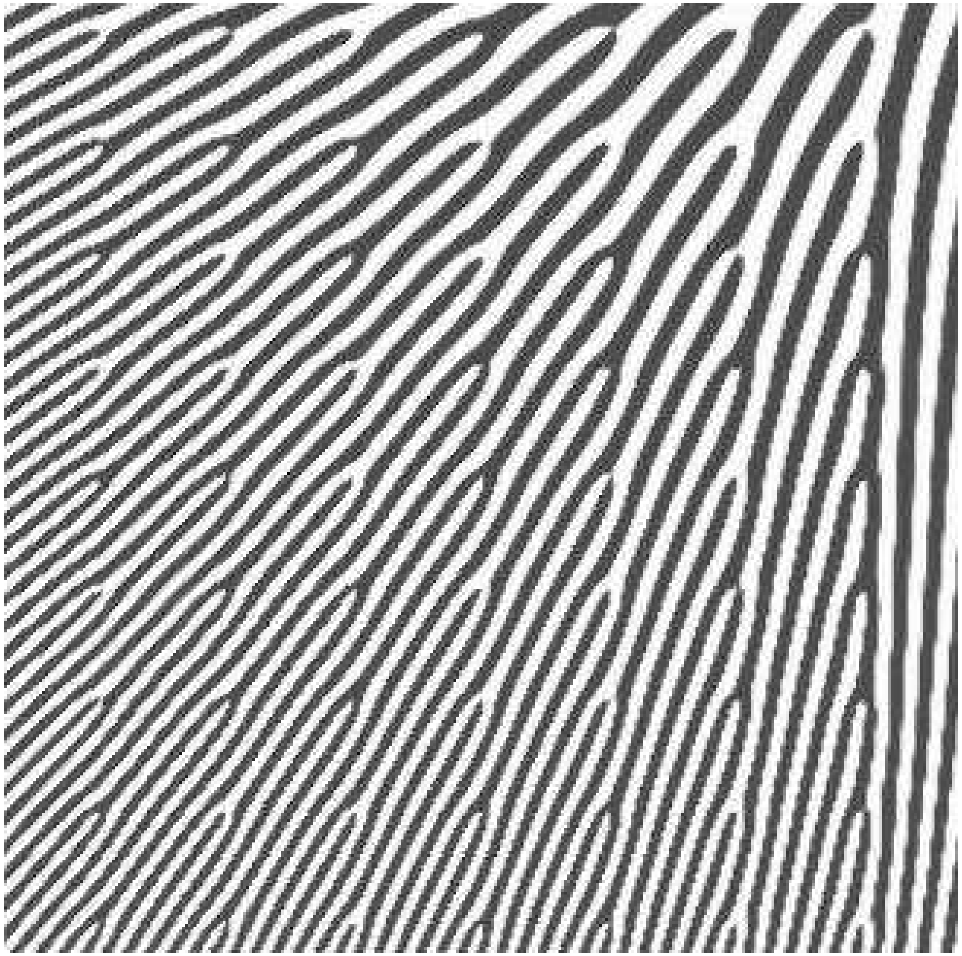
For amplitude transmission function: $0 < |t(x,y)| < 1$

Phase transmission functions directly modulate the phase of the beam: $\Rightarrow t(x,y) = \exp\left[i\theta(x,y)\right]$

our future through science

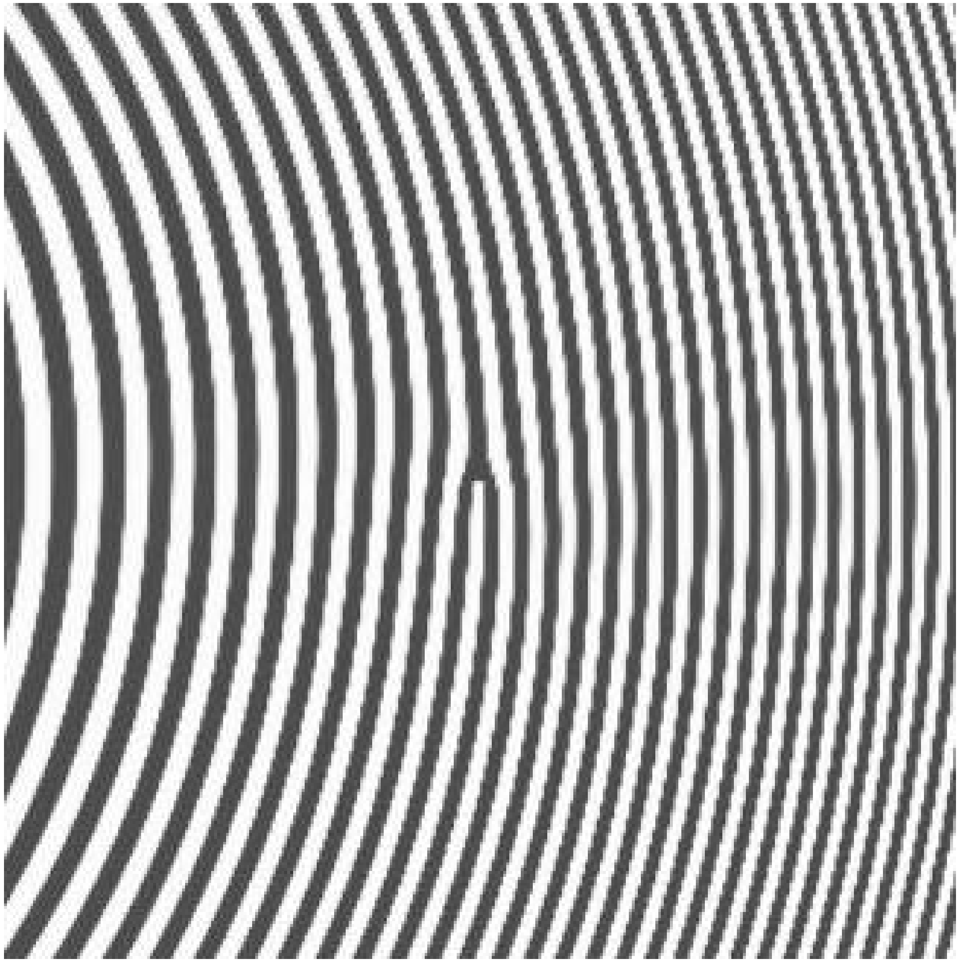
Generating optical vortices

One can thus produce computer generated transmission functions that will produce arbitrary numbers of vortices at specific locations.

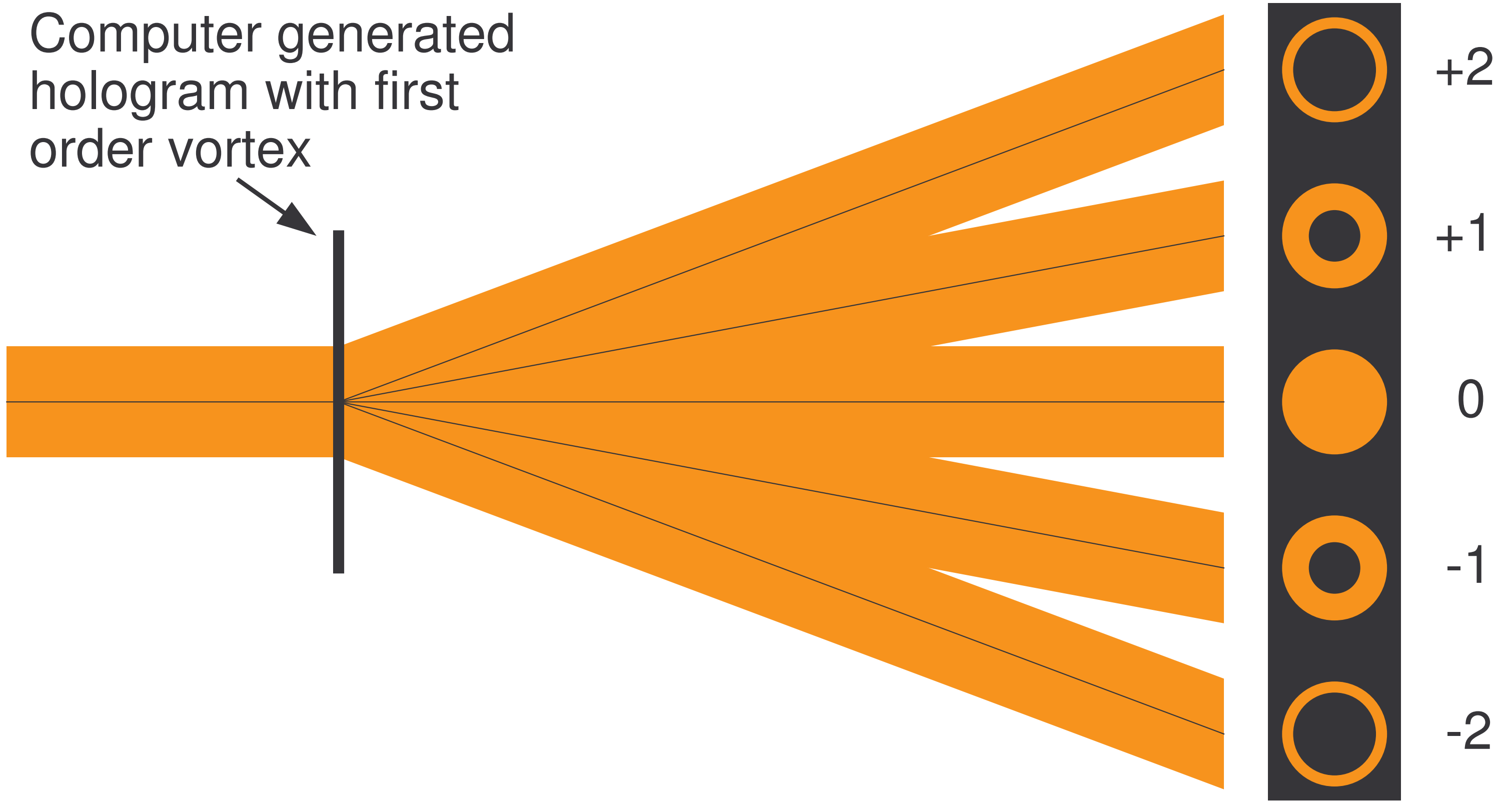


our future through science

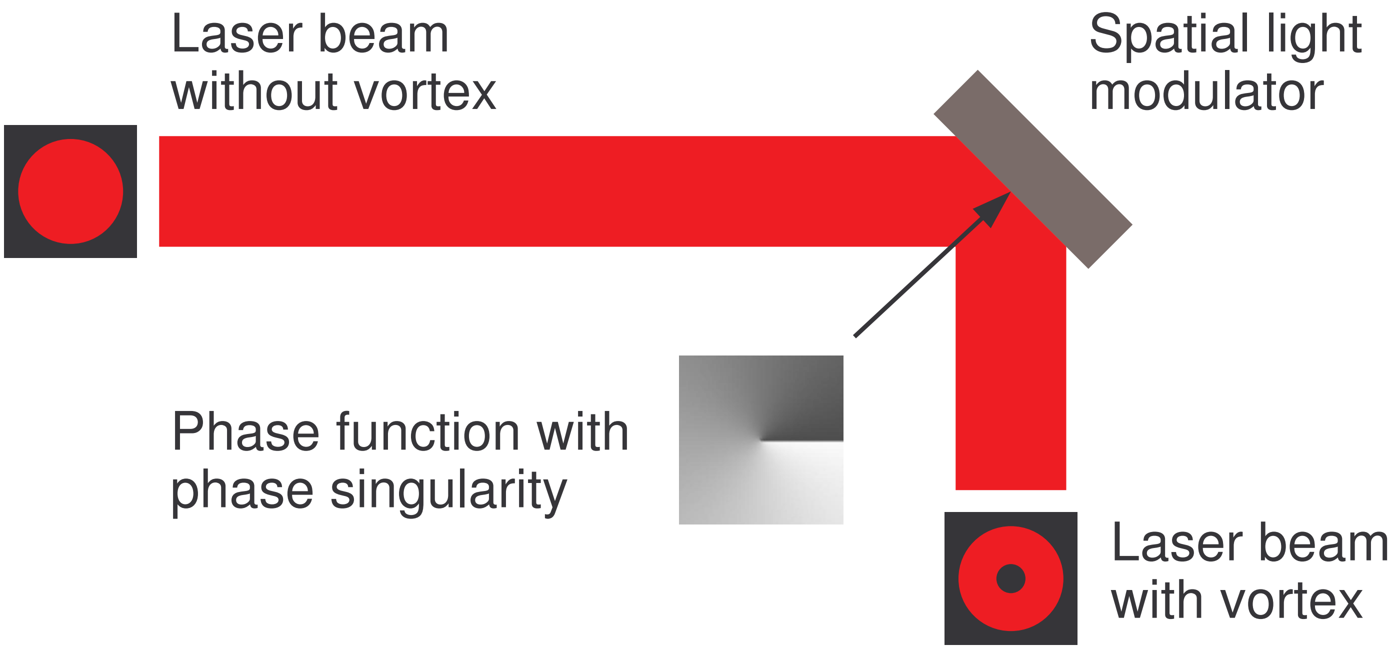
Doughnut lens



Higher diffraction orders



Spatial light modulators



Conclusions

- Optical vortices are phase singularities with integer topological charges.
- Anisotropy and orientation is given by the morphology parameters.
- One can detect optical vortices using interference.
- One can generate optical vortices using computer generated holograms.

our future through science