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Heuristic Space Diversity Management in a Meta-Hyper-Heuristic Framework

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***Abstract*—This paper introduces the concept of heuristic space diversity and investigates various strategies for the management of heuristic space diversity within the context of a meta-hyper-heuristic algorithm. Evaluation on a diverse set of floating-point benchmark problems show that heuristic space diversity has a significant impact on hyper-heuristic performance. The increasing heuristic space diversity strategies performed the best out of all strategies tested. Good performance was also demonstrated with respect to another popular multi-method algorithm and the best performing constituent algorithm.**

I. INTRODUCTION

Over the last decade, research into hyper-heuristics have made an increasing impact on how optimization problems are approached. In contrast to traditional single method optimization algorithms, which search through a space of decision variables for solutions, hyper-heuristics search through a heuristic space of available heuristics or heuristic components [1]. The idea is to either find an "optimal" selection of heuristics, or to construct a heuristic from available heuristic components, to address a specific problem at hand. A meta-hyper-heuristic can be defined as a hyper-heuristic where the constituent or low level algorithms consist of meta-heuristic algorithms referred to as low level meta-heuristics (LLMs) in this work.

Diversity management is another important concept that has received increasing attention recently. Traditionally, the ability of an optimization algorithm to balance exploration and exploitation has been shown to have a significant impact on its performance. If the algorithm converges too quickly, it is more likely to become stuck in a local optimum. If the algorithm focuses too much on exploring new areas of the search space near the end of the optimization run, time is wasted on exploring the search space which could have been used to further refine promising solutions.

Based on the importance of effective management of solution space diversity in traditional optimization algorithms, it is not a major stretch to think that the diversity of the heuristic space and how it is managed throughout the optimization run, could have an important impact on hyper-heuristic performance.

This paper proposes a measurement for quantitatively defining heuristic space diversity (HSD). A number of strategies for managing HSD are also proposed. Algorithm performance was evaluated on a set of varied floating-point benchmark problems and the most promising results were obtained by the hyper-heuristics utilizing an increasing HSD strategy. Good performance was also shown against the population based algorithm portfolio algorithm [2], which is a well known multi-method algorithm, and the covariance matrix adapting evolutionary strategy algorithm (CMAES) [3], the best performing LLM.

To the best of the authors' knowledge, this is the first paper that explicitly introduces the concept of heuristic space diversity and the control of HSD to influence algorithm performance in a meta-hyper-heuristic framework.

The rest of the paper is organized as follows: Section II provides some background with regards to diversity management. Section III provides a brief overview of the HMHH algorithm used as basis for the investigation, while Section IV describes the HSD control strategies which are evaluated. The results are documented in Section V before the paper is concluded in Section VI.

II. DIVERSITY MANAGEMENT IN MULTI-METHOD ALGORITHMS

Although diversity management is not a new concept and is actually relatively common in single method literature, its use in the multi-method algorithm world is relatively limited. Furthermore, if diversity management is considered at all, the focus is mostly on managing solution space diversity (SSD) and not heuristic space diversity. The rest of this section gives a brief overview of diversity management in multi-method algorithms and also discusses a number of related issues.

Examples of controlling SSD to influence hyper-heuristic performance includes Vrugt *et al.*'s AMALGAM [4], Grobler *et al.*'s investigation into the use of local search in a meta-hyper-heuristic framework [5], and Grobler *et al.*'s adaptive local search algorithm [6]. AMALGAM makes use of a species selection mechanism to maintain SSD. In [6] various solution space diversity control strategies based on both adaptive and constant local search and AMALGAM's species selection mechanism were used to evaluate the impact of different SSD profiles on algorithm performance. Both papers showed that multi-method algorithm performance improvements can be obtained by managing SSD effectively.

The use of local search in hyper-heuristics is so closely related to SSD that it is also worth mentioning here. Qu and Burke's graph based hyper-heuristic framework [7] uses a local search algorithm to operate directly on the solution space in conjunction with a hyper-heuristic strategy which operates in heuristic space. Local search algorithms can also be incorporated into the set of available low-level heuristics [8]. This option can be considered an intervention in heuristic space diversity, especially when meta-heuristics are utilized as low-level heuristics, since a more diverse set of algorithms are made available to the high-level strategy.

More closely related to heuristic space diversity is the issue of selecting complementing LLMs. Peng *et al.* [2] proposed a pairwise metric which can be used to determine the risk associated with an algorithm failing to solve the problem in question. Engelbrecht [9] selected complementary swarm behaviours in a heterogeneous PSO by analyzing the exploration-exploitation finger prints of the different PSO updates.

It is clear that a number of authors have considered SSD management and algorithm selection to ensure complementary diverse algorithms. However, to the best of the authors' knowledge, this paper is the first to actively try to influence HSD to improve hyper-heuristic performance.

III. THE HETEROGENEOUS META-HYPER-HEURISTIC ALGORITHM

Due to its excellent performance against other popular multi-method algorithms, the tabu-search based HMHH algorithm of [10] was used as basis for investigating the management of heuristic space diversity. The various algorithmic elements of the HMHH algorithm, including a common population of entities each representing a candidate solution which is evolved over time, a set of LLMs, and a selection strategy, is indicated in Figure 1.

The HMHH algorithm divides the population of entities into a number of subpopulations which are evolved in parallel by a set of LLMs. Each entity is able to access the genetic material of other subpopulations, as if part of a common population of entities. The allocation of entities to LLMs is updated on a dynamic basis throughout the optimization run. The idea is that an intelligent algorithm can be evolved which selects the appropriate LLM at each k^{th} iteration to be applied to each entity within the context of the common parent population, to ensure that the population of entities converge to a high quality solution. The LLM allocation is maintained for k iterations, while the common parent population is continuously updated with new information and better solutions. Throughout this process, the various LLMs are ranked based on their previous performance as

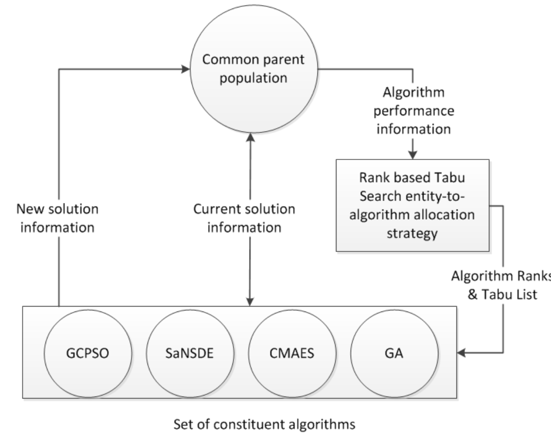

Fig. 1. The heterogeneous meta-hyper-heuristic.

defined by $Q_{\delta m}(t)$ in Algorithm 1. More specifically,

$$Q_{\delta m}(t) = \sum_{i=1}^{|\boldsymbol{I}_m(t)|} \left(f(\boldsymbol{x}_i(t-k)) - f(\boldsymbol{x}_i(t))\right) \quad \forall i \in \boldsymbol{I}_m(t) \tag{1}$$

where $f(\boldsymbol{x}_i(t)$ denotes the fitness function value of entity i at time t and $\boldsymbol{I}_m(t)$ is the set of entities allocated to algorithm m at time t. A tabu list is used to prevent the algorithm from repeatedly using the same poorly performing LLMs. The highest ranking non-tabu operator is then selected for each entity during re-allocation of entities to algorithms as described in [11].

This algorithm uses four common meta-heuristic algorithms as the set of LLMs:

- A genetic algorithm (GA) with a floating-point representation, tournament selection, blend crossover [12], [13], and self-adaptive Gaussian mutation [14].
- The guaranteed convergence particle swarm optimization algorithm (GCPSO) [15].
- The self-adaptive (SaNSDE) algorithm of [16].
- The covariance matrix adapting evolutionary strategy algorithm (CMAES) [3].

IV. INVESTIGATING ALTERNATIVE HEURISTIC SPACE DIVERSITY MANAGEMENT STRATEGIES

The concept of heuristic space diversity is best illustrated by means of an example. In Figure 2 the entities in the population to the left were divided relatively equally between all of the available LLMs during entity to algorithm allocation. This population can be described as having a high HSD. On the other hand, most of the entities in the population of the right were allocated to the genetic algorithm with only one entity each allocated to PSO and ES. This population can be described as having a low HSD.

Algorithm 1: The heterogeneous meta-hyper-heuristic.

Initialize the parent population $\boldsymbol{X}$
$A_i(t)$ denotes the algorithm applied to entity i at iteration t
for *All entities* $i \in \boldsymbol{X}$ **do**
  Randomly select an initial algorithm $A_i(1)$ from the set of LLMs to apply to entity i
end
$t = 0$
$k = 5$
while *A stopping condition is not met* **do**
  for *All entities* i **do**
    Apply LLM $A_i(t)$ to entity i for k iterations
    $t = t + k$
    Calculate $Q_{\delta m}(t)$, the total improvement in fitness function value of all entities assigned to algorithm m from iteration $t - k$ to iteration t.
  end
  for *All entities* i **do**
    Use $Q_{\delta m}(t)$ as input to select LLM $A_i(t)$ according to the rank based tabu search mechanism described in [11]
  end
end

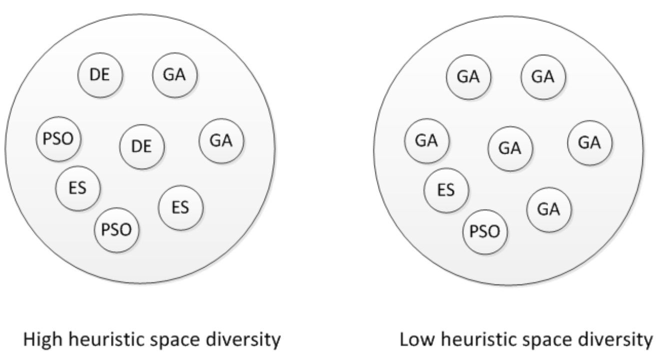

Fig. 2. An example of a population with a high HSD and a population with a low HSD.

A more quantitative metric for heuristic space diversity, $D_h(t)$, the heuristic space diversity at time t, can be defined as follows:

$$D_h(t) = UB_{D_h(t)} \left(1 - \frac{\sum_{i=1}^{I} |T - n_i(t)|}{1.5 n_s} \right) \tag{2}$$

with

$$T = \frac{n_s}{n_a}, \tag{3}$$

where n_a is the number of algorithms available for selection by the hyper-heuristics, n_s is the number of entities in the population, $n_i(t)$ is the number of entities allocated to algorithm i at time t, and $UB_{D_h(t)}$ is the upper bound of the HSD measure. For the purposes of this paper, $UB_{D_h(t)}$ was set to 100 so that $D_h(t) \in$ [0, 100].

Five strategies for controlling HSD throughout the optimization process are explored in this paper:

- **The baseline HMHH algorithm** - This algorithm is the standard HMHH algorithm implemented as described in Section III. No effort is made to manipulate the HSD in this algorithm.
- **Linearly decreasing HSD hyper-heuristic (LDHH)** - This algorithm is characterized by a linearly decreasing HSD. At the start of the optimization run all four LLMs are available for selection. During the optimization run, the worst performing LLM is removed from the set of available algorithms at predefined constant time intervals. As an example, if the maximum allowable function evaluations are 100,000, the worst performing algorithm at that time will be removed respectively at 25,000, 50,000 and 75,000 function evaluations. The idea is to force the hyper-heuristic to explore the heuristic space at the start of the optimization run and exploit the best performing algorithm towards the end of the optimization run.
- **Exponentially decreasing HSD hyper-heuristic (EDHH)** - This algorithm is characterized by an exponentially decreasing HSD. All LLMs are again available for allocation to entities at the start of the optimization run and algorithms are again removed according to their performance at predetermined time intervals. This time, however, the algorithms are removed at exponential time intervals. The result is a slower changeover from exploration to exploitation.
- **Linearly increasing HSD hyper-heuristic (LIHH)** - This algorithm assumes *apriori* knowledge of the LLM performance on the benchmark problem set being solved. The LLMs are ranked from best performing to worst performing. Only the best performing algorithm is made available to the HH at the start of the optimization run. As the optimization process progresses, additional algorithms are made available according to their ranking at predetermined constant time intervals. Here the hyper-heuristic is forced to move from exploitation to exploration. The idea is to obtain maximum gain from the highest ranked algorithm and as the performance gains decrease, the rest of the LLMs become available to diversify the heuristic space and improve the overall algorithm performance.
- **Exponentially increasing HSD hyper-heuristic (EIHH)** - This algorithm is similar to the LIHH algorithm, the only difference being that exponential time intervals are used to add algorithms to the set of available algorithms. The use of exponential time intervals increases the rate of change of HSD leading to a faster changeover from exploitation to exploration.

V. Empirical Evaluation

The various HSD control strategies were evaluated on the first 14 problems of the 2005 IEEE Congress of Evolutionary Computation benchmark problem set [17] in both 10 and 30 dimensions. This benchmark problem set enables algorithm performance evaluation on both unimodal and multimodal functions and includes various expanded and hybridized problems, some with noisy fitness functions. The algorithm control parameter values listed in Table I were found to work well for the algorithms under study during previous research by the authors. $m \longrightarrow n$ indicates that the associated parameter is decreased linearly from m to n over 95% of the maximum number of iterations, I_{max}.

TABLE I. HMHH ALGORITHM PARAMETERS.

Parameter	Value used
Number of entities in common population (n_s)	100
Number of iterations between re-allocation (k)	5
Size of tabu list ($n_a = 4$)	2
Size of tabu list ($n_a = 3$)	1
Size of tabu list ($n_a \leq 2$)	0
PSO parameters	
Acceleration constant (c_1)	$2.0 \longrightarrow 0.7$
Acceleration constant (c_2)	$0.7 \longrightarrow 2.0$
Inertia weight (w)	$0.9 \longrightarrow 0.4$
SaNSDE parameters	As in [16].
GA parameters	
Probability of crossover (p_c)	$0.6 \longrightarrow 0.4$
Probability of mutation (p_m)	0.1
Blend crossover parameter (α)	0.5
GA tournament size (N_t)	13
CMAES parameters	As in [3].

The results of the heuristic space diversity management technique comparison are presented in Table IV, where the results for each algorithm were recorded over 30 independent simulation runs. μ and σ denote the mean and standard deviation associated with the corresponding performance measure and $\#FEs$ denotes the number of function evaluations which were needed to reach the global optimum within a specified accuracy. If the global optimum was reached within the specified accuracy, the run was stopped and the difference between the global optimum and the final fitness function obtained, denoted by FFV, was recorded. Where the global optimum could not be found within the maximum number of iterations, the difference between the final solution at I_{max} and the global optimum, also denoted by FFV, was recorded.

Mann-Whitney U tests were used to evaluate the various strategies according to the number of iterations required to obtain the final fitness function value, as well as the quality of the actual fitness function value. Statistical tests were also used to evaluate the significance of the results. The results in Table II were obtained by comparing each dimension-problem-combination of the strategy under evaluation, to all of the dimension-problem-combinations of the other strategies. For every comparison, a Mann-Whitney U test at 95% significance was performed (using the two sets of 30 data points of the two strategies under comparison) and if the first strategy statistically significantly outperformed the second strategy, a win was recorded. If no statistical difference could be observed a draw was recorded. If the second strategy outperformed the first strategy, a loss was recorded for the first strategy. The total number of wins, draws and losses were then recorded for all combinations of the strategy under evaluation. To illustrate, (11-15-2) in row 1 column 2, indicates that the LDHH strategy outperformed the baseline HMHH algorithm 11 times over the benchmark problem set. Fifteen draws and two losses were recorded.

TABLE II. HYPOTHESES ANALYSIS OF ALTERNATIVE HEURISTIC SPACE DIVERSITY CONTROL MECHANISMS.

Algorithm	HMHH
LDHH	$11-15-2$
EDHH	$11-13-4$
LIHH	$19-5-4$
EIHH	$19-8-1$
TOTAL	$60-41-11$

From the results it is clear that managing the HSD leads to statistically significantly improved hyper-heuristic performance. Table II shows that for 101 cases out of 112, the strategies where the HSD was controlled performed statistically similar or better than the baseline HMHH algorithm. Interestingly the hyper-heuristic's performance was relatively insensitive to the rate of change of diversity. Finally, the increasing HSD strategies outperformed the decreasing HSD strategies quite significantly. It is suspected that the *apriori* LLM performance knowledge is largely responsible for these good results. Unfortunately, this knowledge is not always readily available and then an alternative strategy, for example, randomly selecting the sequence of algorithms to be made available, needs to be used.

In an attempt to further verify the performance of the HSD management strategies, the two best performing solution diversity management strategy from the previous analysis, LDHH and EIHH, were also compared under similar conditions to PAP [2], which was found in a previous study [18] to be the best performing multi-method algorithm currently available (after the HMHH algorithm). Finally, the best performing LLM (CMAES) [3], was also added for comparison purposes. The results are recorded in Table V. In Table III Mann-Whitney U tests were used to compare the performance of PAP and CMAES to the HSD management strategies. This time each HSD control strategy was compared to both CMAES and PAP and the "number of wins-draws-losses" were obtained.

TABLE III. FURTHER HYPOTHESES ANALYSIS OF THE BEST PERFORMING HSD CONTROL MECHANISMS.

	PAP	CMAES
LDHH	$10-11-7$	$7-4-17$
EIHH	$21-7-0$	$9-17-2$

From Table III it can be seen that the HMHH algorithms outperforms PAP in a large number of cases. LDHH performs similar or better to PAP 75% of the time and EIHH performs similar or better 100% of the time. LDHH does not perform quite as well when compared to CMAES. This can, however, be expected since a portion of the function evaluation budget needs to be allocated to solve the algorithm selection problem. This is in contrast to CMAES which can use the whole function evaluation budget on optimization of the actual problem. It is encouraging to note that IEHH significantly outperforms CMAES. Allocating part of the function evaluation budget to other algorithms later during the optimization run clearly has a positive impact on hyper-heuristic performance.

TABLE IV. COMPARISON RESULTS OF THE DIFFERENT HSD MANAGEMENT STRATEGIES ON THE 2005 IEEE CEC BENCHMARK PROBLEM SET.

Prob (Dims)	HMHH				LDHH				EDHH				LIHH				EIHH			
	FFV		# FEs		FFV		# FEs		FFV		# FEs		FFV		# FEs		FFV		# FEs	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
1(10)	$1E-06$	0	13310	694.98	$1E-06$	0	12017	642.78	$1E-06$	0	12090	579.15	$1E-06$	0	8623.3	244.5	$1E-06$	0	8563.3	277.28
1(30)	$1E-06$	0	45040	1783.8	$1E-06$	0	43653	1500.7	$1E-06$	0	46723	2038.7	$1E-06$	0	19253	599.27	$1E-06$	0	19193	549.57
2(10)	$1E-06$	0	31577	2659	$1E-06$	0	13743	959.41	$1E-06$	0	14393	902.84	$1E-06$	0	9173.3	294.7	$1E-06$	0	9106.7	398.21
2(30)	$1E-06$	0	$2.204E+05$	49695	$1E-06$	0	97973	14510	$1E-06$	0	$2.018E+05$	95248	$1E-06$	0	26760	766.36	$1E-06$	0	26410	806.59
3(10)	238.87	910.04	84027	11776	$1E-06$	0	20120	2020.8	$1E-06$	0	19550	1444.1	$1E-06$	0	13350	483.34	$1E-06$	0	13277	397.13
3(30)	$1.352E+05$	73374	$3E+05$	0	55462	48441	$2.970E+05$	16395	70567	75788	$3E+05$	0	$1E-06$	0	61567	1265.3	$1E-06$	0	61603	1201.9
4(10)	$1E-06$	0	43940	9759.9	$1E-06$	0	15550	1344.9	$1E-06$	0	16003	1023.7	$1E-06$	0	9510	347.75	$1E-06$	0	9573.3	374.1
4(30)	0.945	2.854	$3E+05$	0	0.003	0.0183	$1.502E+05$	$1.017E+05$	1.147	3.781	$1.641E+05$	$1E+05$	$1E-06$	0	29330	927.79	$1E-06$	0	29267	986.58
5(10)	$1E-06$	0	17980	1572	$1E-06$	0	17313	1044.1	$1E-06$	0	17077	1054	$1E-06$	0	17520	608.79	$1E-06$	0	17447	567.35
5(30)	1360.6	728.2	$3E+05$	0	1086.8	849.77	$3E+05$	0	1200.2	662.45	$3E+05$	0	$1E-06$	0	$2.572E+05$	68420	0.000	0.002	$2.720E+05$	70931
6(10)	0.119	0.418	65910	19543	0.265	1.010	37347	18132	0.016	0.084	35933	13900	0.001	0.003	18940	744.91	0.000	0.002	19210	716
6(30)	4.275	22.125	$2.510E+05$	45175	1.165	1.906	$2.394E+05$	54813	1.076	1.730	$2.759E+05$	41820	0.133	0.727	$1.229E+05$	46644	0	0	$2.087E+05$	49555
7(10)	0.445	0.342	$1E+05$	0	0.155	0.127	$1E+05$	0	0.106	0.093	93167	17726	0.001	0.003	7533.3	339.71	0.001	0.003	7640	439.91
7(30)	0.005	0.008	$1.615E+05$	$1.153e+05$	0.004	0.010	$1.495E+05$	$1.166E+05$	0.007	0.016	$1.470E+05$	$1.187E+05$	0.000	0.002	16603	673.38	0	0	16657	622.94
8(10)	20.061	0.106	$1E+05$	0	20.034	0.077	99290	3888.8	20.114	0.166	$1E+05$	0	20.07	0.114	$1E+05$	0	20.054	0.097	$1E+05$	0
8(30)	20.166	0.110	$3E+05$	0	20.15	0.130	$3E+05$	0	20.239	0.197	$3E+05$	0	20.209	0.158	$3E+05$	0	20.122	0.106	$3E+05$	0
9(10)	0.038	0.178	38270	19385	0.005	0.005	33440	15441	0.529	0.888	54163	35998	0.495	0.766	79563	30822	0.233	0.419	58827	27886
9(30)	2.245	1.806	$2.896E+05$	30956	9.861	5.367	$2.973E+05$	14953	18.916	8.090	$3E+05$	0	7.852	4.594	$3E+05$	0	3.455	3.710	$2.758E+05$	42827
10(10)	16.157	7.026	$1E+05$	0	19.136	9.445	$1E+05$	0	15.582	9.623	98393	8800.1	1.6145	1.091	88090	30887	1.879	1.205	88200	30602
10(30)	72.135	30.203	$3E+05$	0	73.098	31.88	$3E+05$	0	73.336	28.963	$3E+05$	0	12.719	5.454	$3E+05$	0	10.553	4.577	$3E+05$	0
11(10)	6.876	1.702	$1E+05$	0	6.914	1.313	$1E+05$	0	6.112	2.0343	98277	9439.1	1.223	1.279	74537	39566	1.021	1.149	66540	41709
11(30)	27.311	4.447	$3E+05$	0	21.74	7.740	$3E+05$	0	19.711	5.236	$3E+05$	0	8.902	3.282	$3E+05$	0	10.785	6.415	$3E+05$	0
12(10)	466.17	620.43	88740	19820	156.24	420.61	69417	35661	131.04	401.7	60357	37188	174.03	416.19	72990	41976	317.66	627.11	63770	45137
12(30)	4489.4	5300.9	$3E+05$	0	4646.4	4889	$3E+05$	0	10348	13200	$3E+05$	0	5850.1	3788.7	$2.912E+05$	48127	4822.2	4782.6	$3E+05$	0
13(10)	0.502	0.230	$1E+05$	0	0.547	0.194	$1E+05$	0	0.585	0.227	$1E+05$	0	0.643	0.220	$1E+05$	0	0.442	0.143	$1E+05$	0
13(30)	1.873	0.441	$3E+05$	0	2.794	0.879	$3E+05$	0	2.256	0.632	$3E+05$	0	2.415	0.677	$3E+05$	0	1.836	0.555	$3E+05$	0
14(10)	3.653	0.285	$1E+05$	0	3.457	0.545	$1E+05$	0	3.511	0.425	$1E+05$	0	2.826	0.643	$1E+05$	0	2.695	0.379	$1E+05$	0
14(30)	13.14	0.443	$3E+05$	0	13.204	0.318	$3E+05$	0	13.185	0.381	$3E+05$	0	10.059	0.823	$3E+05$	0	10.288	0.965	$3E+05$	0

TABLE V. COMPARISON RESULTS OF THE BEST PERFORMING HSD MANAGEMENT STRATEGIES VERSUS THE BEST PERFORMING LLM AND MULTI-METHOD ALGORITHM ON THE 2005 IEEE CEC BENCHMARK PROBLEM SET.

Prob (Dims)	LDHH				EIHH				PAP				CMAES			
	FFV		# FEs		FFV		# FEs		FFV		# FEs		FFV		# FEs	
	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ	μ	σ
1(10)	$1E-06$	0	12017	642.78	$1E-06$	0	8563, 3	277.28	$1E-06$	0	13857	630.64	$1E-06$	0	8526, 7	302.78
1(30)	$1E-06$	0	43653	1500.7	$1E-06$	0	19193	549.57	$1E-06$	0	39190	5945.1	$1E-06$	0	19110	447.48
2(10)	$1E-06$	0	13743	959.41	$1E-06$	0	9106, 7	398.21	$1E-06$	0	18760	1030.4	$1E-06$	0	9156, 7	286.1
2(30)	$1E-06$	0	97973	14510	$1E-06$	0	26410	806.59	$1E-06$	0	90063	2729.3	$1E-06$	0	26783	739.1
3(10)	$1E-06$	0	20120	2020.8	$1E-06$	0	13277	397.13	$1E-06$	0	46067	2040.3	$1E-06$	0	13320	379.11
3(30)	55462	48441	$2,97E+05$	16395	$1E-06$	0	61603	1201.9	$1E-06$	0	$2,88E+05$	5481.5	$1E-06$	0	61173	1387.4
4(10)	$1E-06$	0	15550	1344.9	$1E-06$	0	9573, 3	374.1	$1E-06$	0	20483	1424.7	$1E-06$	0	9590	283.27
4(30)	0.003	0.018	$1,50E+05$	$1,02E+05$	$1E-06$	0	29267	986.58	4448.8	5562.2	$2,87E+05$	39164	$1E-06$	0	29357	570.35
5(10)	$1E-06$	0	17313	1044.1	$1E-06$	0	17447	567.35	$1E-06$	0	25010	2790.5	$1E-06$	0	17433	546.04
5(30)	1086.8	849.77	$3E+05$	0	0.000	0.002	$2,72E+05$	70931	1115.5	1446.5	$3E+05$	0	$1E-06$	0	$1,15E+05$	3960.1
6(10)	0.265	1.010	37347	18132	0.000	0.002	19210	716	0.133	0.727	50613	10870	0.001	0.003	18950	744.52
6(30)	1.165	1.906	$2,39E+05$	54813	0	0	$2,09E+05$	49555	0.519	1.346	$2,75E+05$	27166	0.133	0.727	$1,20E+05$	37870
7(10)	0.155	0.127	$1E+05$	0	0.001	0.003	7640	439.91	0.003	0.006	62867	35481	1267	$4,63E-13$	$1E+05$	0
7(30)	0.004	0.010	$1,50E+05$	$1,17E+05$	0	0	16657	622.94	0	0	82047	41811	4696.3	$2,78E-12$	$3E+05$	0
8(10)	20.034	0.077	99290	3888.8	20.054	0.097	$1E+05$	0	20.058	0.086	$1E+05$	0	20.312	0.113	$1E+05$	0
8(30)	20.15	0.130	$3E+05$	0	20.122	0.106	$3E+05$	0	20.264	0.163	$3E+05$	0	20.892	0.175	$3E+05$	0
9(10)	0.005	0.005	33440	15441	0.233	0.419	58827	27886	0.220	0.401	68493	26929	1.946	1.511	88203	30601
9(30)	9.861	5.367	$2,97E+05$	14953	3.455	3.710	$2,76E+05$	42827	2.676	1.92	$2,81E+05$	55032	39.564	6.454	$3E+05$	0
10(10)	19.136	9.445	$1E+05$	0	1.8793	1.205	88200	30602	12.857	6.265	$1E+05$	0	1.647	1.177	90947	27625
10(30)	73.098	31.88	$3E+05$	0	10.553	4.577	$3E+05$	0	70.555	19.764	$3E+05$	0	9.391	3.282	$3E+05$	0
11(10)	6.914	1.313	$1E+05$	0	1.021	1.149	66540	41709	4.085	1.262	$1E+05$	0	1.299	1.365	30310	10496
11(30)	21.74	7.740	$3E+05$	0	10.785	6.415	$3E+05$	0	20.034	2.645	$3E+05$	0	9.026	3.055	$3E+05$	0
12(10)	156.24	420.61	69417	35661	317.66	627.11	63770	45137	231.53	539.16	71737	28064	1546.1	2735.5	70053	43090
12(30)	4646.4	4889	$3E+05$	0	4822.2	4782.6	$3E+05$	0	3573.5	3310.2	$3E+05$	0	20324	19261	$3E+05$	0
13(10)	0.547	0.194	$1E+05$	0	0.442	0.143	$1E+05$	0	0.498	0.159	$1E+05$	0	0.897	0.253	$1E+05$	0
13(30)	2.794	0.879	$3E+05$	0	1.836	0.555	$3E+05$	0	1.74	0.421	$3E+05$	0	3.179	0.561	$3E+05$	0
14(10)	3.457	0.545	$1E+05$	0	2.695	0.379	$1E+05$	0	3.407	0.317	$1E+05$	0	2.585	0.530	$1E+05$	0
14(30)	13.204	0.318	$3E+05$	0	10.288	0.965	$3E+05$	0	13.026	0.319	$3E+05$	0	10.394	0.810	$3E+05$	0

VI. Conclusion

This paper defined the concept of heuristic space diversity and investigated the impact of different heuristic space diversity management strategies on multi-method optimization algorithm performance. The results indicated that a significant performance improvement can be obtained by controlling the HSD of the HMHH algorithm. The control strategies were found to be relatively insensitive to the rate of change of HSD and the increasing HSD strategies were shown to outperform the decreasing and uncontrolled HSD strategies. Finally, the best performing HSD control strategies were shown to perform well against a popular multi-method algorithm and the best performing LLM.

Future research opportunities exist in expanding the analysis to a larger set of benchmark problems and investigating the resulting HSD profiles of popular existing multi-method algorithms such as AMALGAM, PAP and other bandit-based approaches [19] and the subsequent impact these profiles have on algorithm performance. Furthermore, EIHH and LIHH can be modified to avoid the necessity of *apriori* knowledge about LLM performance on the problem being solved.

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