

# Wavefront sensing with all-digital Stokes measurements

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## ABSTRACT

A long-standing question in optics has been to efficiently measure the phase (or wavefront) of an optical field. This has led to numerous publications and commercial devices such as phase shift interferometry, wavefront reconstruction via modal decomposition and Shack-Hartmann wavefront sensors. In this work we develop a new technique to extract the phase which in contrast to previously mentioned methods is based on polarization (or Stokes) measurements. We outline a simple, all-digital approach using only a spatial light modulator and a polarization grating to exploit the amplitude and phase relationship between the orthogonal states of polarization to determine the phase of an optical field. We implement this technique to reconstruct the phase of static and propagating optical vortices.

**Keywords:** wavefront, polarization, Stokes measurements, phase singularities, spatial light modulator

## 1. INTRODUCTION

The quest for efficient and precise measurement techniques of the phase (or wavefront) of an optical field has led to some conventional, state-of-the-art methods, ranging from ray tracing [1], pyramid sensors [2], interferometers [3, 4], the Shack-Hartmann sensor [5], to the use of correlation filters [6] via modal decomposition [7]. However, these techniques are often over-complicated and unable to detect phase singularities.

In this manuscript we have developed a new, digital approach to wavefront sensing [8] based on Stokes polarimetry which makes use of the amplitude and phase relationship between orthogonal states of polarization. With our approach a field of interest is generated by encoding an appropriate hologram on a spatial light modulator (SLM). Since SLMs are diffraction-inefficient, we can exploit the amplitude relationship between the orthogonal polarization states allowing the execution of Stokes polarimetry of the co-linear superposition of the reference beam and the beam of interest to indirectly measure the phase between them. We employ additional optical components, such as a polarization grating (PG), as well as appropriate encoding techniques to mimic the effect of a quarter-waveplate and free-space propagation [9] on our SLM, to construct an adjustment-free, computer-controlled measurement scheme. We illustrate the robustness of our technique by measuring the wavefront of a variety of static and propagating optical fields such as vortex, Bessel, Airy and speckle fields.

## 2. CONCEPT AND THEORY

### Digital Stokes measurements

Here we outline the theory behind the polarization measurements, also known as Stokes measurements, necessary for retrieving the wavefront of an optical field. Concurrently, we will illustrate how two simple additions to the standard Stokes measurements, namely (1) the inclusion of a PG and (2) encoding the SLM to mimic a quarter-waveplate, can transform this manual-based procedure to an all-digital one.

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First, consider the incoherent superposition between two optical fields of orthogonal polarization

$$U(r, \phi) = \cos(\theta/2)U_{\leftrightarrow}(r) + \sin(\theta/2)U_{\updownarrow}(r, \phi) \quad (1)$$

where  $\cos(\theta/2)$  and  $\sin(\theta/2)$  control the amplitudes of the horizontal and vertical components, respectively. We can assign one of the polarization states (vertical) to represent the field of interest (i.e. the field whose wavefront,  $\phi$ , we wish to reconstruct), while the other orthogonal polarization state (horizontal) can represent a reference field with a known (or ‘flat’) wavefront, for example:

$$U_{\leftrightarrow}(r) = \exp(-r^2 / \omega_0) \text{ and } U_{\updownarrow}(r, \phi) = \exp(-r^2 / \omega_0) \exp(i\phi) \quad (2)$$

Here each component consists of a common Gaussian field, while the vertical component has an additional phase term [ $\exp(i\phi)$ ], where  $\phi$  denotes the unknown phase or wavefront we wish to measure. Since most liquid crystal on silicon (LCOS) SLMs only diffract the vertical component of an incident field into the off-axis, diffraction orders, possessing the encoded phase profile, while the horizontal component remains impervious to the encoded phase profile in the on-axis, undiffracted order, this allows us to exploit their diffraction-inefficiency to experimentally realise the fields such as those in Eq. 2. A linearly polarized Gaussian beam illuminating the liquid crystal display (LCD) of a SLM encoded with the phase profile,  $\exp(i\phi)$ , will modify the phase of only the vertical component by  $\phi$ , while the horizontal component remains unchanged as depicted in Eq. 2. Consequently, the incoherent mixture of the two components after the SLM can be described in general by Eq. 1 where the weightings of the two modes can be controlled by adjusting the orientation of the polarization state of the field illuminating the LCD via the use of a polarizer.

The extraction of the phase difference between the horizontal and vertical components ( $\phi$ ) can be achieved via Stokes polarimetry which necessitates four separate intensity measurements to determine the following two Stokes parameters ( $S_2$  and  $S_3$ ) and illustrated in Fig. 1:

$$S_2 = I_{45^\circ} - I_{135^\circ} \text{ and } S_3 = I_{Right} - I_{Left} \quad (3)$$

The two intensity profiles,  $I_{45^\circ}$  and  $I_{135^\circ}$ , pertaining to  $S_2$  can be measured behind a polarizer at angular orientations of  $45^\circ$  and  $135^\circ$  and those pertaining to  $S_3$  ( $I_{Right}$  and  $I_{Left}$ ) by introducing a preceding quarter-waveplate. These four manual measurements can be reduced to two digital measurements, by replacing the quarter-waveplate and polarizer with a PG which acts as a beam-splitter for right- and left-circular polarization, hence extracting  $I_{Right}$  and  $I_{Left}$  in a single measurement as shown in Fig. 2 (a). Furthermore to measure the remaining two intensity profiles,  $I_{45^\circ}$  and  $I_{135^\circ}$ , a quarter-waveplate with its fast-axis set at  $45^\circ$  needs to be introduced before the PG to project  $I_{45^\circ}$  and  $I_{135^\circ}$  into the two detectable ports:  $I_{Right}$  and  $I_{Left}$ , respectively. Since a quarter-waveplate induces a quarter-wavelength phase shift on an incident beam thus converting linearly polarized light to circular and vice versa, this can be achieved by encoding the SLM with an additional  $\pi/2$  phase term, as illustrated in Fig. 2 (b). Previously, adjustments of the polarizer and quarter-waveplate would need to be made while acquiring measurements. However in our approach the necessary optics (PG and SLM) remain static and only a phase change is programmed on the SLM to mimic the required effect of a quarter-waveplate.

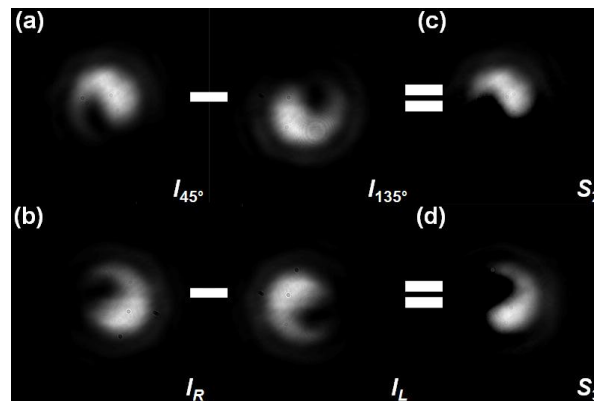


Fig. 1. Experimentally measured intensity profiles (a)  $I_{45^\circ}$ ,  $I_{135^\circ}$  and (b)  $I_R$ ,  $I_L$  (for a vortex beam of  $l = +1$ ) used to calculate the Stokes parameters (c)  $S_2$  and (d)  $S_3$ , respectively.

Similarly as in the case of the manual measurements, the digital measurements are used to extract the wavefront as follows:

$$\phi = 0.5 \arctan(S_3/S_2) \quad (4)$$

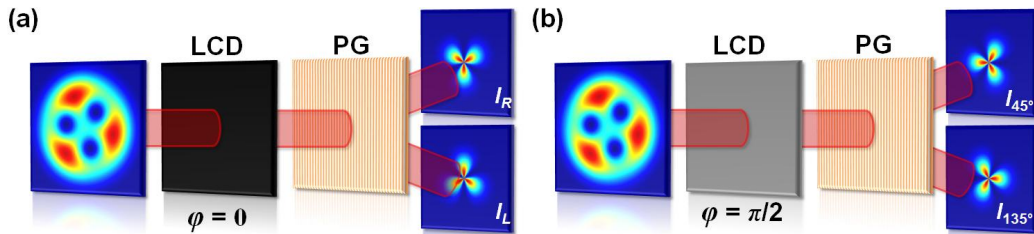


Fig. 2. The digital Stokes polarimetry for extracting (a)  $S_3$  and (b)  $S_2$ . LCD: liquid crystal display of the SLM; and PG: polarization grating.

### Digital ‘free-space’ propagation

Apart from implementing this technique to extract the wavefront of unknown optical fields at a single plane, its use can be extended to investigate the field’s wavefront at multiple planes along the beam’s propagation. This can be achieved by either moving the wavefront sensing device (LCD, PG and detector) transversely along the beam’s propagation axis or a more advantageous approach is to digitally simulate free-space propagation on the LCD. This digital propagation technique [9] manipulates the spatial frequency spectrum of a beam of interest by first Fourier transforming the beam at the plane of the LCD with a lens and second by simultaneously displaying the phase term  $\exp(ik_z z)$  on the LCD while implementing the inverse Fourier transform with a lens to the plane of the detector,

$$\mathfrak{F}^{-1}(\mathfrak{F}[U(r)]\exp[ik_z z]) \quad (5)$$

Merging these two digital methods (wavefront-sensing and propagation) together provides a tool that does not require any information of the optical field under investigation as well as no moving optical components.

## 3. EXPERIMENTAL METHODOLOGY

The experimental setup (Fig. 3) consists of a helium-neon laser that was expanded and collimated to illuminate the LCD of a reflective SLM. A polarizer (P) was used to set the amplitudes of the two orthogonal states depicted in Eq. 1. The LCD was encoded with phase-only azimuthal, conical, cubic or random gray-level holograms to generate vortex, Bessel, Airy or speckle fields, respectively. The fields generated at the plane of the LCD were either Fourier-transformed (lens L3) or relay-imaged (lenses L4 and L5) onto a CCD detector, preceded by a PG, where the Stokes measurements,  $S_2$  and  $S_3$ , were recorded. The Fourier-transforming imaging system allowed the user to digitally simulate free-space propagation on the LCD (as defined in Eq. 5), providing a means to extract the wavefront at multiple planes along the beam’s propagation in real-time.

Since the LCD can be dynamically addressed, we first displayed the hologram required to create our field of interest followed by the same hologram encoded with an additional  $\pi/2$  phase term. For each of the two holograms, the corresponding intensity profiles ( $I_{45^\circ}$ ,  $I_{135^\circ}$  and  $I_{Right}$ ,  $I_{Left}$ ) were recorded to determine the two Stokes parameters,  $S_2$  and  $S_3$ , respectively necessary for reconstructing the wavefront as defined in Eq. 4. Similarly, this measurement was computed multiple times to extract the wavefront as the field was propagated by sequentially encoding the phase term,  $\exp(ik_z z)$ , for various values of  $z$  while the necessary Stokes measurements were recorded.

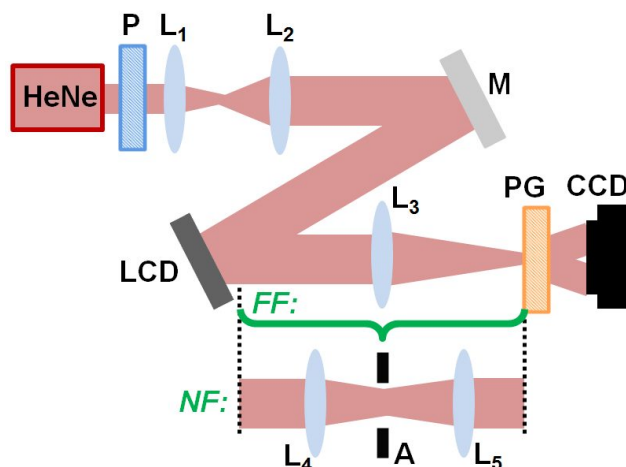


Fig. 3. Schematic of the experimental wavefront-sensing setup. HeNe: helium-neon laser; P: polarizer; L1→5: lenses; M: mirror; LCD: liquid crystal display of the SLM; A: aperture; PG: polarization grating; CCD: CCD detector.

#### 4. RESULTS AND DISCUSSION

##### Digital Stokes measurements

The wavefronts of higher-order optical vortices and Bessel beams were measured via the digital Stokes polarimetry and are depicted in Fig. 4. It is evident that there is extremely good agreement between the experimentally extracted wavefronts, Fig. 4 (a) [(c)], and the theoretically calculated wavefronts, Fig. 4 (b) [(d)], for the vortex (Bessel) beams of azimuthal indices  $l = -3$  to  $+3$ . Apart from identifying unit and higher-order phase singularities, this technique can also distinguish the handedness of the azimuthal phase profile [illustrated by the white arrows in Fig. 4 (a)].

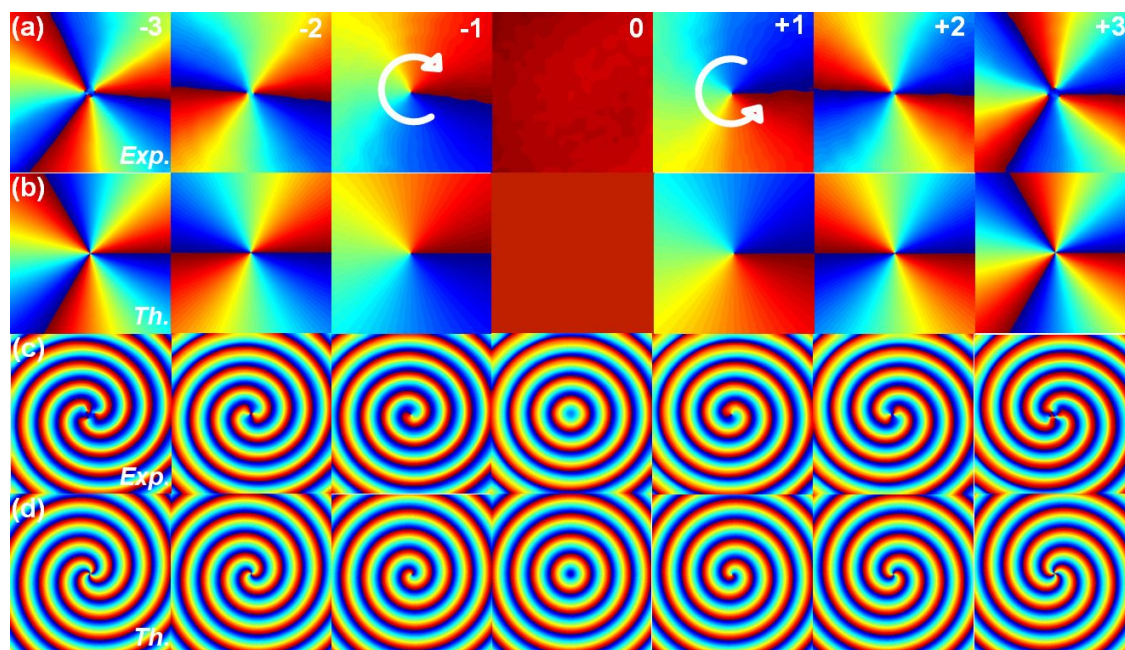


Fig. 4. (a) [(c)] Experimentally measured and (b) [(d)] theoretically calculated wavefronts for vortex (Bessel) beams. Corresponding azimuthal indices are given in the top right corner. The white arrows highlight the handedness of the azimuthal phase.

Similarly, the wavefront was extracted for an Airy beam and a random speckle field depicted in Figs 5 (a) and (b), respectively with a high degree of fidelity with the theoretically calculated wavefront.

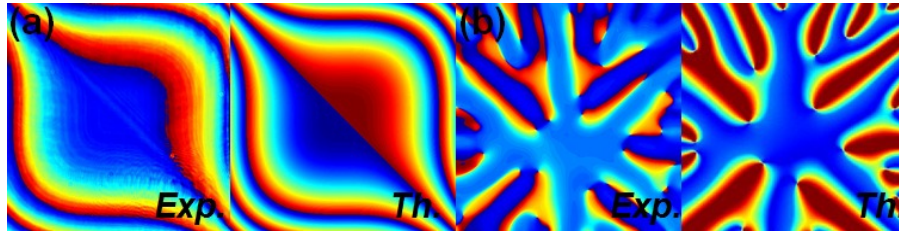


Fig. 5. Experimentally measured (Exp.) and theoretically calculated (Th.) wavefronts for an (a) Airy beam and (b) speckle field.

### Digital ‘free-space’ propagation

The evolution of the wavefront for a vortex beam ( $l = +2$ ) as it propagates was investigated and selected frames are shown in Fig 6. These measurements were made in the far-field of the LCD plane [as illustrated in Fig. 3] for the execution of the virtual propagation described in Eq. 5. One draw-back to performing Stokes polarimetry on the far-field mode is that the mode size is drastically smaller [by a factor of  $f$  ( $L3$ )] than that in the near-field, decreasing the resolution of the measured wavefronts. As the field propagates its wavefront becomes curved in agreement with the encoded phase profile [ $\exp(ik_z z)$ ] which causes the two singularities in the vortex beam to move further away from one another, consequently appearing as if they are spirally around the beam axis.

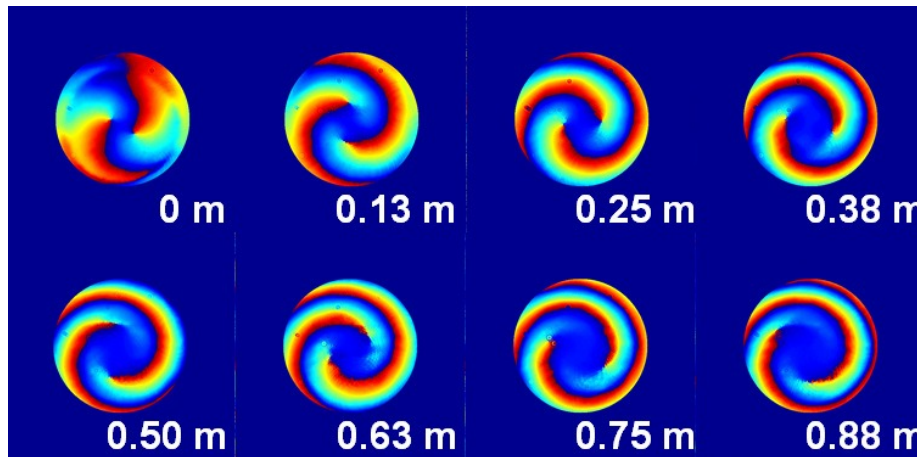


Fig. 6. Experimentally measured wavefronts for a (a) Gaussian and (b) vortex beam ( $l = +2$ ) at different propagation planes. Corresponding propagation distances are given in the bottom right corner.

## 5. CONCLUSION

In conclusion, we presented a detailed adaptation of Stokes measurements for a digital, adjustment-free measurement of the wavefront of unknown optical fields. Even though our technique employs only three components (SLM, PG and detector) we have shown that the wavefront of complex optical fields can be reconstructed with high fidelity. In addition, we also demonstrated that the SLM, a component of our wavefront sensor, can also mimic the effect of free-space propagation on our field of interest, leading to its wavefront being extracted at multiple planes along its propagation. We suggest that our wavefront diagnostic could be a versatile tool in numerous areas such as studies into the creation and annihilation of phase singularities, microscopy and free space communication.

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