Third African Conference on Computational Mechanics - An International Conference - AfriCOMP13
July 30 - August 2, 2013, Livingstone, Zambia
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Treatment of fully enclosed FSI using artificial compressibility

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Key Words: artificial compressibility, partitioned fluid-structure interactions, fully enclosed flow

1 INTRODUCTION

In recent years a great deal of advances have been made with respect to Dirichlet-Neuman (DN) partitioned fluid-structure interaction (FSI) solvers. Problems of varying degrees of complexity can be solved with reasonable efficiencies through the use of the various interface coupling techniques. One particular class of problems which however cannot be solved is fully enclosed FSI problems for incompressible fluids (eg. balloon inflation problems). In this paper we explore the use of partitioned FSI using artificial compressibility (AC), whereby the fluid equations are modified to allow for compressibility which internally incorporates an approximation of the system volume change as a function of pressure.

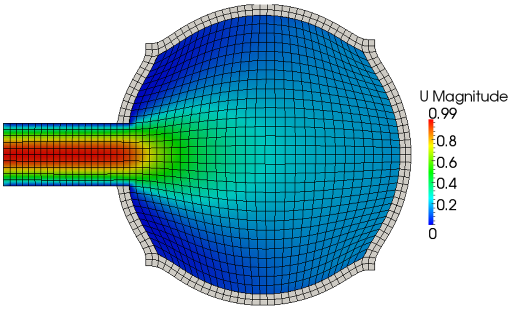
2 ARTIFICIAL COMPRESSIBILITY

In FSI, the use of artificial compressibility is not to couple the pressure and velocity in the Navier-Stokes equations, but rather to include an approximation of the elastic response of the structure within the set of fluid equations. For the sake of brevity, let us consider only the incompressible fluid continuity equation: $\nabla \cdot \boldsymbol{u} = 0$. The basic idea, as has been presented by Järvinen *et al* [1] and Degroote *et al* [2], is to modify the continuity equation by inserting AC in the form

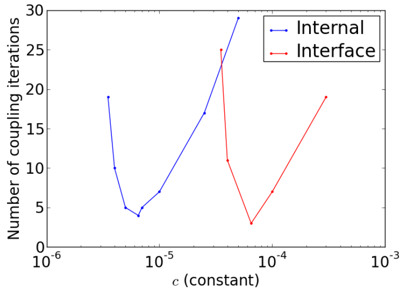
$$\boldsymbol{c}\left(\frac{\partial \boldsymbol{p}}{\partial t}\right) + \nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

where $\boldsymbol{c}$ is an approximation of the system compressibility and is expressed as $\boldsymbol{c} = \frac{1}{\boldsymbol{V}}\frac{d\boldsymbol{V}}{d\boldsymbol{p}}$, where $\boldsymbol{V}$ denotes the internal volume. Equation (1) can be solved implicitly by solving for the pressure derivative as a linear approximation in the form

$$\frac{\boldsymbol{c}}{\Delta t}\left(\boldsymbol{p}_{k+1} - \boldsymbol{p}_k\right) + \nabla \cdot \boldsymbol{u}_k = 0 \tag{2}$$



(a)



(b)

Figure 1: (a) Flow velocity at 10 seconds for simple balloon-like inflation problem ([3]). (b) Number of coupling iterations required for varying constant c for AC through the entire domain (internal) and along the interface only.

where k is the current FSI coupling iteration. The AC term $(\boldsymbol{p}_{k+1} - \boldsymbol{p}_k)$ disappears as the FSI coupling iterations tend to convergence, satisfying the original continuity equation.

It should be noted that the AC term is equivalent to adding an approximation of the volume change to the continuity equation. In other words, modifying the fluid flow velocity within a given iteration to accommodate a predicted volume change $\frac{d\boldsymbol{V}}{dt} + \int_V (\nabla \cdot \boldsymbol{u})\, dV$. It should thus be apparent that the method of AC is limited only to FSI problems where there is indeed a volume change, and more precisely, a volume change based predominantly on a change in interface pressure. The method has been applied with some success to internal blood flow problems ([1] and [2]), but is not suitable for external flow problems (for example flow around a flexible tail which oscillates).

The primary difficulty with AC is how to best approximate $\boldsymbol{c}$. One possibility is to use an analytical expression which approximates the inverse of the system bulk modulus or the square of the system wave modulus. This is however entirely limited to simple geometries. Alternatively both Järvinen [1] and Degroote [2] have suggested using a test load method, whereby the solid interface is stressed with a pressure load to ascertain an approximation to the relationship $\boldsymbol{c} = \frac{1}{\boldsymbol{V}} \frac{d\boldsymbol{V}}{d\boldsymbol{p}}$. The difference between the two authors is based on the domain of effect of the AC. Degroote *et al* [2] suggests limiting the AC to the layer of elements along the interface only, whereas Järvinen *et al* [1] have suggested including AC throughout the entire fluid domain. In this paper we will demonstrate that the effect of this choice has a potentially large impact on the optimal value of $\boldsymbol{c}$, where the choice of a suitable $\boldsymbol{c}$ is often mutually exclusive between AC along the interface only or throughout the entire domain. In Figure 1 we illustrate the numerical performance for both internal and interface AC as a function of varying constant $\boldsymbol{c}$ for a simple balloon filling problem first introduced by Küttler *et al.* [3].

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