Beam-quality measurements using a spatial light modulator

Christian Schulze,[1,*] Daniel Flamm,[1] Michael Duparré,[1] and Andrew Forbes[2,3,4]

[1]*Institute of Applied Optics, Friedrich Schiller University, Fröbelstieg 1, Jena 07743, Germany*

[2]*Council for Scientific and Industrial Research National Laser Centre, P.O. Box 395, Pretoria 0001, South Africa*

[3]*School of Physics, University of KwaZulu-Natal, Private Bag X54001, Durban 4000, South Africa*

[4]*e-mail: aforbes1@csir.co.za*

**Corresponding author: christian.schulze@uni-jena.de*

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We present a fast and easy technique for measuring the beam propagation ratio, M^2, of laser beams using a spatial light modulator. Our technique is based on digitally simulating the free-space propagation of light, thus eliminating the need for the traditional scan in the propagation direction. We illustrate two approaches to achieving this, neither of which requires any information of the laser beam under investigation nor necessitates any moving optical components. The comparison with theoretical predictions reveals excellent agreement and proves the accuracy of the technique.

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Laser beam quality is usually understood as the evaluation of the propagation characteristics of a beam. Because of its simplicity, a very common and widespread parameter has become the M^2 value, which compares the beam parameter product (product of waist radius and divergence half angle) of the beam under test to that of a fundamental Gaussian beam [1]. The definition of the beam propagation ratio M^2 for simple and general astigmatic beams and the instruction for its measurement can be found in the ISO (International Organization for Standardization) standard [2,3]. Here, the measurement of the beam intensity with a camera in various planes is suggested, which allows the determination of the second-order moments of the beam and hence the M^2 value. Several techniques have been proposed to measure the M^2, such as the knife-edge method or the variable aperture method [1,4]. However, despite the fact that these methods might be simple, they do not lead to reliable results [1]. Moreover, the scanning can be a tedious process if many data points are acquired. Another approach to measure the M^2 uses a Shack–Hartmann wavefront sensor, but was shown to yield inaccurate results for multimode beams [5]. ISO-compliant techniques include the measurement of the beam intensity at a fixed plane and behind several rotating lens combinations [6], multiplane imaging using diffraction gratings [7], multiple reflections from an etalon [8], direct measurement of the beam moments by specifically designed transmission filters [9], and field reconstruction by modal decomposition [10–12], respectively.

In essence all approaches to determine the beam propagation ratio require several measurements of varying beam sizes and/or varying curvatures. This has traditionally been achieved by allowing a beam of a given size and curvature to propagate in free space; i.e., nature provides the variation in the beam parameters through diffraction. An obvious consequence of this is that the detector must move with the propagating field, the ubiquitous scan in the propagation direction. Here we illustrate that it is possible to achieve the desired propagation with digital holograms: free-space propagation without the free space.

In this Letter, we follow two different approaches, both applying a spatial light modulator (SLM) to manipulate the phase of the incident light. The two suggested methods include using the SLM, first, as a variable lens, and second, to manipulate the spatial frequency spectrum of the beam. In both cases the intensity is measured with a camera in a fixed position behind the SLM and no moving components are required. Both strategies are shown to enable accurate measurement of the beam quality, which is fast and easy to implement.

In the first method we realized the required changing beam curvature by programming a digital lens of variable focal length. In this case the curvature is changing in a fixed plane (that of the hologram); thus rather than probing one beam at several planes we are effectively probing several beams at one plane (each hologram can be associated with the creation of a new beam). Consider, for example, the geometrical situation described in Fig. 1. Using the laws of Gaussian optics it is straightforward to show that the beam diameter d measured behind a lens of focal length f can be described as

$$d(f) = 2\sqrt{\left(\frac{2\lambda z_2 M^2}{\pi d_L}\right)^2 + \frac{d_L^2}{4}\left(1 + z_2\left(\frac{1}{R_L} - \frac{1}{f}\right)\right)^2}, \quad (1)$$

with the radius of wavefront curvature at the lens position $R_L = z_1\{1 + [\pi d_0^2/(4\lambda z_1 M^2)]^2\}$, the beam diameter at

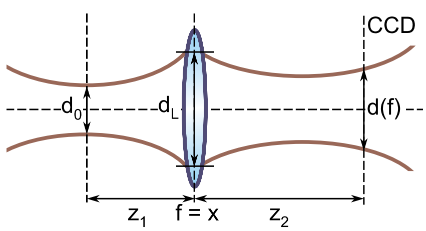

Fig. 1. (Color online) Schematic geometry to determine the M^2 value by measuring the beam diameter $d(f)$ as a function of different lens focal lengths f: d_0, waist diameter; d_L, diameter in plane of the lens; and $z_{1,2}$, distances between waist and lens, and lens and CCD plane, respectively.

the lens position $d_L = \{d_0^2/4 + [2M^2\lambda z_1/(\pi d_0)]\}^{(1/2)}$, the beam waist diameter d_0 in front of the lens, the distances $z_{1,2}$ as defined in Fig. 1, and the wavelength λ. Equation 1 can now be used as a fit function to identify unknown parameters. Accordingly, from the parameter set $(d, d_0, z_1, z_2, f, \lambda, M^2)$, f and λ are known, d and z_2 are measured, and d_0, z_1 and M^2 are used as fit parameters. So using an SLM as a variable lens by displaying a phase pattern $\Psi_f = \pi/\lambda f(x^2 + y^2)$, which is referred to as method A in the following, yields the beam propagation ratio M^2 as a result of fitting the measured data with the theoretical curve of Eq. 1.

The second approach uses the SLM to manipulate the spatial frequency spectrum of the beam. According to the angular spectrum method [10,13], the propagation of an optical field U along a distance z can be described by

$$U(\mathbf{r}, z) = \mathcal{F}^{-1}[\mathcal{F}[U(\mathbf{r}, 0)] \exp(ik_z z)], \tag{2}$$

where $\mathbf{r} = (x, y)$, $k_z(k_x, k_y) = (4\pi^2/\lambda^2 - k_x^2 - k_y^2)^{1/2}$ with the wave vector $\mathbf{k} = [k_x, k_y, k_z]$, and $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and its inverse, respectively. Hence, Fourier transforming a beam plane of interest onto the SLM using a physical lens, displaying the phase pattern $\Psi_{k_z} = k_z z$ on the SLM, and back transforming with a lens to the plane of a CCD camera, enables us to measure the diameter of the artificially propagated beam in a fixed plane. From a hyperbolic fit of these diameters, the M^2 parameter can be determined according to the ISO standard [2]. This procedure is referred to as method B in the following. In consequence, a caustic measurement can be performed, very similar to that of [10], but without any elaborate modal decomposition necessary and without any *a priori* knowledge about the beam under test. Note that both, methods A and B, are, as described above, limited to simple astigmatic beams. However, both methods can be easily extended to handle general astigmatic beams by additionally displaying a cylindrical lens on the SLM. Figure 2 depicts the experimental setup, which is fairly simple, consisting only of the beam source (helium neon laser, $\lambda = 633$ nm, 2 mW), whose beam was expanded to approximate a plane wave (diameter 10 mm), the SLM (Holoeye PLUTO, reflective liquid crystal on silicon device, 1920×1080 pixels, 8 μm pixel pitch), illuminated with the plane wave, a CCD camera (Spiricon), and a lens that is used in method B only, where all optical components remain fixed during the measurement.

To test the two methods, different Laguerre–Gaussian modes LG_{pl} (simple astigmatic) were investigated, since their M^2 value and beam diameter are known to scale with the mode indices p and l according to $M^2 = 2p + l + 1$

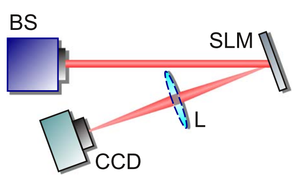

Fig. 2. (Color online) Experimental setup. BS, beam source; SLM, spatial light modulator; L, lens ($f = 400$ mm, only present in method B); CCD, CCD camera.

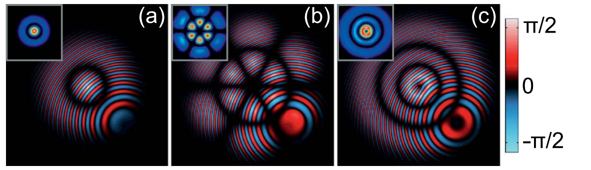

Fig. 3. (Color online) Digital holograms for three sample beams using method A with a focal length of 400 mm. (a) LG_{10}, (b) $LG_{1\pm3}$ (Media 1, Media 2), and (c) LG_{21}. Insets depict resulting measured beam intensities.

and $d_{pl} = d_{00}(2p + l + 1)^{1/2}$. For convenience, only one SLM was simultaneously used for the generation of the sample beams as well as to depict the phase patterns Ψ (in case of application to an unknown source the setup is used for analysis only). The sample beams were generated by displaying the respective LG mode patterns using the method described in [14,15] with an intrinsic beam diameter of $d_{00} = 1.5$ mm, and superposing it for analysis with the lens function Ψ_f (method A) or the propagation factor Ψ_{k_z} (method B) as shown in Fig. 3 for method A by way of example. Using the SLM also for beam generation, method B requires only one lens if the Fourier transform of the sample beam is programmed [see Eq. (2)], which in our case is again a Laguerre–Gaussian mode LG_{pl} since such modes are Fourier transforms of themselves, albeit with a scale difference. As an example, Fig. 4(a) depicts measured and fitted beam diameter as a function of focal length of the lens programmed on the SLM for a Laguerre–Gaussian beam LG_{21} (method A). As can be seen, the

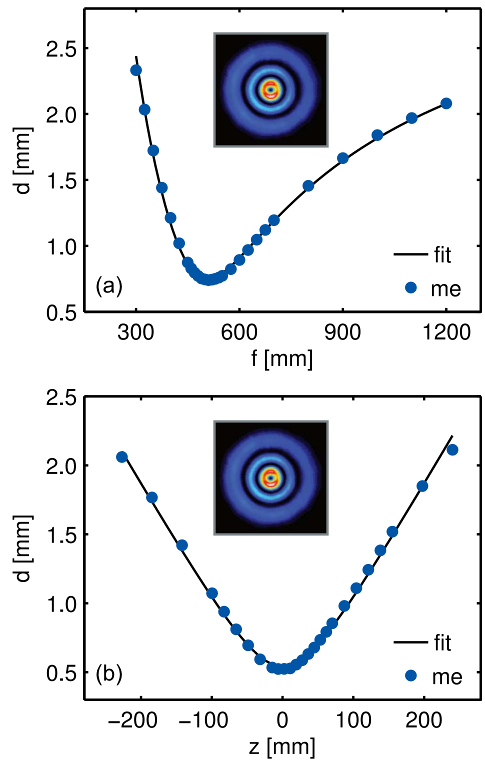

Fig. 4. (Color online) Analysis of a Laguerre–Gaussian LG_{21} beam using (a) method A: measured beam diameter (me) as a function of programmed lens focal length f, yielding an $M^2 = 6.22$ by fitting with Eq. 1 (fit). (b) method B: measured beam diameter (me) as a function of propagation distance z (Media 3, Media 4). Hyperbolic fitting (fit) yields an $M^2 = 6.04$.

Table 1. Measured (A, B) and Expected (th) M^2 Values and Waist Diameters of the Sample Beams

Mode	$d_{A,\mathrm{th}}$ [mm]	d_A [mm]	$d_{B,\mathrm{th}}$ [mm]	d_B [mm]	M^2_{th}	M^2_A	M^2_B
LG_{00}	1.50	1.58	0.43	0.43	1.00	1.03	1.04
LG_{01}	2.12	2.02	0.61	0.64	2.00	2.01	2.09
LG_{10}	2.60	2.56	0.74	0.76	3.00	3.03	3.12
$LG_{0\pm4}$	3.36	3.30	0.96	1.00	5.00	5.25	5.05
$LG_{1\pm3}$	3.68	3.44	1.05	1.11	6.00	6.20	6.13
LG_{21}	3.68	3.45	1.05	1.09	6.00	6.22	6.04

measured diameters follow the theoretical behavior of Eq. 1, yielding an $M^2 = 6.22$, which deviates only by 4% from the theoretical value of 6.0. Characterizing the same beam using method B [Fig. 4(b)] yields an $M^2 = 6.04$ (deviation 1%) by hyperbolic fitting the measured diameters according to the ISO standard [2]. Movies 1–4 in Figs. 3 and 4 depict the changing hologram patterns and the resulting beam intensities for a $LG_{1\pm3}$ (method A) and LG_{21} (method B) beam, respectively; see Fig. 3(b) (Media 1, Media 2) and Fig. 4(b) (Media 3, Media 4).

Table 1 summarizes the results of method A and B for LG modes of different order and two in-phase superpositions ($LG_{0\pm4}$ and $LG_{1\pm3}$), comparing theoretical and measured beam waist diameter and M^2 value. The measurement error is about $\Delta M^2 = 0.15$ and $\Delta d = 10\%\ d$, including errors from the fit and from the uncertainty in the determination of the intensity background level, which is subtracted from all CCD frames. SLM-induced beam deteriorations can be neglected following previous studies [16]. Note that the expected beam waist diameters differ between methods A and B, since in method B the generated mode pattern on the SLM is the far field. Hence, with the intrinsic beam size of the displayed mode patterns of $d_{00} = 1.5$ mm and a lens focal length of $f = 400$ mm, the corresponding theoretical near field beam waist diameter amounts to 0.43 mm. As can be seen from the comparison of theoretically expected and measured waist diameters and M^2 values, all results are in excellent agreement. Deviations from the theoretical values are ≤7% for the waist diameters and ≤5% regarding the M^2.

In conclusion we presented two approaches for fast and easy measurement of the beam propagation ratio M^2 using digital holograms programmed on an SLM. The first approach uses the SLM as a lens of variable focal length, whereas in the second, the SLM was used to manipulate the spatial frequency spectrum of the beam, yielding an artificial propagation. Due to the high SLM frame rate of 60 Hz, an M^2 measurement time below 1 s is achievable. Deviations of the M^2 parameter less than 5% were attained by analyzing different Laguerre–Gaussian modes and mode superpositions of known M^2, revealing a high measurement fidelity.

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