A normal form for hypergraph-based module extraction for $\mathcal{SROIQ}$

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Abstract. Modularization is an important part of the modular design and maintenance of large scale ontologies. Syntactic locality modules, with their desirable model theoretic properties, play an ever increasing role in the design of algorithms for modularization, partitioning and reasoning tasks such as classification. It has been shown that, for the DL $\mathcal{EL}^+$, the syntactic locality module extraction problem is equivalent to the reachability problem for hypergraphs. In this paper we investigate and introduce a normal form for the DL $\mathcal{SROIQ}$ which allows us to map any $\mathcal{SROIQ}$ ontology to an equivalent hypergraph. We then show that standard hyperpath search algorithms can be used to extract modules similar to syntactic locality modules for $\mathcal{SROIQ}$ ontologies.

1 Introduction

The advent of the semantic web presupposes a significant increase in the size of ontologies, their distributive nature and the requirement for fast reasoning algorithms. Modularization techniques not only play an increasingly important role in the design and maintenance of large-scale distributed ontologies, but also in the design of algorithms that increase the efficiency of reasoning tasks such as subsumption testing and classification [11, 1].

Extracting minimal modules is computationally expensive and even undecidable for expressive DLs [2, 3]. Therefore, the use of approximation techniques and heuristics play an important role in the effective design of algorithms. Syntactic locality [2, 3], because of its excellent model theoretic properties, has become an ideal heuristic and is widely used in a diverse set of algorithms [11, 1, 4].

Suntisrivaraporn [11] showed that, for the DL $\mathcal{EL}^+$, $\bot$-locality module extraction is equivalent to the reachability problem in directed hypergraphs. Nortjé et al. [9, 10] extended the reachability problem to include $\top$-locality and introduced bidirectional reachability modules as a subset of $\bot\top$ modules.

In this paper we introduce a normal form for the DL $\mathcal{SROIQ}$, which allows us to map any $\mathcal{SROIQ}$ ontology to an equivalent syntactic locality preserving hypergraph. We show that, given this mapping, the extraction of $\bot$-locality modules is equivalent to the extraction of all B-hyperpaths, $\top$-locality is similar to extracting all F-hyperpaths and $\bot\top^*$ modules to that of extracting frontier graphs. These similarities demonstrate a unique relationship between reasoning tasks, based on syntactic locality, for $\mathcal{SROIQ}$ ontologies, and standard well studied hypergraph algorithms.

2 Preliminaries

2.1 Hypergraphs

Hypergraphs are a generalization of graphs and have been extensively studied since the 1970s as a powerful tool for modelling many problems in Discrete Mathematics. In this paper we adapt the definitions of hypergraphs and hyperpaths from [8, 12].

A (directed) hypergraph is a pair $\mathcal{H} = \langle \mathcal{V}, \mathcal{E} \rangle$, where $\mathcal{V}$ is a finite set of nodes, $\mathcal{E} \subseteq 2^{\mathcal{V}} \times 2^{\mathcal{V}}$ is the set of hyperedges such that for every $e = (T(e), H(e)) \in \mathcal{E}$, $T(e) \neq \emptyset$, $H(e) \neq \emptyset$, and $T(e) \cap H(e) = \emptyset$. A hypergraph $\mathcal{H}' = \langle \mathcal{V}', \mathcal{E}' \rangle$ is a subhypergraph of $\mathcal{H}$ if $\mathcal{V}' \subseteq \mathcal{V}$ and $\mathcal{E}' \subseteq \mathcal{E}$. A hyperedge e is a *B-hyperedge* if $|H(e)| = 1$. A *B-hypergraph* is a hypergraph such that each hyperedge is a B-hyperedge. A hyperedge e is an *F-hyperedge* if $|T(e)| = 1$. An *F-hypergraph* is a hypergraph such that each hyperedge is an F-hyperedge. A *BF-hypergraph* is a hypergraph for which every edge is either a B- or an F-hyperedge.

Let $e = (T(e), H(e))$ be a hyperedge in some directed hypergraph $\mathcal{H}$. Then, $T(e)$ is known as the *tail* of e and $H(e)$ is known as the *head* of e. Given a directed hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$, its symmetric image $\overline{\mathcal{H}}$ is a directed hypergraph defined as: $\mathcal{V}(\overline{\mathcal{H}}) = \mathcal{V}(\mathcal{H})$ and $\mathcal{E}(\overline{\mathcal{H}}) = \{(H, T) \mid (T, H) \in \mathcal{E}(\mathcal{H})\}$. We denote by $BS(v) = \{e \in \mathcal{E} \mid v \in H(e)\}$ and $FS(v) = \{e \in \mathcal{E} \mid v \in T(e)\}$ respectively the *backward star* and *forward star* of a node v. Let n and m be the number of nodes and hyperedges in a hypergraph $\mathcal{H}$. We define the size of $\mathcal{H}$ as $size(\mathcal{H}) = |\mathcal{V}| + \sum_{e \in \mathcal{E}} (|T(e)| + |H(e)|)$.

A simple path $\prod_{st}$ from $s \in \mathcal{V}(\mathcal{H})$ to $t \in \mathcal{V}(\mathcal{H})$ in $\mathcal{H}$ is a sequence $(v_1, e_1, v_2, e_2, ..., v_k, e_k, v_{k+1})$ consisting of distinct nodes and hyperedges such that $s = v_1$, $t = v_{k+1}$ and for every $1 \leq i \leq k$, $v_i \in T(e_i)$ and $v_{i+1} \in H(e_i)$. If in addition $t \in T(e_1)$ then $\prod_{st}$ is a simple cycle. A simple path is *cycle free* if it does not contain any subpath that is a simple cycle.

A node s is B-connected to itself. If there is a hyperedge e such that all nodes $v_i \in T(e)$ are B-connected to s, then every $v_j \in H(e)$ is B-connected to s. A B-hyperpath from $s \in \mathcal{V}(\mathcal{H})$ to $t \in \mathcal{V}(\mathcal{H})$ is a minimal subhypergraph of $\mathcal{H}$ where t is B-connected to s. An F-hyperpath $\prod_{st}$ from $s \in \mathcal{V}(\mathcal{H})$ to $t \in \mathcal{V}(\mathcal{H})$ in $\mathcal{H}$ is a subhyperpath of $\mathcal{H}$ such that $\overline{\prod_{st}}$ is a B-hyperpath from t to s in $\overline{\mathcal{H}}$. A BF-hyperpath from $s \in \mathcal{V}(\mathcal{H})$ to $t \in \mathcal{V}(\mathcal{H})$ in $\mathcal{H}$ is a minimal (in the inclusion sense) subhyperpath of $\mathcal{H}$ such that it is simultaneously both a B-hyperpath and an F-hyperpath from s to t in $\mathcal{H}$. We note that every hypergraph $\mathcal{H}$ can be transformed to a BF-hypergraph $\mathcal{H}'$ by replacing each hyperedge $e = (T(e), H(e))$ with the two hyperedges $e_1 = (T(e), \{n_v\}), e_2 = (\{n_v\}, H(e))$ where n_v is a new node.

Algorithm 1 (Visiting a hypergraph [8])

Procedure Bvisit(s,$\mathcal{H}$)	Procedure Fvisit(t,$\mathcal{H}$)
for each $u \in \mathcal{V}$ **do** $blabel(u) := false$;	**for each** $u \in \mathcal{V}$ **do** $flabel(u) := false$;
for each $e \in \mathcal{E}$ **do** $T(e) := 0$;	**for each** $e \in \mathcal{E}$ **do** $T(e) := 0$;
$Q := \{s\}; blabel(s) := true$;	$Q := \{t\}; flabel(t) := true$;
while $Q \neq \emptyset$ **do**	**while** $Q \neq \emptyset$ **do**
select and remove $u \in Q$;	**select and remove** $u \in Q$;
for each $e \in FS(u)$ **do**	**for each** $e \in BS(u)$ **do**
$T(e) := T(e) + 1$;	$H(e) := H(e) + 1$;
if $T(e) := \vert Tail(e) \vert$ ***then***	**if** $H(e) := \vert Head(e) \vert$ ***then***
for each $v \in Head(e)$ **do**	**for each** $v \in Tail(e)$ **do**
if $blabel(v) = false$ **then**	**if** $flabel(v) = false$ **then**
$blabel(v) = true$	$flabel(v) = true$
$Q := Q \cup \{v\}$	$Q := Q \cup \{v\}$

Given some node s, Algorithm 1 can be used to find all B-connected or F-connected nodes to s in $O(size(\mathcal{H}))$ time. Here, the set of all B-hyperpaths from s and F-hyperpaths to t are respectively represented by all those nodes n such that $blabel(n) = true$ or $flabel(n) = true$, as well as the edges connecting those nodes.

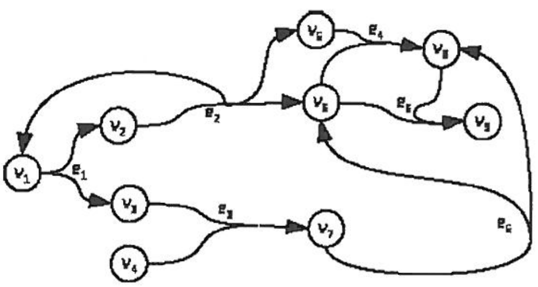

Fig. 1. Example hypergraph $\mathcal{H}_1$

Example 1. In Figure 1 we have $\mathcal{H}_1 = (\mathcal{V}_1, \mathcal{E}_1)$, with $\mathcal{V}_1 = \{v_1, ..., v_9\}$ and $\mathcal{E}_1 = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ such that $e_1 = (\{v_1\}, \{v_2, v_3\})$, $e_2 = (\{v_2\}, \{v_1, v_5, v_6\})$, $e_3 = (\{v_3, v_4\}, \{v_7\})$, $e_4 = (\{v_5, v_6\}, \{v_8\})$, $e_5 = (\{v_7\}, \{v_6, v_8\})$ and $e_6 = (\{v_6, v_8\}, \{v_9\})$. The directed hypergraph $\mathcal{G}_1$ with nodes $\mathcal{V}(\mathcal{G}_1) = \{v_1, v_2, v_3, v_5, v_6, v_8, v_9\}$ and $\mathcal{E}(\mathcal{G}_1) = \{e_1, e_2, e_4, e_6\}$ is a B-hyperpath from v_1 to v_9 in $\mathcal{H}_1$. The hypergraph $\mathcal{G}_2$ with $\mathcal{V}(\mathcal{G}_2) = \{v_3, v_4, v_6, v_7, v_8, v_9\}$ and $\mathcal{E}(\mathcal{G}_2) = \{e_3, e_5, e_6\}$ is an F-hyperpath from v_3 to v_9 in $\mathcal{H}_1$. The hypergraph $\mathcal{G}_3$ with $\mathcal{V}(\mathcal{G}_3) = \{v_6, \dots v_9\}$ and $\mathcal{E}(\mathcal{G}_3) = \{e_5, e_6\}$ is a BF-hyperpath from v_7 to v_9 in $\mathcal{H}_1$.

Definition 1. *Given a hypergraph $\mathcal{H} = (\mathcal{V}, \mathcal{E})$, the* frontier *graph $\mathcal{H}' = (\mathcal{V}', \mathcal{E}', s, t)$ of $\mathcal{H}$, such that $\mathcal{V}' \subseteq \mathcal{V}$, $\mathcal{E}' \subseteq \mathcal{E}$, $s, t \in \mathcal{V}$, is the maximal (in the inclusion sense) BF-graph in which (1) s and t are the origin and destination nodes, (2) if $v \in \mathcal{V}'$ then v is B-connected to s, and t is F-connected to v in $\mathcal{H}'$.*

Algorithm 2 (Frontier graph Extraction Algorithm [8])

`Procedure frontier(`$\mathcal{H}, \mathcal{H}', s, t$`)`

$\mathcal{H}' := \mathcal{H}$; change := true
while *change* $= true$ **do**
  change $= false$
  $\mathcal{H}' = Bvisit(s, \mathcal{H}')$; $\mathcal{H}' = Fvisit(t, \mathcal{H}')$
  for each $v \in \mathcal{V}'$
    if $blabel(v) = false$ **or** $flabel(v) = false$ **then**
      change $:= true$
      $\mathcal{V}' = \mathcal{V}' - \{v\}$; $\mathcal{E}' = \mathcal{E}' - FS(v) - BS(v)$
  if $s \notin \mathcal{V}'$ **or** $t \notin \mathcal{V}'$ **then**
    $\mathcal{H}' := \emptyset$; *change* $:= false$;

Algorithm 2 can be used to extract a frontier graph for any source and destination nodes and runs in $O(n\, size(\mathcal{H}))$ time.

2.2 The DL $\mathcal{SROIQ}$

In this section we give a brief introduction to the DL $\mathcal{SROIQ}$ [5, 7] with its syntax and semantics listed in Table 1. N_C, N_R and N_I denote disjoint sets of atomic concept names, atomic roles names and individual names. The set N_R includes the universal role. Well-formed formulas are created by combining concepts from the table by using the connectives $\neg, \sqcap, \sqcup$ etc.

Given $R_1 \circ \ldots \circ R_n \sqsubseteq R$, where $n \geqslant 1$ and $R_i, R \in N_R$, is a *role inclusion axiom* (RIA). A *role hierarchy* is a finite set of RIAs. Here $R_1 \circ \ldots \circ R_n$ denotes a composition of roles where R, R_i may also be an *inverse role* R^-. A role R is *simple* if it: (1) does not appear on the right-hand side of a RIA; (2) is the inverse of a simple role; or (3) appears on the right-hand side of a RIA only if the left-hand side consists entirely of simple roles. $Ref(R)$, $Irr(R)$ and $Dis(R, S)$, where R, S are roles other than U, are role assertions. A set of role assertions is simple w.r.t. a role-hierarchy H if each assertion $Irr(R)$ and $Dis(R, S)$ uses only simple roles w.r.t. H.

A strict partial order $\prec$ on N_R is a *regular order* if, and only if, for all roles R and S: $S \prec R$ iff $S^- \prec R$. Let $\prec$ be a regular order on roles. A RIA $w \sqsubseteq R$ is $\prec$-regular if, and only if, $R \in N_R$ and w has one of the following forms: (1) $R \circ R$, (2) R^-, (3) $S_1 \circ \ldots \circ S_n$, where each $S_i \prec R$, (4) $R \circ S_1 \circ \ldots \circ S_n$, where each $S_i \prec R$ and (5) $S_1 \circ \ldots \circ S_n \circ R$, where each $S_i \prec R$. A role hierarchy H is *regular* if there exists a regular order $\prec$ such that each RIA in H is $\prec$-regular.

An *RBox* is a finite, regular role hierarchy H together with a finite set of role assertions simple w.r.t. H. If $a_1, \ldots, a_n$ are in N_I, then $\{a_1, \ldots, a_n\}$ is a nominal. N_o is the set of all nominals. The set of $\mathcal{SROIQ}$ *concept descriptions* is the smallest set such that: (1) $\bot, \top$, each $C \in N_C$, and each $o \in N_o$ is a concept description. (2) If C is a concept description, then $\neg C$ is a concept description. (3) If C and D are concept descriptions, R is a role description, S is a simple role description, and n is a non-negative integer, then the following are all concept descriptions: $(C \sqcap D), (C \sqcup D), \exists R.C, \forall R.C, \leqslant nS.C, \geqslant nS.C, \exists S.Self$.

Table 1. Syntax and semantics of $\mathcal{SROIQ}$

Concept	Syntax	Semantics
atomic concept	$C \in N_C$	$C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$
individual	$A \in N_I$	$a^{\mathcal{I}} \in \Delta^{\mathcal{I}}$
nominal	$\{a_1, \ldots, a_n\}, a_i \in N_I$	$\{a_1^{\mathcal{I}}, \ldots, a_n^{\mathcal{I}}\}$
role	$R \in N_R$	$R^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
inverse role	$R^-, R \in N_R$	$R^{-\mathcal{I}} = \{(y,x) \| (x,y) \in R^{\mathcal{I}}\}$
universal role	U	$U^{\mathcal{I}} = \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$
role composition	$R_1 \circ \ldots \circ R_n$	$\{(x,z) \| (x,y_1) \in R_1^{\mathcal{I}} \wedge (y_1,y_2) \in R_2^{\mathcal{I}} \wedge \ldots \wedge (y_n,z) \in R_{n+1}^{\mathcal{I}}\}$
top	$\top$	$\top^{\mathcal{I}} = \Delta^{\mathcal{I}}$
bottom	$\bot$	$\bot^{\mathcal{I}} = \emptyset$
negation	$\neg C$	$(\neg C)^{\mathcal{I}} = \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}}$
conjunction	$C_1 \sqcap C_2$	$(C_1 \sqcap C_2)^{\mathcal{I}} = C_1^{\mathcal{I}} \cap C_2^{\mathcal{I}}$
disjunction	$C_1 \sqcup C_2$	$(C_1 \sqcup C_2)^{\mathcal{I}} = C_1^{\mathcal{I}} \cup C_2^{\mathcal{I}}$
exist restriction	$\exists R.C$	$\{x \| (\exists y)[(x,y) \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}]\}$
value restriction	$\forall R.C$	$\{x \| (\forall y)[(x,y) \in R^{\mathcal{I}} \rightarrow y \in C^{\mathcal{I}}]\}$
self restriction	$\exists R.Self$	$\{x \| (x,x) \in R^{\mathcal{I}}\}$
atmost restriction	$\leqslant nR.C$	$\{x \| \#\{y \| (x,y) \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\} \leqslant n\}$
atleast restriction	$\geqslant nR.C$	$\{x \| \#\{y \| (x,y) \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\} \geqslant n\}$
Axiom	**Syntax**	**Semantics**
concept inclusion	$C_1 \sqsubseteq C_2$	$C_1^{\mathcal{I}} \subseteq C_2^{\mathcal{I}}$
role inclusion	$R_1 \circ \ldots \circ R_n \sqsubseteq R$	$(R_1 \circ \ldots \circ R_n)^{\mathcal{I}} \subseteq R^{\mathcal{I}}$
reflexivity	$Ref(R)$	$\{(x,x) \| x \in \Delta^{\mathcal{I}}\} \subseteq R^{\mathcal{I}}$
irreflexivity	$Irr(R)$	$\{(x,x) \| x \in \Delta^{\mathcal{I}}\} \cap R^{\mathcal{I}} = \emptyset$
disjointness	$Dis(R,S)$	$S^{\mathcal{I}} \cap R^{\mathcal{I}} = \emptyset$
class assertion	$C(a)$	$a^{\mathcal{I}} \in C^{\mathcal{I}}$
inequality assertion	$a \neq b$	$a^{\mathcal{I}} \neq b^{\mathcal{I}}$
role assertion	$R(a,b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \in R^{\mathcal{I}}$
negative role assertion	$\neg R(a,b)$	$(a^{\mathcal{I}}, b^{\mathcal{I}}) \notin R^{\mathcal{I}}$

If C and D are concept description then $C \sqsubseteq D$ is a *general concept inclusion* (GCI) axiom. A *TBox* is a finite set of GCIs. If C is a concept description, $a, B \in N_I$, $R, S \in N_R$ with S a simple role description, then $C(a)$, $R(a,b)$, $\neg S(a,b)$, and $a \neq b$, are individual assertions. An $\mathcal{SROIQ}$ *ABox* is a finite set of individual assertions. All GCIs, RIAs, role assertions, and individual assertions are referred to as axioms. A $\mathcal{SROIQ}$-KB base is the union of a TBox, RBox and ABox.

2.3 Modules

Definition 2. (Module for the arbitrary DL $\mathcal{L}$) *Let $\mathcal{L}$ be an arbitrary description language, $\mathcal{O}$ an $\mathcal{L}$ ontology, and σ a statement formulated in $\mathcal{L}$. Then, $\mathcal{O}' \subseteq \mathcal{O}$ is a* module for σ in $\mathcal{O}$ *(a σ-*module in $\mathcal{O}$*) whenever: $\mathcal{O} \models \sigma$ if and only if $\mathcal{O}' \models \sigma$. We say that $\mathcal{O}'$ is a module for a signature S in $\mathcal{O}$ (an* **S**-module in $\mathcal{O}$*) if, for every $\mathcal{L}$ statement σ with $Sig(\sigma) \subseteq S$, $\mathcal{O}'$ is a σ-module in $\mathcal{O}$.*

Definition 2 is sufficiently general so that any subset of an ontology preserving a statement of interest is considered a module, the entire ontology is therefore a module in itself. An important property of modules in terms of the modular reuse of ontologies is *safety* [2, 3]. Intuitively, a module conforms to a safety condition whenever an ontology $\mathcal{T}$ reuses concepts from an ontology $\mathcal{T}'$ in such a way so that it does not change the meaning of any of the concepts in $\mathcal{T}'$. This may be formalized in terms of the notion of conservative extensions:

Definition 3. (Conservative extension [3]) *Let $\mathcal{T}$ and $\mathcal{T}_1$ be two ontologies such that $\mathcal{T}_1 \subseteq \mathcal{T}$, and let S be a signature. Then (1) $\mathcal{T}$ is an S-conservative extension of $\mathcal{T}_1$ if, for every α with $Sig(\alpha) \subseteq S$, we have $\mathcal{T} \models \alpha$ iff $\mathcal{T}_1 \models \alpha$. (2) $\mathcal{T}$ is a conservative extension of $\mathcal{T}_1$ if $\mathcal{T}$ is an S-conservative extension of $\mathcal{T}_1$ for $\mathcal{S} = Sig(\mathcal{T}_1)$.*

Definition 4. (Safety [3, 6]) *An ontology $\mathcal{T}$ is* safe *for $\mathcal{T}'$ if $\mathcal{T} \cup \mathcal{T}'$ is a conservative extension of $\mathcal{T}'$. Further let S be a signature. We say that $\mathcal{T}$ is safe for S if, for every ontology $\mathcal{T}'$ with $Sig(\mathcal{T}) \cap Sig(\mathcal{T}') \subseteq S$, we have that $\mathcal{T} \cup \mathcal{T}'$ is a conservative extension of $\mathcal{T}'$.*

Intuitively, given a set of terms, or seed signature, S, a S-module $\mathcal{M}$ based on deductive-*conservative extensions* is a minimal subset of an ontology $\mathcal{O}$ such that for all axioms α with terms only from S, we have that $\mathcal{M} \models \alpha$ if, and only if, $\mathcal{O} \models \alpha$, i.e., $\mathcal{O}$ and $\mathcal{M}$ have the same entailments over S. Besides safety, reuse of modules requires two additional properties namely *coverage* and *independence.*

Definition 5. (Module coverage [6]) *Let S be a signature and $\mathcal{T}'$, $\mathcal{T}$ be ontologies with $\mathcal{T}' \subseteq \mathcal{T}$ such that $S \subseteq Sig(\mathcal{T}')$. Then, $\mathcal{T}'$ guarantees coverage of S if $\mathcal{T}'$ is a module for S in $\mathcal{T}$.*

Definition 6. (Module Independence [6]) *Given an ontology $\mathcal{T}$ and signatures S_1, S_2, we say that $\mathcal{T}$ guarantees module independence if, for all $\mathcal{T}_1$ with $Sig(\mathcal{T}) \cap Sig(\mathcal{T}_1) \subseteq S_1$, it holds that $\mathcal{T} \cup \mathcal{T}_1$ is safe for S_2.*

Unfortunately, deciding whether or not a set of axioms is a minimal module is computationally hard or even impossible for expressive DLs [2, 3]. However, if the minimality requirement is dropped, good sized approximations can be defined that are efficiently computable, as in the case of *syntactic locality*, which modules are extracted in polynomial time.

Algorithm 3 (Extract a locality module [2])

`Procedure extract-module(`$\mathcal{T}, S, x$`)`
`Inputs: Tbox` $\mathcal{T}$`; signature` S`;` $x \in \bot, \top$`; Output` x`-module` $\mathcal{M}$

$\mathcal{M} := \emptyset; \mathcal{T}' = \mathcal{T};$
repeat
  change $=$ *false*
  for each $\alpha \in \mathcal{T}'$
    if α not x-local w.r.t. $S \cup$Sig($\mathcal{M}$) **then**
      $\mathcal{M} = \mathcal{M} + \{\alpha\}$
      $\mathcal{T}' = \mathcal{T}' \setminus \{\alpha\}$
      changed $=$ *true*
until *changed* $=$ *false*

Definition 7. (Syntactic locality [3]) *Let* $\mathbf{S}$ *be a signature and* $\mathcal{O}$ *a* $\mathcal{SROIQ}$ *ontology. An axiom* α *is* $\bot$*-local w.r.t.* S *(*$\top$*-local w.r.t* $\mathbf{S}$*) if* $\alpha \in Ax(\mathbf{S})$*, as defined in the grammar:*

$\bot$*-Locality*	
$\mathbf{Ax(S)}$	$::= C^{\bot} \sqsubseteq C \mid C \sqsubseteq C^{\top} \mid w^{\bot} \sqsubseteq R \mid Dis(S^{\bot}, S) \mid Dis(S, S^{\bot})$
$\mathbf{Con}^{\bot}\mathbf{(S)}$	$::= A^{\bot} \mid \neg C^{\top} \mid C^{\bot} \sqcap C \mid C \sqcap C^{\bot} \mid C_1^{\bot} \sqcup C_2^{\bot} \mid \exists R^{\bot}.C \mid \exists R.C^{\bot}$
	$\mid \exists R^{\bot}.Self \mid \geqslant nR^{\bot}.C \mid \geqslant nR.C^{\bot}$
$\mathbf{Con}^{\top}\mathbf{(S)}$	$::= \neg C^{\bot} \mid C_1^{\top} \sqcap C_2^{\top} \mid C^{\top} \sqcup C \mid C \sqcup C^{\top} \mid \forall R.C^{\top} \mid \leqslant nR.C^{\top}$
	$\mid \forall R^{\bot}.C \mid \leqslant nR^{\bot}.C$

$\top$*-Locality*	
$\mathbf{Ax(S)}$	$::= C^{\bot} \sqsubseteq C \mid C \sqsubseteq C^{\top} \mid w \sqsubseteq R^{\top}$
$\mathbf{Con}^{\bot}\mathbf{(S)}$	$::= \neg C^{\top} \mid C^{\bot} \sqcap C \mid C \sqcap C^{\bot} \mid C_1^{\bot} \sqcup C_2^{\bot} \mid \exists R.C^{\bot} \mid \geqslant nR.C^{\bot}$
	$\mid \forall R^{\top}.C^{\bot} \mid \leqslant nR^{\top}.C^{\bot}$
$\mathbf{Con}^{\top}\mathbf{(S)}$	$::= A^{\top} \mid \neg C^{\bot} \mid C_1^{\top} \sqcap C_2^{\top} \mid C^{\top} \sqcup C \mid C \sqcup C^{\top} \mid \forall R.C^{\top} \mid$
	$\exists R^{\top}.C^{\top} \mid \geqslant nR^{\top}.C^{\top} \mid \leqslant nR.C^{\top} \mid \forall R^{\bot}.C \mid \leqslant nR^{\bot}.C$

In the grammar, we have that $A^{\bot}, A^{\top} \notin \mathbf{S}$ *is an atomic concept,* $R^{\bot}, R^{\top}$ *(resp.* $S^{\bot}$*,*$S^{\top}$*) is either an atomic role (resp. a simple atomic role) not in* $\mathbf{S}$ *or the inverse of an atomic role (resp. of a simple atomic role) not in* $\mathbf{S}$*,* C *is any concept,* R *is any role,* S *is any simple role, and* $C^{\bot} \in \mathbf{Con}^{\bot}(\mathbf{S})$*,* $C^{\top} \in \mathbf{Con}^{\top}(\mathbf{S})$*. We also denote by* $w^{\bot}$ *a role chain* $w = R_1 \circ \ldots \circ R_n$ *such that for some* i *with* $1 \leq i \leq n$*, we have that* R_i *is (possibly inverse of) an atomic role not in* $\mathbf{S}$*. An ontology* $\mathcal{O}$ *is* $\bot$*-local (*$\top$*-local) w.r.t.* S *if* α *is* $\bot$*-local (*$\top$*-local) w.r.t.* S *for all* $\alpha \in \mathcal{O}$*.*

Algorithm 3 may be used to extract either $\top$- or $\bot$-locality modules. Alternating the algorithm between $\bot$- and $\top$-locality module extraction until a fixed-point is reached results in $\bot\top^*$ modules.

3 Normal form

In this section we will introduce a normal form for any $\mathcal{SROIQ}$ ontology. The normal form is required to facilitate the conversion process between a $\mathcal{SROIQ}$ ontology and a hypergraph.

Definition 8. *Given* $B_i \in N_C \setminus \{\bot\}$*,* $C_i \in N_C \setminus \{\top\}$*,* $D \in \{\exists R.B, \geq nR.B, \exists R.Self\}$*,* $R_i, S_i \in N_R$ *and* $n \geqslant 1$*, a* $\mathcal{SROIQ}$ *ontology* $\mathcal{O}$ *is in* **normal form** *if every axiom* $\alpha \in \mathcal{O}$ *is in one of the following forms:*

α_1: $B_1 \sqcap \ldots \sqcap B_n \sqsubseteq C_1 \sqcup \ldots \sqcup C_m$ α_2: $\exists R.B_1 \sqsubseteq C_1 \sqcup \ldots \sqcup C_m$
α_3: $B_1 \sqcap \ldots \sqcap B_n \sqsubseteq \exists R.B_{n+1}$ α_4: $B_1 \sqcap \ldots \sqcap B_n \sqsubseteq \exists R.Self$
α_5: $\exists R.Self \sqsubseteq C_1 \sqcup \ldots \sqcup C_m$ α_6: $\geqslant nR.B_1 \sqsubseteq C_1 \sqcup \ldots \sqcup C_m$
α_7: $B_1 \sqcap \ldots \sqcap B_n \sqsubseteq \geqslant nR.B_{n+1}$ α_8: $R_1 \circ \ldots \circ R_n \sqsubseteq R_{n+1}$
α_9: $D_1 \sqsubseteq D_2$

In order to normalize a $\mathcal{SROIQ}$ ontology $\mathcal{O}$ we repeatedly apply the normalization rules from Table 2. Each application of a rule rewrites an axiom into an equivalent normal form. Algorithm 4 illustrates the conversion process.

Algorithm 4 *Given any $\mathcal{SROIQ}$ axiom α:*

1. *Recursively apply rules NR7 - NR11 to eliminate all equivalences, universal restrictions, atmost restrictions and complex role fillers.*
2. *Given that $\alpha = (\alpha_L \sqsubseteq \alpha_R)$, recursively apply the following steps until α_L contains no disjunctions and α_R contains no conjunctions:*
   (a) recursively apply rules NR1, NR3, NR6 to α_L,
   (b) recursively apply rules NR2, NR4, NR5 to α_R.
3. *recursively apply any applicable rules from NR12 through NR21.*

Table 2. $\mathcal{SROIQ}$ normalization rules

NR1	$\neg\hat{C}_2 \sqsubseteq \hat{C}_1 \rightsquigarrow \top \sqsubseteq \hat{C}_1 \sqcup \hat{C}_2$
NR2	$\hat{B}_1 \sqsubseteq \neg\hat{B}_2 \rightsquigarrow \hat{B}_1 \sqcap \hat{B}_2 \sqsubseteq \bot$
NR3	$\hat{B} \sqcap \hat{D} \sqsubseteq \hat{C} \rightsquigarrow \hat{B} \sqcap A \sqsubseteq \hat{C},\ \hat{D} \sqsubseteq A,\ A \sqsubseteq \hat{D}$
NR4	$\hat{B} \sqsubseteq \hat{C} \sqcup \hat{D} \rightsquigarrow \hat{B} \sqsubseteq \hat{C} \sqcup A,\ \hat{D} \sqsubseteq A,\ A \sqsubseteq \hat{D}$
NR5	$\hat{B} \sqsubseteq \hat{C}_1 \sqcap \hat{C}_2 \rightsquigarrow \hat{B} \sqsubseteq \hat{C}_1,\ \hat{B} \sqsubseteq \hat{C}_2$
NR6	$\hat{B}_1 \sqcup \hat{B}_2 \sqsubseteq \hat{C} \rightsquigarrow \hat{B}_1 \sqsubseteq \hat{C},\ \hat{B}_2 \sqsubseteq \hat{C}$
NR7	$\ldots \forall R.\hat{C} \ldots \rightsquigarrow \ldots \neg\exists R.A \ldots,\ A \sqcap \hat{C} \sqsubseteq \bot,\ \top \sqsubseteq A \sqcup \hat{C}$
NR8	$\ldots \exists R.\hat{D} \ldots \rightsquigarrow \ldots \exists R.A \ldots,\ \hat{D} \sqsubseteq A,\ A \sqsubseteq \hat{D}$
NR9	$\ldots \geqslant nR.\hat{D} \ldots \rightsquigarrow \ldots \geqslant nR.A \ldots,\ \hat{D} \sqsubseteq A,\ A \sqsubseteq \hat{D}$
NR10	$\ldots \leqslant nR.\hat{C} \ldots \rightsquigarrow \ldots \neg(\geqslant (n+1)R.\hat{C}) \ldots$
NR11	$\hat{B} \equiv \hat{C} \rightsquigarrow \hat{B} \sqsubseteq \hat{C}, \hat{C} \sqsubseteq \hat{B}$
NR12	$\geqslant 0R.B \sqsubseteq \hat{C} \rightsquigarrow \top \sqsubseteq \hat{C}$
NR13	$\hat{B} \sqsubseteq \exists R.\bot \rightsquigarrow \hat{B} \sqsubseteq \bot$
NR14	$\hat{B} \sqsubseteq \geqslant nR.\bot \rightsquigarrow \hat{B} \sqsubseteq \bot$
NR15	$\hat{B} \sqsubseteq \geqslant 0R.B \rightsquigarrow$
NR16	$\geqslant nR.\bot \sqsubseteq \hat{C} \rightsquigarrow$
NR17	$\exists R.\bot \sqsubseteq \hat{C} \rightsquigarrow$
NR18	$\hat{B} \sqcap \bot \sqsubseteq \hat{C} \rightsquigarrow$
NR19	$\bot \sqsubseteq \hat{C} \rightsquigarrow$
NR20	$\hat{B} \sqsubseteq \hat{C} \sqcup \top \rightsquigarrow$
NR21	$\hat{B} \sqsubseteq \top \rightsquigarrow$

Above $A \notin N_C$, $\hat{B}_i$ and $\hat{C}_i$ are possibly complex concept descriptions and $\hat{D}$ a complex concept description. $R \in N_R$, $n \geqslant 0$. We note that rules NR18 and NR20 makes use of the commutativity of $\sqcap$ and $\sqcup$.

Theorem 1. *Algorithm 4 converts any $\mathcal{SROIQ}$ ontology $\mathcal{O}$ to an ontology $\mathcal{O}'$ in normal form, such that $\mathcal{O}'$ is a conservative extension of $\mathcal{O}$. The algorithm terminates in linear time and adds at most a linear number of axioms to $\mathcal{O}$.*

For every normalized ontology $\mathcal{O}'$ the definition of syntactic locality from Definition 7 may now be simplified to that of Definition 9. This is possible since for every axiom $\alpha = (\alpha_L \sqsubseteq \alpha_R) \in \mathcal{O}'$, $\bot$-locality of α is dependent solely on α_L and $\top$-locality is dependent solely on α_R.

Definition 9. (Normal form syntactic locality) *Let* **S** *be a signature and* $\mathcal{O}$ *a normalized* $\mathcal{SROIQ}$ *ontology. Any axiom* α *is* $\bot$*-local w.r.t.* **S** *(*$\top$*-local w.r.t* **S***) if* $\alpha \in Ax(\mathbf{S})$*, as defined in the grammar:*

$\bot$-*Locality*	
$\mathbf{Ax(S)}$	$::= C^{\bot} \sqsubseteq C \mid w^{\bot} \sqsubseteq R \mid Dis(S^{\bot}, S) \mid Dis(S, S^{\bot})$
$\mathbf{Con}^{\bot}\mathbf{(S)}$	$::= A^{\bot} \mid C^{\bot} \sqcap \mid C \sqcap C^{\bot} \mid \exists R^{\bot}.C \mid \exists R.C^{\bot} \mid \exists R^{\bot}.Self \mid$ $\geqslant nR^{\bot}.C \mid\geqslant nR.C^{\bot}$
$\top$-*Locality*	
$\mathbf{Ax(S)}$	$::= C \sqsubseteq C^{\top} \mid w \sqsubseteq R^{\top}$
$\mathbf{Con}^{\top}\mathbf{(S)}$	$::= A^{\top} \mid C^{\top} \sqcup C \mid C \sqcup C^{\top} \mid \exists R^{\top}.C^{\top} \mid\geqslant nR^{\top}.C^{\top} \mid$ $\exists R^{\top}.Self$

In the grammar, we have that $A^{\bot}, A^{\top} \notin \mathbf{S}$ *is an atomic concept,* $R^{\bot}$*,*$R^{\top}$ *(resp.* $S^{\bot}$*,*$S^{\top}$*) is either an atomic role (resp. a simple atomic role) not in* **S** *or the inverse of an atomic role (resp. of a simple atomic role) not in* **S***,* C *is any concept,* R *is any role,* S *is any simple role, and* $C^{\bot} \in \mathbf{Con}^{\bot}(\mathbf{S})$*,* $C^{\top} \in \mathbf{Con}^{\top}(\mathbf{S})$*. We also denote by* $w^{\bot}$ *a role chain* $w = R_1 \circ \ldots \circ R_n$ *such that for some* i *with* $1 \leq i \leq n$*, we have that* R_i *is (possibly inverse of) an atomic role not in* **S***. An ontology* $\mathcal{O}$ *is* $\bot$*-local(*$\top$*-local) w.r.t.* **S** *if* α *is* $\bot$*-local(*$\top$*-local) w.r.t.* **S** *for all* $\alpha \in \mathcal{O}$*.*

We note that we may denormalize a normalized ontology if we maintain a possibly many-to-many mapping from normalized axioms to their original source axioms. Formally, define a function $denorm : \hat{\mathcal{O}} \rightarrow 2^{\mathcal{O}}$, with $\mathcal{O}$ an $\mathcal{SROIQ}$ ontology and $\hat{\mathcal{O}}$ its normal form. For brevity, we write $denorm(\Phi)$, with Φ a set of normalized axioms, to denote $\bigcup_{\alpha \in \Phi} denorm(\alpha)$.

4 $\mathcal{SROIQ}$ hypergraph

Suntisrivaraporn [11] showed that for the DL $\mathcal{EL}^{+}$, extracting $\bot$-locality modules are equivalent to the reachability problem in directed hypergraphs. This was extended in [9, 10] to include a reachability algorithm for $\top$-locality modules. In this section we show that a $\mathcal{SROIQ}$ ontology $\mathcal{O}$ in normal form can be mapped to a hypergraph which preserves both $\bot$-locality and $\top$-locality.

Definition 10. *Let* α *be a normalized axiom and* $\alpha_{\bot}$ *a minimum set of symbols from* $Sig(\alpha)$ *required to ensure that* α *is not* $\bot$*-local, and let* $\mathcal{H} = (\mathcal{V}, \mathcal{E})$ *be a hypergraph. We say that an edge* $e \in \mathcal{E}$ *preserves* $\bot$*-locality iff* $\alpha_{\bot} = T(e)$*. Similarly,* $e \in \mathcal{E}$ *preserves* $\top$*-locality whenever* $\alpha_{\top} = H(e)$*.*

For each normal form axiom α_i in Definition 8 we show that α_i may be mapped to a set of hyperedges, with nodes denoting symbols from $\mathrm{Sig}(\alpha_i)$, such that both $\bot$-locality and $\top$-locality are simultaneously preserved.

- Given $\alpha_1 : B_1 \sqcap \ldots \sqcap B_n \sqsubseteq C_1 \sqcup \ldots \sqcup C_m$ we map it to the hyperedge $e_{\alpha_1} = (\{B_1, \ldots, B_n\}, \{C_1, \ldots, C_m\})$. We transform the hyperedge e_{α_1} to two new hyperedges $e^{B}_{\alpha_1} = (\{B_1, \ldots, B_n\}, \{H_1\})$ a B-hyperedge, $e^{F}_{\alpha_1} =$

$(\{H_1\}, \{C_1, \ldots, C_m\})$ an F-hyperedge and with H_1 a new node. By definition each C_j is B-connected to H_1 if all B_i are B-connected to H_1. From Definition 9 we know that this preserves $\bot$-locality for α_1 since it is $\bot$-local, w.r.t. a signature S, exactly when any of the conjuncts $B_i \notin S$. In other words it is non $\bot$-local exactly when all $B_i \in S$. The same also holds for $\top$-locality, since $e^F_{\alpha_1}$ requires every $C_i \in \alpha_1$ to be in S for H_1 to be F-connected. From Definition 9 we see that, w.r.t. a signature S, $e^F_{\alpha_1}$ is $\top$-local exactly when any of the disjuncts $C_i \notin S$.

- Given $\alpha_2 : \exists R.B_1 \sqsubseteq C_1 \sqcup \ldots \sqcup C_m$ or $\alpha_6 : \geqslant nR.B_1 \sqsubseteq C_1 \sqcup \ldots \sqcup C_m$ we map it to the two hyperedges $e^B_{\alpha_{2/6}} = (\{B_1, R\}, \{H_2\})$, $e^F_{\alpha_{2/6}} = (\{H_2\}, \{C_1, \ldots, C_m\})$ an F-hyperedge and with H_2 a new node. This mapping preserves $\bot$-locality for $\alpha_{2/6}$ since by Definition 9 it is $\bot$-local, w.r.t. a signature S, exactly when either B_1 or R is not in S. The argument for $\top$-locality follows that of α_1.
- Given $\alpha_3 : B_1 \sqcap \ldots \sqcap B_n \sqsubseteq \exists R.B_{n+1}$ or $\alpha_7 : B_1 \sqcap \ldots \sqcap B_n \sqsubseteq \geqslant nR.B_{n+1}$ we map it to the hyperedges $e^B_{\alpha_{3/7}} = (\{B_1, B_2, \ldots, B_{n-1}, B_n\}, \{H_3\})$, $e^{F_1}_{\alpha_{3/7}} = (\{H_3\}, \{B_{n+1}\})$, $e^{F_2}_{\alpha_{3/7}} = (\{H_3\}, \{R\})$. This mapping preserves $\bot$-locality for $\alpha_{3/7}$ similarly to $e^B_{\alpha_1}$ for α_1. From Definition 9 we know that $\top$-locality for either of these axioms, w.r.t. a signature S, is dependent on neither R nor B_{n+1} being elements of S. Therefore, they are non $\top$-local exactly when either or both of these are in S. This is represented by the two edges $e^{F_1}_{\alpha_{3/7}}$ and $e^{F_2}_{\alpha_{3/7}}$ for which H_3 becomes F-connected exactly when either R or B_{n+1} is F-connected.
- Given $\alpha_4 : B_1 \sqcap \ldots \sqcap B_n \sqsubseteq \exists R.Self$ and $\alpha_5 : \exists R.Self \sqsubseteq C_1 \sqcup \ldots \sqcup C_m$ we see that $\exists R.Self$ is both $\bot$ or $\top$ local exactly when $R \notin S$. Therefore we map α_4 to the hyperedge $e^B_{\alpha_4} = (\{R\}, \{C_1, \ldots, C_m\})$, and α_5 to the hyperedge $e^F_{\alpha_5} = (\{B_1, \ldots, B_n\}, \{R\})$.
- Given $\alpha_8 : R_1 \circ \ldots \circ R_n \sqsubseteq R_{n+1}$, we see that α_8 is $\bot$-local exactly when any $R_i \notin S, i \leq n$ and is $\top$-local exactly when $R_{n+1} \notin S$. We therefore map α_8 to the hyperedge $e^B_{\alpha_8} = (\{R_1, \ldots, R_n\}, \{R_{n+1}\})$.
- For α_9 we have many forms, all variants of those discussed in the previous mappings. Therefore α_9 is mapped to any of the following: $e^{B_1}_{\alpha_9} = (\{R, B_1\}, \{H_9\})$, $e^{F_1}_{\alpha_9} = (\{H_9\}, \{R\})$, $e^{F_2}_{\alpha_9} = (\{H_9\}, \{B\})$, or $e^1_{\alpha_9} = (\{R, B_1\}, \{R\})$, or $e^{F_1}_{\alpha_9} = (\{R_1\}, \{R_2\})$, $e^{F_2}_{\alpha_9} = (\{R_1\}, \{B\})$, or $e^1_{\alpha_9} = (\{R_1\}, \{R_2\})$.

Given a $\mathcal{SROIQ}$ ontology $\mathcal{O}$ in normal form we may now map every axiom $\alpha \in \mathcal{O}$ to its equivalent set of hyperedges. For each of these mappings there are at most three hyperedges introduced, therefore mapping the whole ontology $\mathcal{O}$ to an equivalent hypergraph $\mathcal{H}_\mathcal{O}$ will result in a hypergraph with the number of edges at most linear in the number of axioms in $\mathcal{O}$. It is easy to show that the mapping process can be completed in linear time in the number of axioms in $\mathcal{O}$.

We note that, similar to the normalization process, we may maintain a possibly many-to-many mapping from normalized axioms to their associated hyperedges. Formally, define a function $deedge : \mathcal{H}_\mathcal{O} \rightarrow 2^\mathcal{O}$, with $\mathcal{O}$ a $\mathcal{SROIQ}$ ontology and $\mathcal{H}_\mathcal{O}$ its hypergraph. For brevity, we write $deedge(\Phi)$, with Φ a set of hyperedges, to denote $\bigcup_{e \in \Phi} deedge(e)$.

5 Hypergraph module extraction

In this section we show that, given a hypergraph $\mathcal{H}_\mathcal{O}$ for a $\mathcal{SROIQ}$ ontology $\mathcal{O}$, we may extract a frontier graph from $\mathcal{H}_\mathcal{O}$ which is a subset of a $\bot\top^*$ module. We show that some of these modules guarantee safety, module coverage and module independence. The hypergraph algorithms presented require one start node s and a destination node t. In order to extend these algorithms to work with an arbitrary signature S, we introduce a new node s with with an edge $e_{s_i} = (s, s_i)$ for each $s_i \in S \cup \top$, as well as a new node t with an edge $e_{t_i} = (s_i, t)$ for each $s_i \in S \cup \bot$.

Theorem 2. *Let $\mathcal{O}$ be a $\mathcal{SROIQ}$ ontology and $\mathcal{H}_\mathcal{O}$ its associated hypergraph and S a signature. Algorithm 1 - Bvisit extracts a set of B-hyperpaths $\mathcal{H}_\mathcal{O}^B$ corresponding to the $\bot$-locality module for S in $\mathcal{O}$. Therefore, these modules also guarantees safety, module coverage and module independence.*

Theorem 3. *Let $\mathcal{O}$ be a $\mathcal{SROIQ}$ ontology and $\mathcal{H}_\mathcal{O}$ its associated hypergraph and S a signature. Algorithm 1 - Fvisit extracts a set of F-hyperpaths $\mathcal{H}_\mathcal{O}^F$ corresponding to a subset of the $\top$-locality module for S in $\mathcal{O}$.*

Theorem 4. *Let $\mathcal{O}$ be a $\mathcal{SROIQ}$ ontology and $\mathcal{H}_\mathcal{O}$ its associated hypergraph and S a signature. Algorithm 2 extracts a frontier graph $\mathcal{H}_\mathcal{O}^{BF}$ corresponding to a subset of the $\bot\top^*$-locality module for S in $\mathcal{O}$.*

The module extracted in Theorem 3 is a subset of the $\top$-locality module for a given seed signature. It is as yet unclear whether or not these modules provide all the model-theoretic properties associated with $\top$-locality modules. However, from the previous work done for the DL $\mathcal{EL}^+$ [10], it is evident that these modules preserve all entailments for a given seed signature S. Further, they also preserve and contain all justifications for any given entailment. Similarly, the exact module theoretic properties of modules associated with frontier graphs is something we are currently looking into.

6 Conclusion

We have introduced a normal form for any $\mathcal{SROIQ}$ ontology, as well as the necessary algorithms in order to map any $\mathcal{SROIQ}$ ontology to a syntactic locality preserving hypergraph. This mapping process can be accomplished in linear time in the number of axioms with at most a linear increase in the number of hyperedges in the hypergraph.

Standard path searching algorithms for hypergraphs may now be used to find: (1) sets of B-hyperpaths — this is equivalent to finding $\bot$-syntactical locality modules; (2) sets of F-hyperpaths — these are subsets of $\top$-locality modules, and (3) frontier graphs — these are subsets of $\bot\top^*$ modules. Whilst the modules associated with B-hyperpaths share all the module theoretic properties of $\bot$-locality modules, it is unclear at this point which module-theoretic properties modules associated with F-hyperpaths and frontier graphs possess.

The ability to map $\mathcal{SROIQ}$ ontologies to hypergraphs, such that hyperedges preserve syntactic locality conditions, allows us to investigate the relationship between DL reasoning algorithms and the vast body of standard hypergraph algorithms in greater depth.

Our primary focus for future research is to investigate and define the module-theoretic properties of modules associated with F-hyperpaths and frontier graphs as well as their relative performance with respect to existing locality methods. Thereafter, we aim to expand our research and investigate other hypergraph algorithms and how they may be applied to DL reasoning problems.

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