

Functionally Graded Material: An Overview

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Abstract— Functionally Graded Material (FGM) belongs to a class of advanced material characterized by variation in properties as the dimension varies. The overall properties of FMG are unique and different from any of the individual material that forms it. There is a wide range of applications for FGM and it is expected to increase as the cost of material processing and fabrication processes are reduced by improving these processes. In this study, an overview of fabrication processes, area of application, some recent research studies and the need to focus more research effort on improving the most promising FGM fabrication method (solid freeform SFF) is presented. Improving the performance of SFF processes and extensive studies on material characterization on components produced will go a long way in bringing down the manufacturing cost of FGM and increase productivity in this regard.

Keywords— Applications of FGM, Functionally graded material, Material characterization of FGM, Processing technique of FGM, Solid freeform fabrication.

I. INTRODUCTION

Pure metals are of little use in engineering applications because of the demand of conflicting property requirement. For example, an application may require a material that is hard as well as ductile, there is no such material existing in nature. To solve this problem, combination (in molten state) of one metal with other metals or non-metals is used. This combination of materials in the molten state is termed alloying (recently referred to as conventional alloying) that gives a property that is different from the parent materials. Bronze, alloy of copper and tin, was the first alloy that appears in human history [1]. Bronze really impacted the world at that time, it was a landmark in human achievement and it is tagged the 'Bronze Age' in about 4000 BC [1]. Since then, man has been experimenting with one form of alloy or the other with

the sole reason of improving properties of material. There is limit to which a material can be dissolved in a solution of another material because of thermodynamic equilibrium limit [2]. When more quantity of the alloying material is desired, then the traditional alloying cannot be used. Another limitation of conventional alloying is when alloying two dissimilar materials with wide apart melting temperatures; it becomes prohibitive to combine these materials through this process. Powdered Metallurgy (PM) is another method of producing part that cannot be produced through the conventional alloying, as alloys are produced in powdered form and some of the problems associated with the conventional alloying are overcome. Despite the excellent characteristics of powdered metallurgy, there exist some limitations, which include: intricate shapes and features that cannot be produced using PM; the parts are porous and have poor strength [3]. Although these limitations are of advantage to some applications (e.g. filter and non structural applications) but are detrimental to others. Another method of producing materials with combination of properties is by combining materials in solid state, which is referred to as composite material.

Composite material are a class of advanced material, made up of one or more materials combined in solid states with distinct physical and chemical properties. Composite material offers an excellent combination of properties which are different from the individual parent materials and are also lighter in weight. Wood is a composite material from nature which consists of cellulose in a matrix of lignin [4]. Composite materials will fail under extreme working conditions through a process called delamination (separation of fibers from the matrix) [5]. This can happen for example, in high temperature application where two metals with different coefficient of expansion are used. To solve this problem, researchers in Japan in the mid 1980s, confronted with this challenge in an hypersonic space plane project requiring a thermal barrier (with outside temperature of 2000K and inside temperature of 1000K across less than 10 mm thickness), came up with a novel material called Functionally Graded Material (FGM) [6,7].

Functionally Graded Material (FGM), a revolutionary material, belongs to a class of advanced materials with varying properties over a changing dimension [8, 9]. Functionally graded materials occur in nature as bones, teeth etc. [10], nature designed this materials to meet their expected service requirements. This idea is emulated from nature to solve engineering problem the same way artificial neural network is used to emulate human brain. Functionally graded material, eliminates the sharp

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interfaces existing in composite material which is where failure is initiated [5]. It replaces this sharp interface with a gradient interface which produces smooth transition from one material to the next [6, 7]. One unique characteristics of FGM is the ability to tailor a material for specific application [8].

There are different kinds of fabrication processes for producing functionally graded materials. Functionally graded materials can be divided into two broad groups namely: thin and bulk FGM. Thin FGM are relatively thin sections or thin surface coating, while the bulk FGM are volume of materials which require more labour intensive processes. Thin section or surface coating FGM are produced by Physical or Chemical Vapour Deposition (PVD/CVD), Plasma Spraying, Self-propagating High-temperature Synthesis (SHS) etc [11]. Bulk FGM is produced using powder metallurgy technique, centrifugal casting method, solid freeform technology etc [10]. Functionally graded materials find there applications in aerospace, automobile, medicine, sport, energy, sensors, optoelectronic etc [12]. As the fabrication process is improved, cost of powder is reduced and the overall process cost is reduced, hence expanding the application of FGM. Owing to the importance of FGM, there are lots of research efforts at improving the material processing, fabrication processing and properties of the FGM.

This paper presents an overview of fabrication methods and application areas of functionally graded materials. Some research works on functionally graded materials in recent times are presented and the future research needs are proposed. The rest of the paper is organized as follows: processing techniques of functionally graded materials is presented in section II, section III gives areas of application of FGM. Recent research efforts are presented in section IV, while future research needs is presented in section V. The paper ends with concluding remarks in section VI.

II. PROCESSING TECHNIQUES OF FUNCTIONALLY GRADED MATERIALS (FGM)

Thin functionally graded materials are usually in the form of surface coatings, there are a wide range of surface deposition processes to choose from depending on the service requirement from the process.

A. Vapour Deposition Technique

There are different types of vapour deposition techniques, they include: sputter deposition, Chemical Vapour Deposition (CVD) and Physical Vapour Deposition (PVD). These vapour deposition methods are used to deposit functionally graded surface coatings and they give excellent microstructure, but they can only be used for depositing thin surface coating. They are energy intensive and produce poisonous gases as their byproducts [13].

Other methods used in producing functionally graded coating include: plasma spraying, electrodeposition, electrophoretic, Ion Beam Assisted Deposition (IBAD), Self-Propagating High-temperature Synthesis (SHS), etc

[10]. All the above mentioned processes cannot be used to produce bulk FGM because they are generally slow and energy intensive, therefore they are uneconomical to be used in producing bulk FGM. Some of the fabrication methods for producing bulk functionally graded materials are as follows:

B. Powder Metallurgy (PM)

Powder metallurgy (PM) technique is used to produce functionally graded material [14, 15] through three basic steps namely: weighing and mixing of powder according to the pre-designed spatial distribution as dictated by the functional requirement, stacking and ramming of the premixed-powders, and finally sintering [16]. PM technique gives rise to a stepwise structure. If continuous structure is desired, then centrifugal method is used.

C. Centrifugal Method

Centrifugal method is similar to centrifugal casting where the force of gravity is used through spinning of the mould to form bulk functionally graded material [17]. The graded material is produced in this way because of the difference in material densities and the spinning of the mould. There are other similar processes like centrifugal method in the literature (e.g. gravity method, etc.). Although continuous grading can be achieved using centrifugal method but only cylindrical shapes can be formed. Another problem of centrifugal method is that there is limit to which type of gradient can be produced [18] because the gradient is formed through natural process (centrifugal force and density difference). To solve these problems, researchers are using alternative manufacturing method known as solid freeform.

D. Solid Freeform (SFF) Fabrication Method

Solid freeform is an additive manufacturing process that offers lots of advantages that include: higher speed of production, less energy intensive, maximum material utilization, ability to produce complex shapes and design freedom as parts are produced directly from CAD (e.g. AutoCAD) data [19]. SFF involves five basic steps [20]: generation of CAD data from the software like AutoCAD, Solid edge etc, conversion of the CAD data to Standard Triangulation Language (STL) file, slicing of the STL into two dimensional cross-section profiles, building of the component layer by layer, and lastly removal and finishing. There are various types of SFF technologies, laser based processes are mostly employed in fabrication of functionally graded materials [21]. Laser based SFF process for FGM [22, 23] include: laser cladding based method [9-14, 22, 24-30], Selective Laser Sintering (SLS) [31, 32], 3-D Printing (3-DP) [26, 29], and Selective Laser Melting (SLM) [25, 28]. Laser cladding based system and selective laser melting are capable of producing fully dense components. Solid freeform provide manufacturing flexibility amongst other advantages but the technology is characterized by poor surface finish making it necessary to carry out a secondary finishing operation. There are lots of

research efforts in this direction to improve surface finish, dimensional accuracy etc.

There are other fabrication methods for functionally graded materials; readers can refer to the review studies by Kieback and Neubrand; and Gasik, [18, 33]. These authors presented comprehensive processing techniques of functionally graded materials.

III. AREAS OF APPLICATION OF FGM

Some of the applications of functionally graded materials are highlighted below:

A. Aerospace

Functionally graded materials can withstand very high thermal gradient, this makes it suitable for use in structures and space plane body, rocket engine component etc [34]. If processing technique is improved, FGM are promising and can be used in wider areas of aerospace.

B. Medicine

Living tissues like bones and teeth are characterized as functionally graded material from nature [35], to replace these tissues, a compatible material is needed that will serve the purpose of the original bio-tissue. The ideal candidate for this application is functionally graded material. FGM has find wide range of application in dental [36] and orthopedic applications for teeth and bone replacement [37].

C. Defense

One of the most important characteristics of functionally graded material is the ability to inhibit crack propagation. This property makes it useful in defense application, as a penetration resistant materials used for armour plates and bullet-proof vests [38].

D. Energy

FGM are used in energy conversion devices. They also provide thermal barrier and are used as protective coating on turbine blades in gas turbine engine [39, 40].

E. Optoelectronics

FGM also finds its application in optoelectronics as graded refractive index materials and in audio-video discs magnetic storage media.

Other areas of application are: cutting tool insert coating, automobile engine components, nuclear reactor components, turbine blade, heat exchanger, Tribology, sensors, fire retardant doors, etc [41-43]. The list is endless and more application is springing up as the processing technology, cost of production and properties of FMG improve [44].

IV. RECENT RESEARCH EFFORTS IN FGM

Lots of studies have been conducted on behaviour of functionally graded materials [45-55], and the literature is very rich on this because of the wide areas of application of this novel material. Performance of FGM under localized

transverse loading was investigated by Woodward and Kashtalyan [41] and property estimation study was conducted by Lu *et al.*, [38]. A comprehensive review on performance of FGM was published in 2007 by Birman and Byrd, [56]. An overview on fracture behaviour of FGM was conducted by Shanmugavel *et al.*, [8]. Other reviews on functionally graded materials available in the literature are: review study on research and development by Cherradi *et al.*, [57], Tilbrook *et al.*, also conducted review study on crack propagation in functionally graded materials [58]. A number of researches have also been conducted in the areas of analysis and modeling work on functionally graded material; some of these work can be found in [59-67]. There are still more to be done in terms of research to improve the performance of manufacturing processes of FGM.

V. FUTURE RESEARCH DIRECTION

Functionally graded material is an excellent advanced material that will revolutionize the manufacturing world in the 21st century. There are a number of roadblocks for realizing this objective. Cost is a major problem, with substantial part of the cost expended on powder processing and fabrication method. Solid freeform fabrication technique offers a greater advantage for producing FGM, but there are still lots of issues that need to be resolved with this promising technology. More research needs to be conducted on improving the performance of SFF processes through extensive characterization of functionally graded material in other to generate a comprehensive database and to develop a predictive model for proper process control. Further work should also be done to improve the process control through development of more powerful feedback control for overall FMG fabrication process improvement (i.e. full automation). This will improve the overall performance of the process, bring down the cost of FGM and improve reliability of the fabrication process.

VI. CONCLUSION

Functionally graded materials are very important in engineering and other applications but the cost of producing these materials makes it prohibitive in some applications. This study presents an overview on FGM, various fabrication methods were highlighted with solid freeform providing the best advantage over other processes because of the manufacturing flexibility it offers. An overview of different application areas is also presented and how the application area can further be enhanced and also extended by bringing down the fabrication cost through improving the most promising fabrication method (SFF).

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