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Real Interest Rate Persistence in South Africa: Evidence and Implications

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Abstract

The real interest rate is a very important variable in the transmission of monetary policy. It features in vast majority of financial and macroeconomic models. Though the theoretical importance of the real interest rate has generated a sizable literature that examines its long-run properties, surprisingly, there does not exist any study that delves into this issue for South Africa. Given this, using quarterly data (1960:Q2-2010:Q4) for South Africa, our paper endeavors to analyze the long-run properties of the EPRR by using tests of unit root, cointegration, fractional integration and structural breaks. In addition, we also analyze whether monetary shocks contribute to fluctuations in the real interest rate based on test of structural breaks of the rate of inflation, as well as, Bayesian change point analysis. Based on the tests conducted, we conclude that the South African EPPR can be best viewed as a very persistent but ultimately mean-reverting process. Also, the persistence in the real interest rate can be tentatively considered as a monetary phenomenon.

Keywords: Real Interest Rate, Monetary Policy, Persistence, Mean Reversion.
JEL Classification: C22, E21, E44, E52, E62, G12.

1 Introduction

Macroeconomic and financial theoretical models, *e.g.* the consumption-based asset pricing model (Lucas, 1978; Breeden, 1979; Hansen and Singleton, 1982, 1983), neoclassical growth models (Cass, 1965; Koopmans, 1965), central bank policy models (Taylor, 1993) and many monetary transmission mechanism models include the real interest rate (interest rate less expected or realized inflation rate) as a key variable. There are two types of real interest rates: the *ex ante* real interest rate (EARR) and the *ex post* real rate (EPRR). Economic agents base their decisions on their expectations about the inflation level over the decision horizon. As such, the EARR turns out to be the appropriate gauge for assessing economic decisions. However, given that the EARR cannot be directly observable, we cannot evaluate its time-series properties.

Though the theoretical importance of the real interest rate has generated a sizable literature[1] that examines its long-run properties, surprisingly, to the best of our knowledge, there does not exist

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[1]See Neely and Rapach (2008) for a detailed literature review.

any study that delves into this issue for South Africa.[2] Against this backdrop, using quarterly data (1960:Q2-2010:Q4) for South Africa, our paper endeavors to analyze the long-run properties of the EPRR by using tests of unit root, cointegration, fractional integration and structural breaks. In addition, we also analyze whether monetary shocks contribute to fluctuations in the real interest rate based on the Bai and Perron (1998) test of structural breaks of the rate of inflation, as well as, Bayesian change point analysis proposed by Barry and Hartigan (1993). The remainder of the paper is organized as follows: Section 2 discusses the theoretical background on the long-run behavior of the real interest rate. Section 3 lays out the difference between the EARR and EPRR and presents the results from the unit root, cointegration and fractional integration tests; while Section 4 analyzes structural breaks in the real interest rate. Section 5 investigates the monetary explanation of persistence, and finally Section 6 concludes the paper.

2 Theoretical Background

Consumption-Based Asset Pricing Model

Lucas (1978), Breeden (1979), and Hansen and Singleton (1982, 1983)'s canonical consumption based asset pricing model hypothesizes a representative household choosing a real consumption sequence, $\{c_t\}_{t=0}^{\infty}$, to solve the problem:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t),$$

subject to an intertemporal budget constraint. β is a discount factor and $u(c_t)$ represents an instantaneous utility function. The first-order condition yields the intertemporal Euler equation,

(1) $\qquad\qquad\qquad E_t\{\beta[u'(c_{t+1})/u'(c_t)](1 + r_t)\} = 1,$

where $1 + r_t$ represents the gross one-period real interest rate with payoff at period $t + 1$ and E_t is the conditional expectation operator. Many studies often consider the utility function to have the constant relative risk aversion form, $u(c_t) = c_t^{1-\gamma}/(1 - \gamma)$ Where γ is the coefficient of relative risk aversion. Therefore, making use of the assumption of joint log-normality of consumption growth and the real interest rate, the log-linear version of equation (1) can be written as (Hansen and Singleton, 1982, 1983):

(2) $\qquad\qquad\qquad \kappa - \gamma E_t[\Delta log(c_{t+1})] + E_t[log(1 + r_t)] = 0,$

where $\Delta log(c_{t+1}) = log(c_{t+1}) - log(c_t) = log(\beta) + 0.5\sigma^2$, and σ^2 is the constant conditional variance of $log[\beta(c_{l+1}/c_l)^{-\alpha}(1 + r_l)]$. Equation (2) relates the conditional expectations of the *per capita* real consumption growth rate $[\Delta log(c_{t+1})]$ with the (net) real interest rate $[log(1 + r_t) \approx r_t]$. According to Rose (1988), if equation (2) is to hold, then the *per capita* real consumption growth rate and the (net) real interest rate series must have the same integration properties. Bearing in mind that $[\Delta log(c_{t+1})]$ is nearly without doubt a stationary process, Rose (1988) shows that the real interest rate is non-stationary *i.e.* $r_t \backsim I(1)$ in many industrialized countries. A unit root in the real interest rate together with stationary consumption growth imply that permanent changes in the level of the real rate will be unmatched by such changes in consumption growth. Therefore, equation (2) seemingly cannot hold. The problem identified by Rose (1988) is vindicated in Figure 1, which

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[2]Studies that exists for South Africa only deals with the uncovered interest rate parity condition, or in other words, the interest rate behavior of South Africa relative to other developed or emerging economies. See for example Kahn and Farrell (2002), Kryshko (2006), Lacerda et al., (2010) and de Bruyn et al., (2011).

plots South Africa's ex-post 3-month Treasury bill based real interest rate and annualized *per capita* consumption growth rate for the period from 1960:Q2 to 2010:Q4. Figure 1 shows that the two series seem to be divergent for most of the period prior to the 1980s whereas the series appear to track each other quite well during the early 1980s and from the late 1980s to the last date of the sample.[3]

Fig. 1: **Ex Post Real Interest Rate and Real *per capita* Consumption Growth, 1960:Q2-2010:Q4**
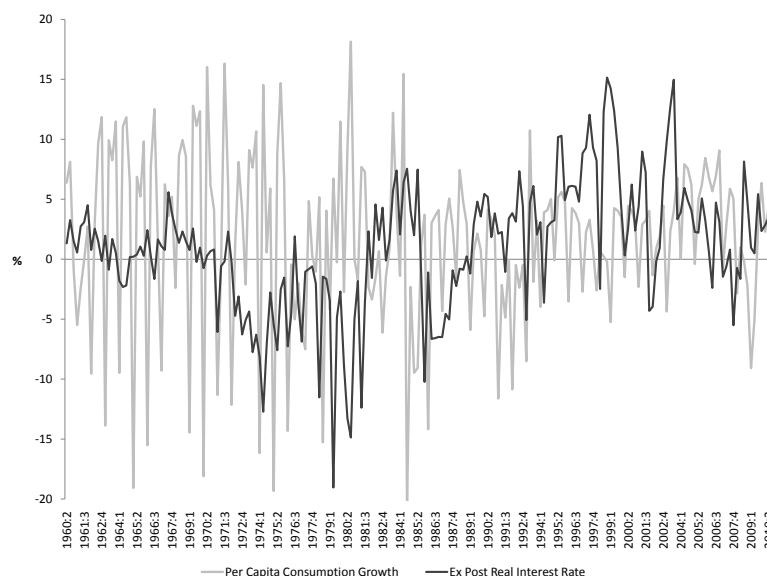
NOTE: The figure plots SA ex post 3-month real interest rate and annualized *per capita* consumption growth. Consumption includes nondurable goods and services consumption.

3   Testing the Integration Properties of Real Interest Rates

Ex Ante versus *Ex Post* Real Interest Rates

The *ex ante* real interest rate (EARR) refers to the nominal interest rate less the expected inflation rate. On the other hand, the *ex post* real rate (EPRR) refers to the nominal interest rate less actual inflation. Economic agents base their decisions on their expectations about the inflation level over the decision horizon. To illustrate, the Euler equations (1) and (2) link the expected marginal utility of consumption to the expected real return. Consequently, the EARR turns out to be the appropriate gauge for assessing economic decisions. However, given that the EARR cannot be directly observable, we cannot evaluate its time-series properties.

Two alternative approaches to the problem of unobserved expectations. The first approach uses econometric forecasting methods to construct inflation forecasts (Mishkin, 1981, 1984; Huizinga and Mishkin,1986). However, all the appropriate information used by economic agents when forming their inflation expectations are not included in econometric forecasting. In addition, these forecasting models fail to change with the structure of the economy.

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[3]The most elementary consumption-based asset pricing model is based on an endowment economy with a representative household and constant preferences. However, more elaborate theoretical models allowing for changes in fiscal or monetary policy to affect the steady-state real interest rate, while leaving the steady-state consumption growth rate unaffected have been proposed in the literature. These models allow a disparity in the integration properties of the consumption growth and real interest rates. See Neely and Rapach (2008) for extensive review of such models.

The second approach uses the actual inflation rate as a proxy for inflation expectations. By definition, the actual inflation rate at time $t(\pi_t)$ is the sum of the expected inflation rate and a forecast error term (ε_t):

$$\text{(3)} \qquad \pi_t = E_{t-1}\pi_t + \varepsilon_t.$$

If expectations are formed rationally, $E_{t-1}\pi_t$ should be an optimal forecast of inflation (Nelson and Schwert, 1977), and εt should therefore be a white noise process. The EARR can approximately be expressed as:

$$\text{(4)} \qquad r_t^{ea} = i_t - E_t\pi_{t+1},$$

where i_t is the nominal interest rate. Solving equation (3) for $E_t(\pi_{t+1})$ and substituting it into equation (4), we obtain

$$\text{(5)} \qquad \begin{aligned} r_t^{ea} &= i_t - (\pi_{t+1} - \varepsilon_{t+1}) \\ &= i_t - \pi_{t+1} + \varepsilon_{t+1} = r_t^{ep} + \varepsilon_{t+1}, \end{aligned}$$

where $r_t^{ep} = i_t - \pi_{t+1}$ is the EPRR. Equation (5) implies that, under rational expectations, only a white noise component distinguishes the EPRR from the EARR. Consequently, the EPRR and EARR will have the same long-run (integration) properties. This result holds if the expectation errors (ε_{t+1}) are stationary and does not necessarily requires rational expectations to hold (Peláez, 1995; Andolfatto *et al.*, 2008).

The literature usually assesses the integration properties of the EPRR through a decision rule. First, the individual components of the EPRR *i.e.* i_t and π_{t+1} are analyzed. If unit root tests reveal that both i_t and π_{t+1} are $I(0)$, then EPRR is stationary. On the other hand, if i_t and π_{t+1} have different orders of integration *e.g.* $i_t \sim I(1)$ and $\pi_{t+1} \sim I(0)$, then EPRR must be non-stationary, as a linear combination of an $I(0)$ process and an $I(1)$ process results in an $I(1)$ process. Lastly, if both i_t and π_{t+1} are $I(1)$, then stationarity of the EPRR is assessed through a cointegration test of i_t and π_{t+1} *i.e.* testing if the linear combination $i_t - [\theta_0 + \theta_1\pi_{t+1}]$ is stationary. Two approaches are used in the literature: First, a cointegrating vector of $(1, -\theta_1)' = (1, -1)'$ is imposed and thereafter a a unit root test of $r_t^{ep} = i_t - \pi_{t+1}$ is applied. Such an approach usually has more power to reject the null of cointegration when the true cointegrating vector is $(1, -1)'$. Alternatively, the second approach involves freely estimating the cointegrating vector between i_t and π_{t+1} thereby allowing for tax effects (Darby, 1975). If i_t and π_{t+1} are $I(1)$ processes then EPRR requires $\theta_1 = 1$ or $\frac{1}{1-\tau}$ with τ being the marginal tax rate on the nominal interest income. Generally, estimates of θ_1 in the range of 1.30 to 1.40 is considered plausible when allowing for tax effects, since this would imply amarginal tax rate of 20 to 30 percent. Note that cointegration between i_t and π_{t+1} does not necessarily imply that the EPRR is stationary, one requires $\theta_1 = 1$ or $\frac{1}{\tau}$ in addition, since other values of θ_1 would imply that the equilibrium real interest rate varies with inflation.

Unit Root and Cointegration Tests

There exist a vast literature on unit root and cointegration tests applied to assessing the time series properties of the real interest rate.[4] Table 1 illustrates the unit root tests based on the augmented Dickey and Fuller (1979) [ADF] and the MZ_α test proposed by Ng and Perron (2001) for the South African 3-month Treasury bill rate, the Consumer Price Index (CPI) inflation, the *ex post* real interest rate and the *per capita* consumption growth rate. The MZ_α statistic is designed to have better size and power properties than the ADF test. Note that the data for the Treasury bill rate

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[4]This can be found in Neely and Rapach (2008).

and CPI are obtained from International Monetary Fund's International Financial statistics (IFS), while, the data on consumption is obtained from the Quarterly Bulletins of the South African Reserve Bank, and are converted to *per capita* form using population figures obtained from the World Bank's World Development Indicators (WDI).[5] The raw data covers the quarterly period of 1960:Q1-2010:Q4, which post transformations yields a data set that starts in 1960:Q2.[6]

Consistent with the literature, neither test rejects the null hypothesis of unit root for the nominal interest rate, when we look at the critical values at the 5 percent level. Note that, the MZ_α statistic fails to reject the null of unit root at the 10 percent level of significance for the nominal Treasury bill rate.[7] For the inflation rate though, evidence against stationarity is exceptionally strong. The failure of the MZ_α statistic to reject the null of unit root for either inflation and the nominal interest rate at the conventional (5 percent) level of significance argues for cointegration analysis of these two variables to determine the EPRR's integration properties. When we prespecify a (1,-1)$'$ cointegrating vector and apply unit root tests to the EPRR, we reject the null at the 5 percent level of significance for the MZ_α statistic and at the 10 percent level for the ADF statistic. Thus, the EPRR appears to be stationary for South Africa.

Table 1: **Unit Root Test Statistics, 1960:Q2-2010:Q4**

Variable	ADF	MZ_α
3-Month Treasury Bill Rate	-2.48 [1]	-7.00* [1]
Inflation Rate	-1.80 [7]	-2.51 [7]
Ex Post Real Interest Rate	-2.60* [8]	-10.27** [7]
Per Capita Consumption Growth	-4.14*** [7]	-9.24** [8]

NOTE: the ADF and MZ_α statistics correspond to a one-sided (lower-tail) test of the null hypothesis that the variable has a unit root against the alternative hypothesis that the variable is stationary. -2.58, -2.89 and -3.51 are the 10 percent, 5 percent and 1 percent critical values respectively for the ADF statistic, while, -5.70, -8.10 and -13.80 are the 10 percent, 5 percent and 1 percent critical values for the MZ_α statistic respectively. The lag order for the regression model used to compute the test statistic is reported in brackets. *, ** and *** indicate significance at the 10 percent, 5 percent and 1 percent levels.

To test the null hypothesis of no cointegration without pre-specifying a cointegrating vector, Table 2 reports the single-equation augmented Engle and Granger (1987) [AEG] and MZ_α statistic from Perron and Rodriguez (2001), and trace statistic from Johansen (1991). All these statistics fail to reject the null hypothesis that the Treasury bill rate and inflation are not cointegrated.[8] Though

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[5]The annual population figures are interpolated to quarterly values.

[6]Specifically as far as the data is concerned, the 3-month Treasury constant maturity rate monthly data are converted to quarterly frequency by averaging over the three months comprising a quarter. The annualized CPI inflation rate is based on the seasonally adjusted (at an annual rate) CPI with the base year of 2005. *Ex post* real interest rate is defined as the three-month Treasury bill rate minus the realized inflation rate in the subsequent quarter. Finally, the annualized consumption growth rate is based on real personal consumption expenditures (base year of 2005) adjusted seasonally at an annual rate. Annualized inflation and consumption growth rates are computed by taking 400 times the first differences of the natural logs of the CPI deflator and consumption.

[7]Given this, we also conducted the Phillips and Perron (1988) [PP] and the Kwiatkowski et al., (1992) [KPSS] unit root tests on the treasury bill rate. The PP test could not reject the null of unit root even at the 10 percent level, while the KPSS test rejected the null of stationarity at one percent level of significance. In addition, we also tested the EPRR based on the nonlinear unit root test proposed by Kapetanios et al., (2003) [KSS] and the Bayesian unit root test developed by Sims (1988). Both the KSS and the Bayesian tests rejected the null of unit root. These results are available upon request from the authors.

[8]Following Gregory and Hansen (1996), we also tested for cointegration using residual based tests that allows for regime shifts, but the null of no cointegration could not be rejected even at 10 percent level of significance. Similar conclusions were also reached based on both the Bierens (1997) and Breitung (2002) nonparametric cointegration tests, as these tests too failed to reject the null of no cointegration. Note that both these tests allow for nonlinearity of an unknown form in the short-run dynamics of the the two variables. Interestingly, even though no cointegration could be detected, estimates of θ_1 based on the dynamic ordinary least squares (DOLS) or the Johansen (1991) methods are not significantly different from unity. However, this result, perhaps, explains as to why we detect stationarity for the

the evidence on stationarity of the EPRR is mixed depending on whether we use unit root or cointegration tests,[9] *per capita* consumption growth is clearly stationary based on the ADF and MZ_α statistics, thus highlighting the difference that may exist in the persistence properties of these two variables. As unit root and cointegration tests have low power to reject the null if the true model is a highly persistent but stationary process (Dejong *et al.*, 1992), the next step will be to analyze the specifically the persistence property of the EPRR.

Table 2: **Cointegration Test Statistics, 3-Month Treasury Bill Rate and Inflation Rate (1960:Q2-2010:Q4)**

Cointegration tests		
AEG	MZ_α	Trace
-2.39 [7]	-10.46 [3]	13.53 [4]

NOTE: The AEG and MZ_α statistics correspond to a one-sided lower tail test of the null hypothesis that the 3-month T-bill and inflation rate are not cointegrated against the alternative hypothesis that the variables are cointegrated. The 10%, 5% and 1% critical values for the AEG statistic are -3.14, -3.37 and -3.95. The 10%, 5% and 1% critical values for the MZ_α are -12.80, -15.84 and -22.84. The trace statistic corresponds to a one-sided upper-tail test of the null hypothesis that the 3-month treasury bill rate and inflation are not cointegrated against the alternative that the variables are cointegrated. The 10%, 5% and 1% critical values for the trace statistic are 17.77, 20.87 and 27.70. The lag order for the regression model used to compute the test statistic is reported in bracket.

Confidence Intervals for the Sum of the Autoregressive Coefficients

The sum of the AR coefficients, ρ, in the AR representation of $i_t - \pi_{t+1}$ equals unity for an $I(1)$ process, while $\rho < 1$ for an $I(0)$ process. However, it is inherently difficult to differentiate an $I(1)$ process from a highly persistent $I(0)$ process, as the two types of processes can be observationally equivalent (Blough, 1992; Faust, 1996). We need to determine a range of values for ρ that are consistent with the data to analyze the theoretical implications of the time-series properties of the real interest rate, over and above whether ρ is ≤ 1. That is, a series with a ρ value of 0.95 is highly persistent even if it does not contain a unit root as such, and it is much more persistent than a series with a ρ value of 0.4 for example (Neely and Rapach, 2008).

The Hansen (1999) grid bootstrap and the Romano and Wolf (2001) sub-sampling procedures were used to compute a 95 percent confidence interval for ρ in the $i_t - \pi_{t+1}$ process. The grid bootstrap for the EPRR is (0.70, 0.95) while the sub-sampling is (0.68, 0.92). The upper bounds are consistent with a highly persistent process. The grid bootstrap and sub-sampling intervals for *per capita* consumption growth are (-0.63, -0.09) and (-1.18, 0.45). The discrepancy between the upper bounds of the ρ for per capita growth rate of consumption and those of the lower bounds of the ρ for the EPRR confirm the difference in persistence properties between the two variables again.

EPRR based on the unit root tests with a prespecified cointegrating vector of (1,-1)′. Further, using the nonparametric nonlinear co-trending analysis developed by Bierens (2000), the null hypothesis that there exists one co-trending vector between the nominal Treasury bill and the inflation rate could not be rejected. These results are available upon request from the authors.

[9]Until recently, researchers used models of cointegration that assumed both the cointegrating relationship and short-run dynamics to be linear. But now studies have started to relax these linearity assumptions based on nonlinear cointegration or threshold dynamics, which in turn, allow the cointegrating relationship and mean reversion to be contingent on the current values of the variables. Given this, when we tested for nonlinear cointegration between the 3-month Treasury bill rate and the the inflation rate based on the test proposed by Li and Lee (2010), the null of no cointegration could not be rejected even at the 10 percent level of significance. This result is available upon request from the authors. It must be pointed out that, although evidence of threshold behavior could be interesting, these models do not obviate the persistence in the EPRR, since there are still regimes where the real interest rate behaves like an unit root process.

Testing for Fractional Integration

Unit root and cointegration tests determine whether a process is stationary or non-stationary (i.e., $I(0)$ or $I(1)$). The distinction between $I(0)$ and $I(1)$ implicitly restricts the types of allowed dynamic processes. As a result, studies such as Granger (1980), Granger and Joyeux (1980) and Hosking (1981) test for fractional integration in the EARR and EPRR. A fractionally integrated series is denoted by $I(d)$, $0 \leq d \leq 1$. If $d = 0$, then the series is $I(0)$ and shocks decay at a geometric rate. If $d = 1$, then the series is $I(1)$, and shocks have permanent effects. If $0 < d < 1$ then the series is mean-reverting as in the $I(0)$ case. However, shocks vanish at a much slower hyperbolic rate. Such series can be considerably more persistent than a very persistent $I(0)$ series (Neely and Rapach, 2008).

Testing for fractional integration in this paper is carried out by estimating the d parameter using the Shimotsu (2008) semi-parametric two-step feasible exact local Whittle estimator, which allows for an unknown mean in the series. The estimate of d for the South African EPRR is found to be 0.69 with a 95 percent confidence interval of (0.49, 0.89), suggesting long-memory, but mean-reverting behavior. So we can reject the hypothesis that $d = 0$ or $d = 1$.[10] The estimate of d for *per capita* consumption growth is equal to 0.18, with a 95 percent confidence interval of (-0.02, 0.38), suggesting that we cannot reject the null hypothesis of $d = 0$ at conventional significance levels.[11] The fractional integration results further corroborate the difference in persistence between the EPRR and *per capita* consumption growth.

4   Testing for Regime Switching and Structural Breaks in Real Interest Rate

Some studies test for structural breaks in real interest rate following Huizinga and Mishkin (1986). Taking into account the possibility for structural breaks can significantly lower the persistence within the identified regimes (Perron, 1989). Also, Jouini and Nouira (2006) argue that ignoring the possibility of the presence of structural breaks can generate spurious evidence of fractional integration (Neely and Rapach, 2008). Researchers, in general, have relied on Hamilton's (1989) Markov-switching approach to test for regime switching in the EPRR, whereby the model is assumed to be ergodic, implying that the current state will eventually cycle back to any possible state. Just as these models, structural breaks too have similar properties without being ergodic, suggesting that they do not necessarily tend to revert to previous conditions. Given that the literature on real interest rate tend to show no obvious tendency to return to previous state, structural breaks are considered to be more appropriate for modeling the EPRR than Markov-switching (Neely and Rapach, 2008).

We test for structural breaks in the unconditional mean of the EPRR using use the powerful methodology of Bai and Perron (1998) for testing for multiple structural breaks in a regression model. The results for the EPRR are reported in table 3. The procedure based on the sequential estimation finds one change in the mean of EPRR that has occurred in 1989:Q1. This break is associated with a significant change in the average annualized real interest rate in the two regimes.

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[10]Similar results were obtained for the fractional integration parameter of EPRR using the Geweke and Porter-Hudak (1983) and the Robinson (1992) methods, though the latter estimation strategy produced a relatively lower value of d. In addition, when we used the modified R/S statistic developed by Lo (1991), we confirmed the EPRR to be a long-memory process. These results are available upon request from the authors.

[11]This short-memory behavior of the *per capita* consumption growth rate was also confirmed by the Geweke and Porter-Hudak (1983) and the Robinson (1992) estimation of d, as well as by the modified R/S statistic which failed to reject the null hypothesis of the *per capita* consumption growth being a short-memory process. These results are available upon request from the authors.

Table 3: **Bai and Perron (1988) Test Statistics and Estimation Results for ex post Real Interest Rate (1960:Q2-2010:Q4)**

Test statistic		Regime	Estimated ex post real interest rate mean
UD_{max}	32.08***	1960:Q2-1989:Q1 [1985:Q2-1994:Q1]	-1.47*** (0.43)
WD_{max} (5%)	38.13**	1989:Q2-2010:Q4	4.15*** (0.49)
$F(1\|0)$	22.57***		
$F(2\|1)$	10.45**		
$F(3\|2)$	4.86		
$F(4\|3)$	4.39		
$F(5\|4)$	0		

NOTE: *, **, *** indicate significance at the 10%, 5%, 1% levels. The bracketed dates in the Regime column denote a 90% confidence interval for the end of the regime. Numbers in parentheses in the last column denote standard errors for the estimated mean.

For the period between 1960:Q2 and 1989:Q1, the average real interest rate is at -1.47 percent, and then increases significantly at 4.15 percent for the period between 1989:Q2 and 2010:Q4.[12] The finding of a significant break implies only a reduction in within-regime (local) persistence, but high degree of global persistence for the EPRR still remains.[13]. Figure 2 plots the South African EPRR and the means for the regimes defined by the structural break estimated using the Bai and Perron (1998) methodology.

Fig. 2: *Ex Post* **Real Interest Rate and Regime Specific Means, 1960:Q2-2010:Q4**
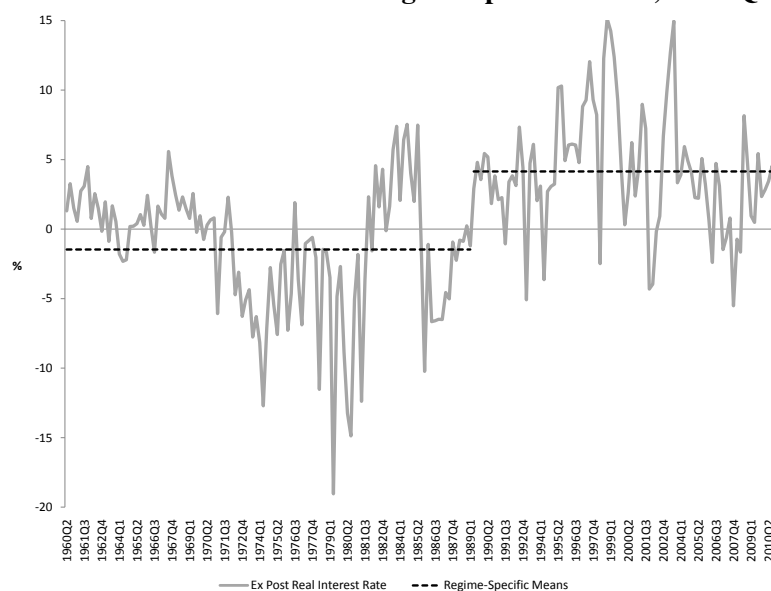
NOTE: Based on the Bai and Perron (1998) methodology, the figure plots the SA *Ex post* real interest rate and means for the regimes defined by the structural breaks.

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[12]In contrast to this evidence of a unique break in the EPRR, the Bai and Perron (1998) methodology obtained no structural break in the mean of *per capita* consumption growth. These results are available upon request from the authors.

[13]Note that, when we conducted the Zivot and Andrews (1992) unit root test allowing for a break in the intercept, it rejected the null of unit root at the 10 percent level of significance for the EPRR. This test identified a break for the series at 1987:Q4.

5   Is the Persistence of EPRR a Monetary Phenomenon?

Based on the tests conducted above, we can conclude that the South African EPPR can be best viewed as a very persistent but ultimately mean-reverting process. This section considers what types of shocks are most likely to produce the persistence in the South African real interest rate. Let us recall the underlying motivation for learning about real interest rate persistence: In a simple endowment economy, the real interest rate should have the same persistence properties as consumption growth. However, our empirical analyses reveal that the real rate is much more persistent than consumption growth. Given that, permanent technology growth shocks can create a non-stationary real rate, but also affects consumption growth in the same way, they cannot account for the mismatch in persistence. So now, we consider fiscal, monetary, and transient technology shocks as potential causes of persistent fluctuations in the real interest rate.

Figures 1 and 2 and the Bai and Perron (1998) structural break tests reveal a sharp increase in the real rate since 1989:Q2. The South African government did not undertake the sort of expansionary fiscal policy that would be necessary for such a rise in real rates. In fact, the budget deficits have been consistently declining over this period (until the "Great Recession" recently), as deduced from the data on budget deficit as a percentage of Gross Domestic Product published by the South African Reserve Bank. In fact, the average deficits over 1960:Q2-1989:Q1 and 1989:Q2-2010:Q4 have been virtually the same. Hence, fiscal shocks cannot be really viewed as a plausible candidate for the increase in real rates during this period. Turning to technology shocks, the lack of independent data on technology shocks makes it difficult to relate such changes with real interest rates. Besides, technology growth has been traditionally viewed as reasonably stable. Having said that, using a structural vector autoregressive framework to estimate the long-run effects of exogenous changes in the inflation rate on output and the real interest rate, Amusa et al., (forthcoming) indicate that persistence in real interest rate could be explained to some extent by technology shocks. Further, one could suggest that oil price increases, might influence the real rate, as can be seen from the negative real interest rate that prevailed over the decade of 1970. However, it is less likely for oil price shocks alone to account for the pronounced swings in the South African real interest rate. This leaves us with the monetary policy shocks as an explanation for the persistence of EPRR.

Table 4: **Bai and Perron (1988) Test Statistics and Estimation Results for Inflation Rate (1960:Q2-2010:Q4)**

Test statistic		Regime	Estimated inflation rate mean
UD_{max}	113.64***	1960:Q2-1971:Q1 [1970:Q2-1971:Q2]	2.79*** (0.55)
WD_{max} (5%)	163.60***	1971:Q2-1993:Q2 [1990:Q3-1994:Q4]	12.40*** (0.39)
$F(1\|0)$	64.78***	1993:Q3-2010:Q4	6.23*** (0.43)
$F(2\|1)$	36.88***		
$F(3\|2)$	8.75		
$F(4\|3)$	2.14		
$F(5\|4)$	0		

NOTE: *, **, *** indicate significance at the 10%, 5%, 1% levels. The bracketed dates in the Regime column denote a 90% confidence interval for the end of the regime. Numbers in parentheses in the last column denote standard errors for the estimated mean.

When one delves into the history of monetary policy of South Africa, one realizes that the financial markets were regulated till the mid 1980, resulting in low nominal rates. Coupled with high rates of inflation, the negative real rates over the decade of the 1970 was obvious. After, the financial markets got liberalized, the South African Reserve Bank started pursuing a contractionary

Fig. 3: **Inflation Rate and Regime Specific Means, 1960:Q2-2010:Q4**
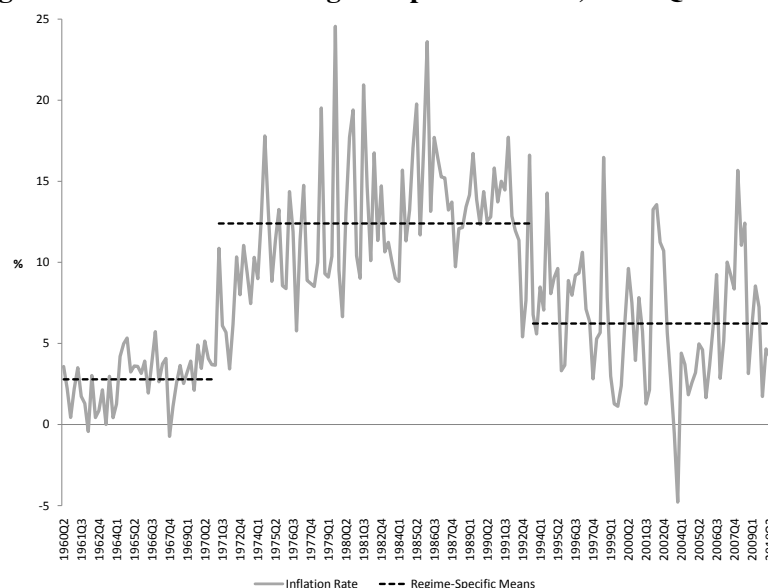
NOTE: Based on the Bai and Perron (1998) methodology, the figure plots the SA inflation rate and means for the regimes defined by the structural breaks.

monetary policy since the 1990s as part of an informal inflation targeting framework.[14] So, the increase in the average EPRR seems natural post 1989. While we interpret the timing of the major swing in the real rate to suggest a monetary explanation, we ultimately need to provide formal evidence for this line of reasoning. Hence, following Rapach and Wohar (2005), we use the Bai and Perron (1998) methodology to try and explain the deterministic regime shifts in the EPRR by considering regime shifts in the mean inflation rate. According to Friedman (1963), changes in the mean inflation rate can be considered to be a monetary phenomenon. Table 4 reports the results of the Bai and Perron (1998) methodology for the inflation rate. Two significant changes in the mean of inflation rate are identified to occur in 1971:Q1 and in 1993:Q2. For the period between 1960:Q2 and 1971:Q1, the average inflation rate was at 2.79 percent; for the period between 1971:Q2 and 1993:Q2 it averaged at 12.40 percent and for the period between 1993:Q3 and 2010:Q4, the average mean decreased to 6.23 percent. Figure 3 plots the South African inflation rate and the means for the regimes defined by the structural breaks estimated. In general, it can be seen that periods of low inflation are associated with periods of high rates of the EPRR. However, the break in the inflation rate in 1990:Q3 is about four years later than the only identified break in the real interest rate. Though, it must be said that the lower limit of the 90 percent confidence band for the structural break of the inflation rate falls within the corresponding 90 percent confidence interval of the EPRR. Thus, the evidence that a regime shift in the mean inflation rate tend to correspond to the regime shift in the mean real interest rate is weak.[15]

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[14]Refer to Ludi and Ground (2006) for a detail discussion of the history of monetary policy in South Africa.

[15]Using a measure of monetary policy surprise developed by Reid (2009), we analyzed the impulse response function of the EPRR obtained from an autoregressive distributed lag (ARDL) model following a contractionary monetary policy shock over the period of 2002:Q2-2010:Q4. The results, based on Romer and Romer's (2004) Monte Carlo methods to generate confidence bands for the impulse response functions, indicated a delayed very short-lived (1 quarter) positive effect on the EPRR, suggesting again a weak monetary explanation of the persistence of the real rate – a result in line with the findings of Amusa et al., (forthcoming). The impulse response functions from the ARDL model is available upon request from the authors. Note, Reid (2009) used the change in the 3-months Banker's Acceptance rate on the day after the monetary policy committee makes its statement as a proxy for the surprise component of monetary policy. This event-based data was converted to a quarterly frequency by averaging the shocks over a three month period.

Bayesian Change Point Analysis

Given the weak evidence that the EPRR persistence is a monetary phenomenon based on the structural break test, we decided to look into this issue further using a Bayesian change point analysis for the real rate and the inflation rate. This method allows us to obtain posterior probabilities of a change for each point of the two series, thus, (possibly) allowing us to better relate inflation with the persistence of the EPRR. In the following paragraphs, we describe the Barry and Hartigan (1993) algorithm for the Bayesian change point methodology.

The Barry and Hartigan (1993) algorithm models the series generating process by assuming that there is an underlying sequence of parameters partitioned into contiguous blocks of equal parameter values. The beginning of each block is then a change point, while observations are assumed to be independent in different blocks given the sequence of parameter. Let X_1, X_2, ...,X_n be independent observations, with each X_i having a density dependent on θ_i, $i = 1, ..., n$. Furthermore, there exists an unknown number of contiguous blocks with partitions $\rho = \{i_0, i_1, ..., i_b\}$ such that $0 = i_0 < i_1 < ... < i_n = n$ and $\theta_i = \theta_{i_b}$ when $i_{r-1} < i \leq i_r$.

In the notation used by Barry and Hartigan (1993), let X_{ij} denote the sequence of points $X_{i+1}, ..., X_j$ in time. Let $f_{ij}(x_{ij}|\theta_j)$ denote the density of x_{ij} when $\theta_{i+1} = \theta_{i+2} = ... = \theta_j$. For constant blocks, a transition distribution is defined as follows: Given θ_i, θ_{i+1} equals θ_i with probability $1 - p_i$ or has density $f(\theta_{i+1}|\theta_i)$ with probability p_i. This in essence implies that smaller values of p_i will result in longer θ_i blocks.

Further, the probability of a partition $\rho = i_0, i_1, ..., i_b$ is given by $f(\rho) = Kc_{i_0 i_i} c_{i_1 i_2} ... c_{i_{b-1} i_b}$ where c_{ij} are prior cohesion for adjacent blocks ij. In notations, let X_1, X_2, ...,X_n are assumed to be an independent for a given sequence of μ_i, such that $X_i \backsim N(\mu_i, \sigma^2)$, $i = 1, 2, ..., n$. A prior cohesion c_{ij} following Yao (1984) is introduced as follows:

$$(6) \qquad c_{ij} = \begin{cases} (1 - p)^{j-i-1} p, & j < n \\ (1 - p)^{j-i-1} & j = n \end{cases}$$

Also, a block prior is introduced as follows:

$$(7) \qquad f_{ij}(\mu_j) \backsim N(\mu_0, \frac{\sigma_0^2}{j - i})$$

The priors distribution for each of μ_0, σ_0^2, p, $w = \frac{\sigma^2}{(\sigma_0^2 + \sigma^2)}$ are as follows:

$$\begin{aligned} f(\mu_0) &= 1, -\infty \leq \mu_0 \leq \infty \\ f(\sigma^2) &= 1/\sigma^2, 0 \leq \sigma^2 \leq \infty \\ f(p) &= 1/p_0, 0 \leq p \leq p_0 \\ f(w) &= 1/w_0, 0 \leq w \leq w_0 \end{aligned}$$

After experimenting with different values of w_0 and p_0, we initialize (p_0, w_0) with $(0.002, 0.2)$ to ensure that the change points in the two series matches major local and global events that took place over the period of 1960:Q2-2010:Q4. As can be seen from Figure 4, a major change point in the inflation rate in 1972:Q1 preceded a major change point in the EPRR in 1972:Q2. Also, a major change point in the EPRR in 1992:Q3 followed a change point in the inflation rate in 1989:Q1, and is followed by couple of major changes in the inflation rate in 19954:Q4 and 1995:Q1. The other major change points in inflation in 1981 and 1985, seems to be in line with the move to a flexible

exchange rate regime and liberalization of the domestic financial markets. While, the changes in the inflation rate in 1995 and late 1999 corresponds closely to the creation of democratic South Africa and its move to an inflation targeting regime in 2000.[16] Relative to the evidence provided by the Bai and Perron (1998) tests of structural breaks, the Bayesian change point analysis seems to provide a slightly more stronger evidence in favor of the real interest rate persistence being a monetary phenomenon. However, it is important to emphasize, that the evidence is suggestive and more research is, perhaps, required (based on structural identification) to determine the veracity of the monetary explanation for the persistence of the real interest rate of South Africa.

Fig. 4: Posterior Probabilities of Change for the *Ex post* Real Rate and the Inflation Rate
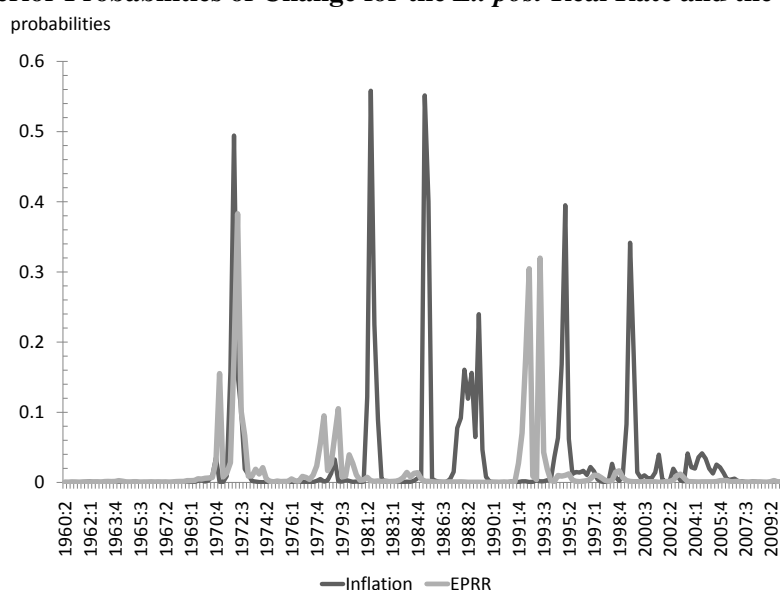
6 Conclusion

Rose's (1988) influential work stimulated a considerable empirical literature that investigates the time-series properties of real interest rates. Surprisingly, there does not exist any study that delves into this issue for South Africa. Against this backdrop, using quarterly data (1960:Q2-2010:Q4) for South Africa, our paper endeavors to analyze the long-run properties of the ex post real rate (EPRR) using tests of unit root, cointegration, fractional integration and structural breaks. In addition, we also analyze whether monetary shocks contribute to fluctuations in the real interest rate based on the Bai and Perron (1998) test of structural breaks of the rate of inflation, as well as, Bayesian change point analysis proposed by Barry and Hartigan (1993). We find evidence that the 3-month Treasury bill rate based interest rate seems stationary. with estimated 95 percent confidence intervals for the sum of the AR coefficients having an upper bound that is very near unity. Further, the EPRR appear to display long-memory behavior; shocks are very long-lived, but the real interest rate is estimated to be ultimately mean-reverting. In addition, structural breaks in unconditional means characterize the EPRR. Although the breaks reduce within-regime persistence, the real interest rate remains highly persistent because the regimes have different means. Finally, we suggest a tentative monetary explanation of the South African real interest rate persistence based on timing, lack of persistence in consumption growth, and tests for structural breaks, as well

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[16]We used the bcp package in R, developed by Erdman and Emerson (2007), to implement the algorithm of Barry and Hartigan (1993).

as, Bayesian change point analysis. In the future, further analysis of the relative importance of different types of shocks in explaining real interest rate persistence could be of immense help to this literature.

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