On Synthesis and Optimization of Steam System Networks. 3. Pressure Drop Consideration

Tim Price[†] and Thokozani Majozi*[,†,‡]

Department of Chemical Engineering, University of Pretoria, South Africa, and Modelling and Digital Science, CSIR, Pretoria, South Africa

Heat exchanger networks in steam systems are traditionally designed to operate in parallel. Coetzee and Majozi (*Ind. Eng. Chem. Res.* **2008**, *47*, 4405−4413) found that by reusing steam condensate within the network the steam flow rate could be reduced. This was achieved by restructuring the networks into a series design, with the consequence of greatly increasing the pressure drop of the system. The boiler return condensate temperature was also reduced, which was found to decrease the boiler efficiency. Maintaining the boiler efficiency has been considered in the first two papers in this series, and pressure drop is introduced in this paper. The formulations from the previous two papers are used to find the minimum steam flow rate for a HEN while maintaining the boiler efficiency. The network exhibiting the minimum pressure drop for this flow rate is then designed using the critical path algorithm. The boiler efficiency is maintained using the constraints explored in papers I (*Ind. Eng. Chem. Res.* DOI: 10.1021/ie1007008) and II (*Ind. Eng. Chem. Res.* DOI: 10.1021/ie1008579) of this series. The minimum pressure drop for the network exhibiting the minimum flow rate found in paper I was 344.4 kPa; however, the flow rate was reduced by 29.6% as shown in paper I.

1. Introduction

Heat integration using the principles of reuse and recycle is very effective at reducing the utility requirement of a system. One of the major consequences of this reuse and recycle is the consequent increase in the pressure drop of the system, as in cooling systems investigated by Kim and Smith.[2] The increased pressure drop could lead to the need for additional fluid movers in retrofit design and additional capital expenditure in grassroot design. Thus, pressure drop is often included as an optimization objective in many heat exchanger network (HEN) design techniques.

Several attempts at minimizing pressure drop have been done by the likes of Jegede[3] and Jegede and Polley.[4] These authors attempted to optimize the heat transfer coefficient, or h values, and then calculate pressure drop based on these h values because they are closely related in heat exchanger design.[5] A flaw in this approach can be picked up instantly as the optimal pressure drop is in fact not determined, but it is simply the byproduct of another optimization. Also this work did not consider the intrinsic relationship between the pressure drop and the network structure or even the number of heat transfer units, or shells.

Nie and Zhu[6] identified the need to consider pressure drop in a HEN retrofit design. The changes to the HEN could be as a result of changes to the process or even plant expansion. They found that pressure drop is complex as it is dependent not only on the process streams themselves but also on the network topology. Nie and Zhu[7] consider another aspect of heat exchanger network design that adds to capital cost. Pressure drop occurs in all fluid movement since fluids are most commonly transported from a point of high pressure to one of lower pressure. Sometimes this pressure drop becomes too high and reduces the effectiveness of the HEN. Pumps and compressors are used to overcome pressure drop, but these add to the capital cost of the design. As such, the authors seek to minimize pressure drop for the network. Pressure drop is largely interconnected with other design variables. As a result, the authors determine the optimal $\Delta T_{\min}$ for the network by a three-way trade-off with heat transfer area, utilities, and pressure drop.

Kim and Smith[8] worked on a heuristic approach to cooling system design that considered both the cooling tower and the heat exchanger network. They improved the cooling tower performance by decreasing the cooling water flow rate to the HEN. This was achieved by reordering the parallel HEN into a series structure. Kim and Smith[2] then proceeded to examine the effects of water reuse on the pressure drop for the system. Additional pressure drop creates a problem in retrofit designs as additional fluid movers may be required to compensate. This increases the capital cost of the retrofit operation. Thus, the authors attempted to a find a means of minimizing pressure drop in a systematic manner. The authors develop a mathematical model by using a novel node representation and simple yet accurate approximations for pressure drop. The resulting MINLP is then linearized using various relaxation or approximation techniques so as to give a feasible solution. The pressure drop for the network is then minimized using mathematical programming. These steps are intended to guide designers to retrofit designs that could eliminate the need for additional cooling capacity or fluid-moving capacity.

2. Problem Statement

The steam flow rate to a HEN can be reduced using the techniques developed by Coetzee and Majozi.[1] The boiler efficiency can be maintained with the minimum flow rate by incorporating the appropriate preheaters mentioned in the first paper of this series. The restructuring of the HEN has the adverse effect of increasing the pressure drop for the network. The problem addressed in this investigation can be formally stated as follows.

Given:

• a steam boiler with known efficiency,

• a set of heat exchangers linked to the boiler with limiting temperatures and fixed duties,

* Corresponding author. E-mail: thoko.majozi@up.ac.za. Fax: +27 86 633 5729.
[†] University of Pretoria.
[‡] CSIR.

10.1021/ie1008585
Published on Web 08/20/2010

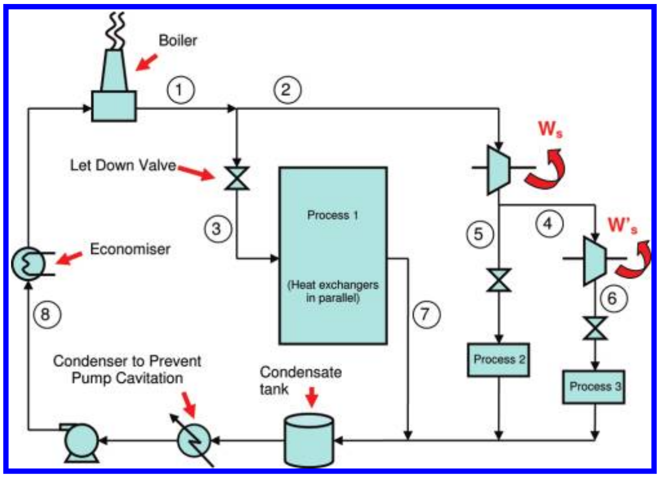

Figure 1. Typical steam system layout.

• turbines and background heat exchangers also linked to the boiler, and

• the predetermined minimum flow rate for the heat exchanger network

determine the minimum network pressure drop for a system with the minimum steam flow rate and design a network that achieves this pressure drop without compromising the boiler efficiency (BE).

3. System Description

The system consists of a steam system as shown in the previous two papers in this series. The focus will be on the HEN itself and how it can be rearranged to minimize the pressure drop. The BE constraints will need to be included, and as such, the system includes all turbine and boiler elements.

Figure 1 shows a typical steam system in the context of this investigation. Superheated high-pressure steam is produced inside the steam boiler and is shown as stream 1. A portion of this steam is sent to a high-pressure steam turbine, stream 2, where energy is recovered from the steam in the form of shaft work. The rest of the steam from the boiler is sent directly to a process through a let-down valve, stream 3, for pressure reduction and to remove the sensible heat so that the latent heat can be used in the appropriate set of heat exchangers, shown as process 1. The exhaust from the high-pressure turbine is used further in a medium-pressure turbine, stream 4, and/or taken directly to another set of heat exchangers, represented by process 2, through a let-down valve that regulates the pressure in accordance with process requirements as stream 5. The exhaust from the medium-pressure turbine can also be used in a set of heat exchangers, represented by process 3 and shown as stream 6.

When the steam leaves the processes, it is either saturated or subcooled condensate, shown as stream 7. This condensate is then collected in a condensate tank before passing through a condenser en route to the boiler. The condenser is to prevent cavitation in the pump. Before entering the boiler, the condensate, stream 8, passes through a preheater. Some make-up water is also added to the condensate return, but this has been omitted from the diagram for the sake of simplicity.

4. Pressure Drop through HEN Elements

In a HEN, pressure can be lost through the likes of pipes and heat exchangers. Pressure drop correlations are therefore needed in the form of constraints so as to incorporate this into the optimization framework. Many correlations exist in literature; however, those used by Kim and Smith[2] are the most appropriate as they are readily integrated with flow rate, which is already a variable in the current HEN model developed in the first paper in this series. The correlations are derived from the work of Nie[9] where pressure drop is calculated for conventional shell-and-tube heat exchangers.

4.1. Heat Exchanger Pressure Drop. According to the rigorous derivations by Nie,[9] the tube-side pressure drop for a heat exchanger is much more easily determined than the shell-side pressure drop. As such, it will be assumed that the steam and condensate pass through the tube side of the heat exchanger as this will help integrate pressure drop for the HEN optimization problem.

The function for pressure drop presented by Kim and Smith[2] is shown in constraint 1.

$$\Delta P_t = N_{t1} V_t^{1.8} + N_{t2} V_t^2 \tag{1}$$

This constraint was originally derived based on fluid velocity but was adapted by Kim and Smith[2] for volumetric flow rate. According to Nie,[9] the two terms account for the pressure drop as a result of the friction loss in the tubes and the loss as a result of sudden contractions, expansions, and flow reversals. The two factors N_{t1} and N_{t2} are shown in constraints 2 and 3 below.

$$N_{t1} = \frac{1.115567}{\pi^{2.8}} \frac{\rho^{0.8} \mu^{0.8} n_{tp}^{2.8} A}{N_t^{2.8} d_o d_i^{4.8}} \tag{2}$$

$$N_{t2} = \frac{20}{\pi^2} \frac{n_{tp}^{3} \rho}{N_t^2 d_i^4} \tag{3}$$

In constraints 1−3, ΔP_t is the tube-side pressure drop, V_t is the tube-side volumetric flow rate, ρ is the fluid density, μ is the fluid viscosity, n_{tp} is the number of tube passes, A is the heat transfer area, N_t is the number of tubes, d_o is the outside tube diameter, and d_i is the inside tube diameter.

Many of the terms in these constraints such as the heat transfer area, the number of tubes, and the tube dimensions are very much interrelated in the heat exchanger design. As such, Kim and Smith[2] use certain industrial design guidelines to demonstrate pressure drop in their case studies. These guidelines, as well as those used for this work, will be discussed in Appendix A.

Condensers also appear in the networks developed thus far, and this pressure drop must be catered for. According to Sinnot,[5] the pressure drop through condensers where total condensation occurs can be approximated by calculating the pressure drop in the conventional fashion using the inlet vapor conditions and multiplying this by a factor. Two factors are put forward, the first by Kern,[10] who suggests a 50% factor, and the second by Frank,[11] who suggests 40%. Since 50% will be the more conservative option, this will be used throughout the model. Thus, the condenser pressure drop will be approximated by constraint 4, where N_{t1} and N_{t2} are equivalent to those for constraint 1.

$$\Delta P_{t,c} = 0.5(N_{t1} V_t^{1.8} + N_{t2} V_t^2) \tag{4}$$

4.2. Piping Pressure Drop. Kim and Smith[2] define the piping pressure drop according to constraint 5. This is derived from commonly used pressure drop correlations, as well as a friction factor by Hewit et al.[12] to approximate the Fanning friction factor. In constraint 5, N_P^{EX} is a factor to relate fluid properties and the pipe structure as shown in constraint 6.

$$\Delta P_P = N_P{}^{EX} V_P{}^{1.8} \tag{5}$$

$$N_P{}^{EX} = \frac{1.11557}{\pi^{1.8}} \frac{\rho^{0.8}\mu^{0.2}L}{D_i^{4.8}} \tag{6}$$

In constraint 6, L is the pipe length and D_i is the pipe inside diameter. Since the diameter is a design choice, Kim and Smith[2] use an economic trade-off of the optimal pipe size suggested by Peters and Timmerhaus[13] where the optimal pipe diameter is given as a function of volumetric flow rate and fluid density. Using this relation, constraints 5 and 6 are rewritten as constraints 7 and 8, respectively.

$$\Delta P_P = N_P{}^{NW} \frac{1}{V_P{}^{0.36}} \tag{7}$$

$$N_P{}^{NW} = \frac{188.318}{\pi^{1.8}} \rho^{0.176}\mu^{0.2}L \tag{8}$$

In constraint 7, it can be seen that pressure drop is now an inverse function to volumetric flow rate, which seems counter-intuitive. The relation by Peters and Timmerhaus,[13] however, ensures that every time a new velocity is chosen the optimal pipe diameter is used, giving the inverse relation. This is advantageous, however, because now all of the piping and heat exchanger pressure drops are a function of volumetric flow rate and constants relating to the heat exchanger structure.

4.3. Pressure Drop With Respect to Mass Flow Rate. The heat exchanger and piping pressure drops shown in constraints 1−4, 7, and 8 can be changed so that they can accommodate the fluid mass flow rate, which is used in the first paper in this series. Constraints 9−12 show these adjustments for the heat exchangers and condensers, whereas constraints 13 and 14 show the adjustment for the piping correlations. These correlations will be used for pressure drop through equipment in the following section.

$$\Delta P_t = N_{t1}^* \dot{m}_t{}^{1.8} + N_{t2}^* \dot{m}_t{}^2 \tag{9}$$

$$N_{t1}^* = \frac{1.115567}{\pi^{2.8}} \frac{\mu^{0.8} n_{tp}{}^{2.8} A}{\rho N_t{}^{2.8} d_o d_i{}^{4.8}} \tag{10}$$

$$N_{t2}^* = \frac{20}{\pi^2} \frac{n_{tp}{}^3}{\rho N_t{}^2 d_i^4} \tag{11}$$

$$\Delta P_{t,c} = 0.5(N_{t1}^* \dot{m}_t{}^{1.8} + N_{t2}^* \dot{m}_t{}^2) \tag{12}$$

$$\Delta P_P = N_P{}^{NW*} \frac{1}{\dot{m}_P{}^{0.36}} \tag{13}$$

$$N_P{}^{EX*} = \frac{188.318}{\pi^{1.8}} \rho^{0.536}\mu^{0.2}L \tag{14}$$

5. Constraints

Three separate aspects must be modeled in this formulation. The first two aspects are the HEN design model and the BE constraints for both the sensible heat preheater and the dedicated preheater. These constraints are developed and discussed in far greater detail in the first paper in this series. As such, only the primary constraints will be shown below for completeness.

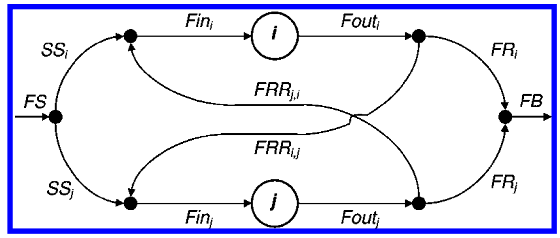

Figure 2. Superstructure used for mass and energy balances.

The third aspect is that of network pressure drop. The minimum steam flow rate is found using the first two models. The pressure drop constraints are then used to create a network that exhibits the minimum pressure drop for that particular flow rate. Therefore, the HEN and BE models will be used to find the flow rate. This flow rate and the boiler efficiency will then be fixed while all three models are incorporated to find the minimum flow rate. The HEN model is needed to design the minimum pressure drop network, whereas the efficiency constraints are used to ensure the boiler efficiency is maintained.

5.1. HEN Model. The first paper in this series contains the description of the derivation of the HEN model, which was first derived by Coetzee and Majozi.[1] Essentially, this model is used to find the minimum steam flow rate for a particular HEN. It then also constructs the network to correspond to that flow rate. Figure 2 is used to derive the mass balances for the individual heat exchangers as well as the entire network, which are shown by constraints 15−35 below.

$$FS = \sum_{i \in I} SS_i \tag{15}$$

$$F_{in_i} = SS_i + \sum_{j \in I} F_{RR_{j,i}} \quad \forall i \in I \tag{16}$$

$$F_{out_i} = F_{R_i} + \sum_{j \in I} F_{RR_{i,j}} \quad \forall i \in I \tag{17}$$

$$FB = \sum_{i \in I} F_{R_i} \tag{18}$$

$$F_{in_i} = F_{out_i} \quad \forall i \in I \tag{19}$$

$$F_S = FB \tag{20}$$

Binary variables are used to specify which heat exchangers are heated by latent or sensible energy. A single heat exchanger can have a dedicated source or be allowed to use either. These are dealt with by using constraints 21−25 or constraints 21−24 as well as constraints 26 and 27, respectively.

$$SS_i^U = \frac{Q_i}{\lambda} \quad \forall i \in I \tag{21}$$

$$F^{\mathrm{U}}_{\mathrm{RR}_i} = \frac{Q_i}{c_{\mathrm{p}}(T^{\mathrm{L}}_{\mathrm{in}_i} - T^{\mathrm{L}}_{\mathrm{out}_i})} \quad \forall i \in I \tag{22}$$

$$SS_i \le SS_i^{\mathrm{U}} y_i \quad \forall i \in I \tag{23}$$

$$\sum_{j \in i} F_{\mathrm{RR}_{j,i}} \le F^{\mathrm{U}}_{\mathrm{RR}_i} x_i \quad \forall i \in I \tag{24}$$

$$y_i + x_i = 1 \quad \forall i \in I \tag{25}$$

$$\sum_{i \in I} y_i + \sum_{i \in I} x_i \ge |i| \tag{26}$$

$$\sum_{i \in I} y_i + \sum_{i \in I} x_i \le |i| + n \tag{27}$$

The energy to heat a process stream can consist of latent energy, designated by constraint 28, or by sensible energy shown in constraint 29. Constraint 30 is then the total energy used to satisfy the duty of a process stream.

$$Q_i^{\mathrm{S}} = SS_i \lambda \quad \forall i \in I \tag{28}$$

$$Q_i^{\mathrm{L}} = \sum_{j \in i} (c_{\mathrm{p}} SL_{j,i} T_{\mathrm{sat}}) + \sum_{j \in I} (c_{\mathrm{p}} L_{j,i} T_{\mathrm{out}_j}) - (c_{\mathrm{p}} F_{\mathrm{out}_i} T_{\mathrm{out}_i}) \quad \forall i \in I \tag{29}$$

$$Q_i = Q_i^{\mathrm{S}} + Q_i^{\mathrm{L}} \quad \forall i \in I \tag{30}$$

The saturated and subcooled condensate flow rates throughout the network must also be controlled. As such, constraints 31−34 are employed as mass balances for the different types of condensate.

$$F_{\mathrm{RR}_{j,i}} = SL_{j,i} + L_{j,i} \quad \forall i,j \in I \tag{31}$$

$$F_{\mathrm{R}_i} = F_{\mathrm{RS}_i} + F_{\mathrm{RL}_i} \quad \forall i \in I \tag{32}$$

$$SS_i = \sum_{j \in I} SL_{i,j} + F_{\mathrm{RS}_i} \quad \forall i \in I \tag{33}$$

$$\sum_{j' \in I} SL_{j',i} + \sum_{j' \in I} L_{j',i} = \sum_{j \in I} SL_{i,j} + F_{\mathrm{RL}_i} \quad \forall i \in I \tag{34}$$

Constraint 35 is used to prevent local recycle, which is inappropriate for thermodynamic reasons.

$$L_{i,j} = 0 \quad \forall i,j \in I, i = j \tag{35}$$

The objective function is then to minimize the steam flow rate throughout the network; this is shown by constraint 36. The model in its current form is of the MINLP variety, with constraint 29 being the only nonlinear constraint. As in the first paper of this series, the condition of optimality demonstrated by Savelski and Bagajewicz[14] will be used to linearize this constraint and transform the HEN model into an MILP (shown in eq 4.22).

$$\min Z = FS \tag{4.22}$$

5.2. Boiler Efficiency Model. The relevant BE constraints derived in the first paper in this series will be shown below. Figure 3 shows the steam system and the origin of associated stream flow rates and temperatures. In the first paper in this series, the boiler efficiency is maintained by preheating the boiler return stream using a sensible heat preheater or a dedicated preheater in the HEN. These formulations will be used again and investigated separately when considering the various network pressure drops.

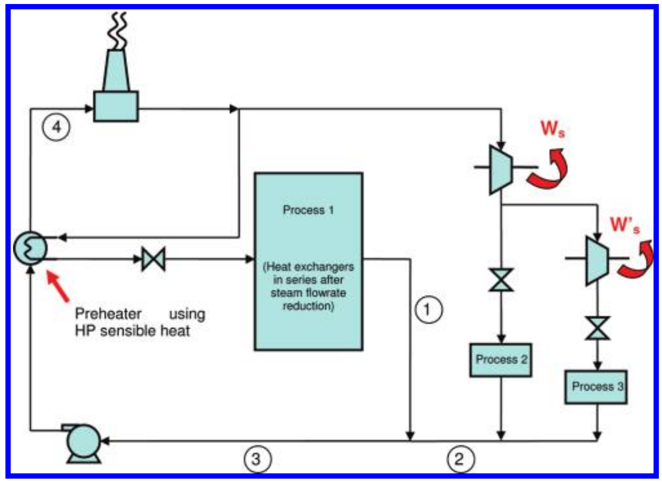

Figure 3. Steam system with streams for boiler efficiency.

The main process outlet flow rate, shown as stream 1, is calculated by the HEN model, whereas the temperature is calculated by constraint 36. Stream 2 originates from the turbines, and the flow rate and temperature are assumed constant due to the fixed power ratings. Streams 1 and 2 join to create stream 3, which proceeds to the pump. The temperature of this stream is calculated by constraint 37. This stream is then preheated to create stream 4. This stream then proceeds to the boiler, and its temperature is calculated by constraint 38. The boiler efficiency is then calculated by constraint 39.

$$T_{\mathrm{proc}} = \frac{\sum_{i \in I} F_{\mathrm{RS}_i} T_{\mathrm{sat}} + \sum_{i \in I} F_{\mathrm{RL}_i} T^{\mathrm{L}}_{\mathrm{out}_i}}{FS} \tag{36}$$

$$T_{\mathrm{pump}} = \frac{(T_{\mathrm{proc}} FS) + (T_{\mathrm{turb}} F_{\mathrm{turb}})}{(FS + F_{\mathrm{turb}})} \tag{37}$$

$$T_{\mathrm{boil}} = T_{\mathrm{pump}} + \frac{FS(h_{\mathrm{sup}} - h_{\mathrm{sat}})\theta}{(FS + F_{\mathrm{turb}})c_{\mathrm{p}}} \tag{38}$$

$$\eta_{\mathrm{b}} = \frac{q[(FS + F_{\mathrm{turb}})/F^{\mathrm{U}}]}{[c_{\mathrm{p}}(T_{\mathrm{sat}} - T_{\mathrm{boil}}) + q][(1 + b)((FS + F_{\mathrm{turb}})/F^{\mathrm{U}}) + a]} \tag{39}$$

In the event that the boiler efficiency cannot be maintained with the available sensible heat, two scenarios exist. First, the boiler efficiency can be maintained with a compromise in the steam flow rate. In this case, the minimum steam flow rate is relaxed by adding a slack variable to the flow rate components of constraint 39. This variable is then minimized as the objective function. The second scenario involves the minimum flow rate being fixed with a compromise in boiler efficiency. In this case, the boiler efficiency is relaxed by adding a slack variable to this term in constraint 39. This slack variable is then also minimized.

The second means to maintain the boiler efficiency is using a dedicated preheater, also discussed in the first paper in this series. Figure 4 shows the steam system with the dedicated preheater.

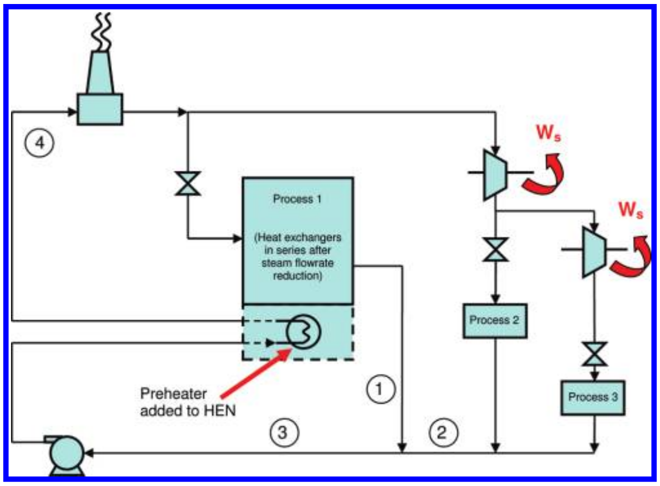

Figure 4. Steam system with dedicated preheater.

Constraints 36 and 37 are used to calculate the process outlet temperature and pumping temperature, respectively. As before, stream 3 then passes through a preheater; however, this preheater is part of the process. The boiler return temperature is calculated by constraint 40. The subscript $i*$ in the constraints refers to the additional dedicated preheater. The duty Q_{i*} is calculated by constraint 41. The limiting temperatures for the preheater are then the pumping and boiler return temperatures that are adjusted by the minimum approach temperature as shown in constraints 42 and 43.

$$T_{\mathrm{boil}} = T_{\mathrm{pump}} + \frac{Q_{i*}}{(FS + \mathrm{slack}^{+} + F_{\mathrm{turb}})c_{\mathrm{p}}} \qquad (40)$$

$$Q_{i*} = SS_{i*}\lambda + \sum_{j\in I}(c_{\mathrm{p}}SL_{j,i*}T_{\mathrm{sat}}) + \sum_{j\in I}(c_{\mathrm{p}}L_{j,i*}T_{\mathrm{out}_j}) - [c_{\mathrm{p}}F_{\mathrm{out}_{i*}}(T_{\mathrm{boil}} + \Delta T_{\mathrm{min}})] \quad \forall i* \in I \qquad (41)$$

$$T^{\mathrm{L}}_{\mathrm{in}_{i*}} = T_{\mathrm{pump}} + \Delta T_{\mathrm{min}} \quad \forall i* \in I \qquad (42)$$

$$T^{\mathrm{L}}_{\mathrm{out}_{i*}} = T_{\mathrm{boil}} + \Delta T_{\mathrm{min}} \quad \forall i* \in I \qquad (43)$$

Using these formulations, the boiler efficiency can be maintained for a given steam flow rate. The network is then rearranged to find the minimum pressure drop that corresponds to that flow rate.

5.3. Pressure Drop. Recycle and reuse of condensate in a HEN may reduce the utility flow rate; however, the series nature of the network increases the pressure drop. Kim and Smith[2] identified the need to minimize pressure drop in such systems from their work in cooling systems. By following a similar approach, pressure drop will be minimized for the HEN in steam systems.

5.3.1. Pressure As an Intensive Property. It has been widely established that pressure drop in HENs is not only dependent on stream variables such as flow rate, but also on the network layout. To account for the network layout, Kim and Smith[2] use the concept of the critical path algorithm (CPA) developed by Gass,[15] which is well understood in mathematical programming. The total pressure drop of the network is essentially represented by the largest pressure drop of a connection of streams. This critical path should then be minimized to find the minimum pressure drop of the system.

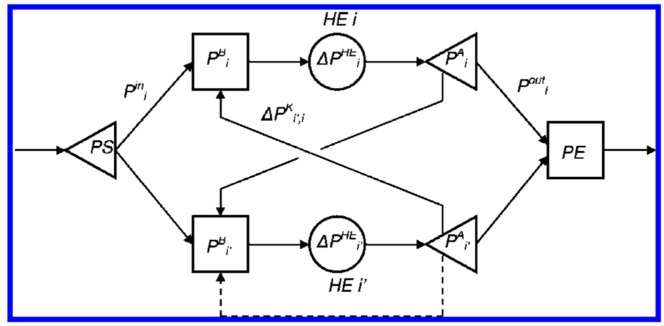

Figure 5. Node superstructure used by Kim and Smith.[2]

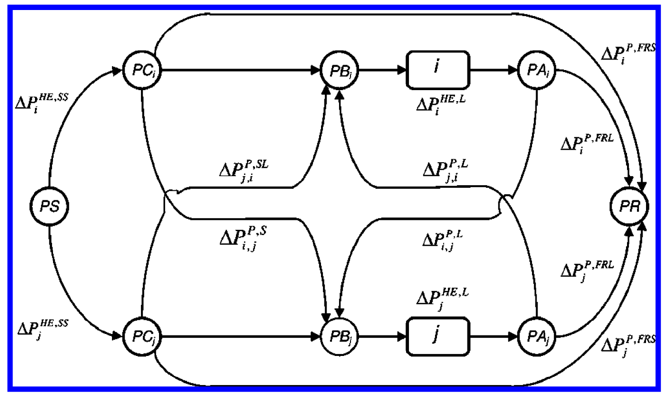

Figure 6. New node superstructure that accommodates phase change.

Kim and Smith[2] use a node superstructure as a framework to establish the CPA model. Nodes represent mixers that combine streams before heat exchangers as well as splitters that redirect streams after heat exchangers. Pressure is lost between nodes in various HEN elements such as heat exchangers and pipes. The mixers are linked to a source node, and the splitters are linked to a sink node. The source node represents the maximum pressure of the system, whereas the sink node represents the minimum pressure of the system. The objective is then to find the maximum pressure drop through the network and then minimize this using mathematical programming.

The superstructure developed by Kim and Smith[2] to model the pressure drop can be seen in Figure 5. The superstructure greatly resembles that used for mass and energy balances in the work on heating systems by Coetzee and Majozi[1] shown in Figure 2.

This superstructure accurately caters for cooling systems, where it is assumed the cooling water stays in the liquid phase. To create the same type of node superstructure for steam systems, multiple phases must be incorporated to deal with steam and condensate. The pressure drop for condensers is different to that of ordinary heat exchangers where the streams remain in the same phase. Constraint 12 is therefore used for the pressure drop for the initial part of the network. A new node superstructure was developed to cater for steam systems and is shown in Figure 6.

In the figure, it can be seen that the condensers are connected to the main source node at source pressure P_{S}. Since it is assumed that the steam loses minimal pressure in pipes, $\Delta P^{HE,SS}$ is only as a result of the pressure drop in the condensers. The distributing or splitting node after the condensers then has pressure P_{C}.

This node is then connected to each condensate heat exchanger's source node or mixer P_{B}, as well as the final sink node for return to the boiler P_{R}. The pressure drop in the pipes between P_{C} and P_{B} is a function of the saturated condensate flow rate, and the pressure drop is designated $\Delta P^{\mathrm{P},SL}$. It must also be noted that condensate can be reused by the same process stream, and as such the $\Delta P_{i,i}{}^{\mathrm{P},SL}$ and $\Delta P_{j,j}{}^{\mathrm{P},SL}$ terms can exist. The direct return to the boiler is accomplished by the saturated return flow rate F_{RS}; thus, this pressure drop is designated $\Delta P^{\mathrm{P},F_{\mathrm{RS}}}$.

Subcooled liquid, designated L, is used for the reuse of condensate between heat exchangers. This has the associated pressure drop $\Delta P^{\mathrm{P},L}$. The pressure drop for each heat exchanger is a function of the sum of saturated and subcooled liquid entering it. This is shown as $\Delta P^{HE,L}$. Each splitter node has the pressure P_{A}. The return stream to the boiler, F_{RL}, then proceeds to the return mixer with pressure P_{R}. The pressure drop in the pipes of this return stream is subsequently labeled $\Delta P^{\mathrm{P},F_{\mathrm{RL}}}$.

The CPA is adapted by Kim and Smith[2] to cater for pressure drop. This is in the form of a difference in pressure between nodes and the pressure drop between the nodes. This is represented in a manner similar to constraint 44 below.

$$P_{\mathrm{A}} - P_{\mathrm{B}} \geq \Delta P_{\mathrm{A,B}} \tag{44}$$

In constraint 44, it is implied that fluid flows from node A to node B. The pressure difference between the nodes is essentially a result of the piping pressure loss or the pressure loss through a heat exchanger. Since the mixing nodes before a heat exchanger may receive fluid from multiple sources, this constraint will occur a number of times with different source splitter nodes and consequently different pressure drop values. The inequality in constraint 44 ensures that P_{B} assumes the lowest pressure value that satisfies all of the constraints. In this way, the node pressure is always at this low value when the model is solved.

This constraint, though simple, is very elegant and effective at finding the largest pressure drop path for the network. However, since it will be used in conjunction with the HEN model, a means of eliminating those nodes that do not exist must be made. Kim and Smith[2] use binary variables to accomplish this. They establish a connection existence binary variable for each connection in the network. This binary variable takes the value of 1 if the connection exists and 0 if it does not. An additional term is then added to constraint 44 to exclude it for the cases where no connection exists. This is done by adding a large pressure term such that it is satisfied for these cases. The large pressure is represented by the term BP in constraint 45.

$$P_{\mathrm{A}} - P_{\mathrm{B}} + BP(1 - y_{\mathrm{A,B}}) \geq \Delta P_{\mathrm{A,B}} \tag{45}$$

The binary variables are created using the actual flow variables that are part of the network design model. The only nodes affected by this phenomenon are those that could possibly receive multiple inputs. These are the P_{B} and P_{R} nodes, the mixers before the condensate heat exchangers and the boiler return, respectively. Only four flow variables are associated with these nodes and they are SL, L, F_{RL}, and F_{RS}. Each of the binary variables requires two constraints as well as known upper and lower bounds for the flow rates. Constraints 46 and 47 are used to demonstrate this for the variable SL.

$$SL - SL^{\mathrm{U}}(ySL) \leq 0 \tag{46}$$

$$SL - SL^{\mathrm{L}}(ySL) \geq 0 \tag{47}$$

The binary variable will assume the value of 1 if the appropriate connection is active. In constraints 46 and 47, SL^{U} and SL^{L} are the upper and lower flow rate limits, respectively. Constraints 48−53 represent the CPA constraints that exist based on the superstructure in Figure 6.

$$P_{\mathrm{S}} - P_{\mathrm{C}_i} = \Delta P_i^{HE,SS} \quad \forall i \in I \tag{48}$$

$$P_{\mathrm{C}_j} - P_{\mathrm{B}_i} - BP(1 - ySL_{j,i}) = \Delta P_{j,i}^{\mathrm{P},SL} \quad \forall i,j \in I \tag{49}$$

$$P_{\mathrm{A}_j} - P_{\mathrm{B}_i} - BP(1 - yL_{j,i}) = \Delta P_{j,i}^{\mathrm{P},L} \quad \forall i,j \in I \tag{50}$$

$$P_{\mathrm{A}_i} - P\mathrm{R} - BP(1 - yF_{\mathrm{RL}_i}) = \Delta P_i^{\mathrm{P},F_{\mathrm{RL}}} \quad \forall i \in I \tag{51}$$

$$P_{\mathrm{C}_i} - P_{\mathrm{R}} - BP(1 - yF_{\mathrm{RS}_i}) = \Delta P_i^{\mathrm{P},F_{\mathrm{RS}}} \quad \forall i \in I \tag{52}$$

$$P_{\mathrm{B}_i} - P_{\mathrm{A}_i} = \Delta P_i^{HE,L} \quad \forall i \in I \tag{53}$$

The constraints shown above, as well as those showing the various pressure drops, are combined to form the pressure drop model. All that remains is to state the objective function. Since the CPA finds the maximum pressure drop through the network, the objective function then simply minimizes this pressure drop. Constraint 54 is thus the objective function.

$$\min Z = P_{\mathrm{S}} - P_{\mathrm{R}} \tag{54}$$

A number of heuristic methods can be used to remove some of the constraints. Topological restrictions are common, as are simple aspects such as pressure gradient, because for a given heat exchanger P_{A} is less than or equal to P_{B} according to constraint 53, which removes the option of local recycle.

5.3.2. Pressure Drop Constraints in Terms of Mass Flow Rates. The HEN and pressure drop models use mass as opposed to volumetric flow rates. As such, constraints 9−14 will be linked to the appropriate mass flow rate variables from the HEN model. Constraints 55−60 show the appropriate pressure drop elements used in the final formulation.

$$\Delta P_i^{HE,SS} = 0.5(N_{\mathrm{t1}}^{*}SS_i^{1.8} + N_{\mathrm{t2}}^{*}SS_i^{2}) \quad \forall i \in I \tag{55}$$

$$\Delta P_i^{HE,L} = N_{\mathrm{t1}}^{*}\Big(\sum_{j\in I} SL_{j,i} + \sum_{j\in I} L_{j,i}\Big)^{1.8} + N_{\mathrm{t2}}^{*}\Big(\sum_{j\in I} SL_{j,i} + \sum_{j\in I} L_{j,i}\Big)^{2} \quad \forall i \in I \tag{56}$$

The piping pressure drop relations for these flow rates will be shown below. The pressure drop for the steam flowing from the boiler to the condensers is considered negligible due to the low density of steam and the density dependence of constraint 9.

$$\Delta P_{j,i}^{\mathrm{P},SL} = N_{\mathrm{P}}^{\mathrm{NW}*}\frac{1}{(SL_{j,i})^{0.36}} \quad \forall i \in I \tag{57}$$

$$\Delta P_{j,i}^{\mathrm{P},L} = N_{\mathrm{P}}^{\mathrm{NW}*}\frac{1}{(L_{j,i})^{0.36}} \quad \forall i \in I \tag{58}$$

$$\Delta P_i^{\mathrm{P},F_{\mathrm{RS}}} = N_{\mathrm{P}}^{\mathrm{NW}*}\frac{1}{(F_{\mathrm{RS}_i})^{0.36}} \quad \forall i \in I \tag{59}$$

$$\Delta P_i^{\mathrm{P},F_{\mathrm{RL}}} = N_{\mathrm{P}}^{\mathrm{NW}*}\frac{1}{(F_{\mathrm{RL}_i})^{0.36}} \quad \forall i \in I \tag{60}$$

5.3.3. Solution Strategy. The mathematical intricacies of the CPA require that it is solved independently from the other

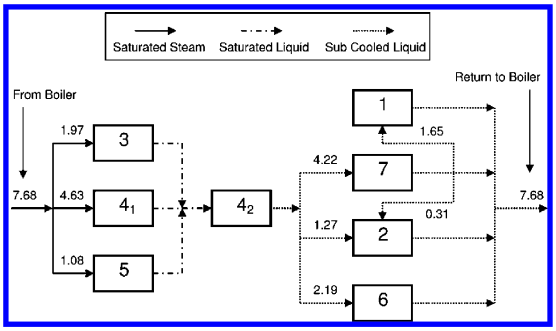

Figure 7. HEN structure for minimum steam flow rate.

models. Thus, the boiler efficiency models will be used to find a minimum flow rate. This flow rate will then be used in the pressure drop model as a parameter. The actual boiler efficiency constraints will also have to be included in the pressure drop section to ensure that the efficiency is maintained, even if the network is rearranged in finding the minimum pressure drop. The number of heat exchanger splits can be included as a variable or set manually. Since fewer heat exchanger splits are favorable in terms of cost, the effect of varying this will also be examined. The pressure drop models are nonlinear; as such, these will be solved as a MINLP, whereas the flow rate minimization models remain the combined MILP and MINLP.

6. Case Study

The case study first used by Coetzee and Majozi[1] was used in the first paper in this series and, as such, is also used for the pressure drop formulation. The results pertaining to the boiler efficiency will be stated briefly and can be viewed in more detail in the first paper in the series. The two preheating strategies are explored separately, and the pressure drop is then determined for the relevant steam system arrangement.

With reference to the first paper in the series, the HEN model was able to reduce the steam flow rate from 10.90 to 7.69 kg/s. This is a reduction of 29.3%. As a result of the reduced flow rate and return temperature, the boiler efficiency was reduced from 63.5% to 59.9%. The HEN structure corresponding to the minimum flow rate can be seen in Figure 7 below. Clearly, one heat exchanger split has been employed. A split heat exchanger is indicated by the main heat exchanger number being accompanied by a subscript. Heat exchanger 4 in Figure 7 is made up of 4_1 and 4_2, representing two separate physical heat exchangers. All flow rates are shown in kg/s.

6.1. Sensible Heat Preheater. It was found that using the sensible heat preheater made it possible to maintain the boiler efficiency using 79.2% of the available sensible heat. The original network shown in Figure 7 could thus be used. This network has one heat exchanger split; thus, it was attempted to use a single split for the pressure drop model. By incorporating the pressure drop constraints, the minimum pressure drop using the minimum flow rate was found to be 344.4 kPa, and the HEN can be seen in Figure 8 below. By comparing the networks, it can be seen that additional reuse occurs between the units. By varying the number of heat exchanger splits in the pressure drop section, it was found that a minimum pressure drop of 344.4 kPa could not be improved.

In the first paper in the series, the available sensible heat is limited so as to evaluate the HEN and BE models' abilities to meet the objective function. If the available sensible heat is limited to 30%, the boiler efficiency can be maintained with an increase of 8.3% in the minimum flow rate. This still corresponds to a saving of 23.6% of the original parallel network

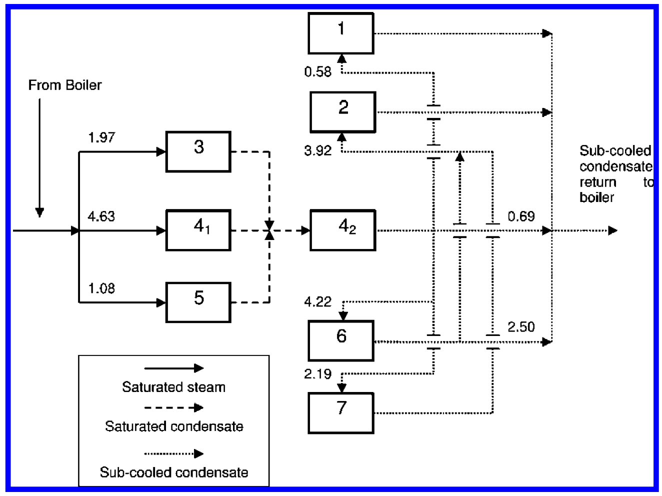

Figure 8. Minimum pressure drop while maintaining boiler efficiency.

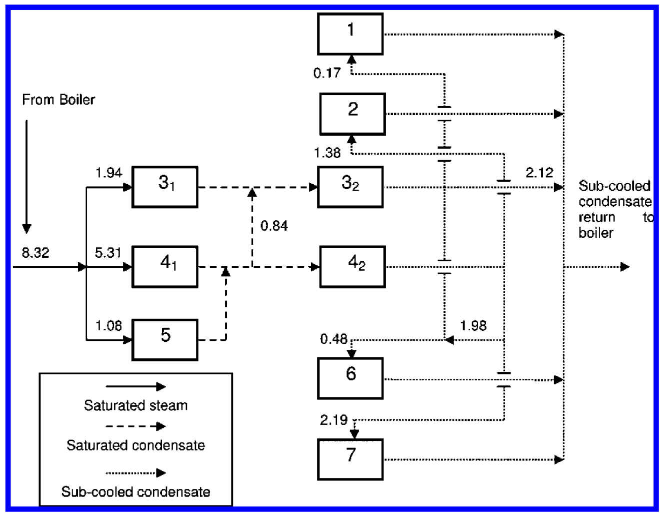

Figure 9. Network for minimum pressure drop for reduced sensible heat.

flow rate. This new minimum flow rate was found using 1 heat exchanger split; thus, this was used for the pressure drop model. No feasible starting point could be found with a single split heat exchanger, and as such, the solution was infeasible. By increasing the number of splits to 2, a solution of 272.3 kPa was found and the network associated with this can be seen as Figure 9 below. The additional heat exchanger did not influence the minimum flow rate. By varying the number of heat exchanger splits in the pressure drop section, it was found that a minimum flow rate of 274.8 kPa could not be improved. Setting the number of heat exchanger splits as a variable resulted in a similar situation as previously mentioned, with an infeasible solution resulting from a lack of a feasible starting point. From this result, it can be deduced that manually changing the number of heat exchanger splits improves the performance of the model by removing a largely influential variable.

6.2. Dedicated Preheater. By adding a dedicated heat exchanger to preheat the boiler return stream, it was found that the boiler efficiency could be maintained with a flow rate 13.9% higher than the minimum, but still 19.6% lower than the original steam flow rate. This network can be seen in Figure 10. This minimum flow rate was achieved with 1 heat exchanger split, as well as the dedicated preheater. By minimizing the pressure drop using this flow rate and number of heat exchanger splits, the pressure drop was found to be 231.9 kPa, and the associated HEN can be seen in Figure 11. By comparing this to the original

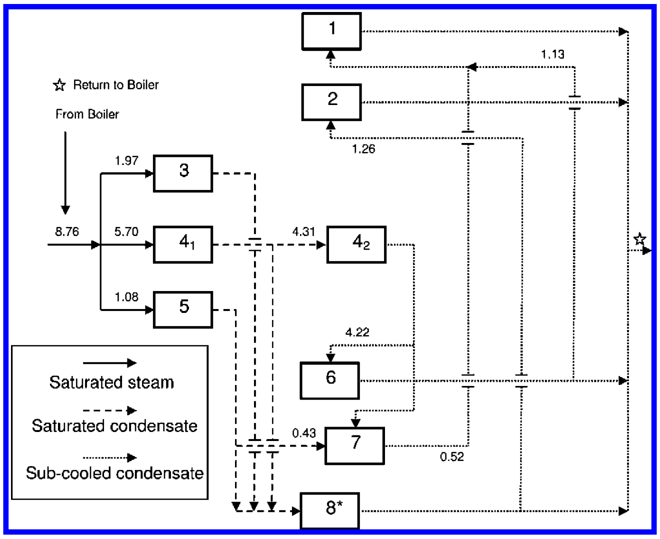

Figure 10. HEN with extra heat exchanger.

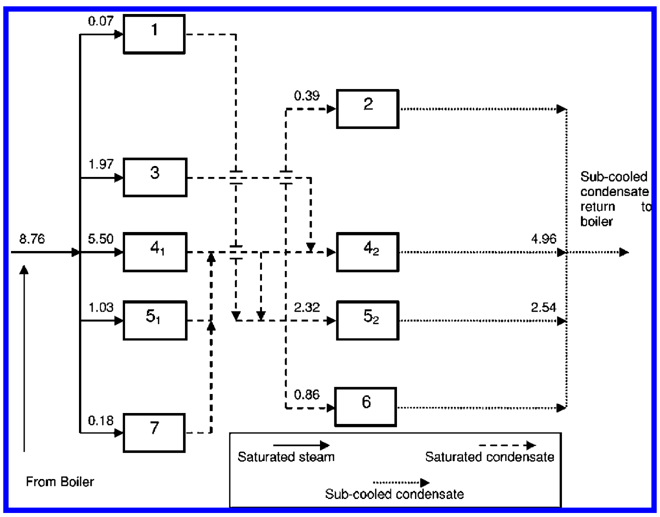

Figure 11. Network for extra preheater.

network, it can be seen that the dedicated preheater is eliminated. The same amount of heat exchangers exist, however, because saturated condensate is recycled to heat exchanger 5. This creates a situation where the stream that is pumped has a very high temperature and creates a risk of cavitation. By varying the number of splits for the pressure drop model, the minimum pressure drop of 231.9 kPa could not be improved.

Because the return temperature is very high for this solution, it was decided to force the model to use the dedicated preheater. This was achieved by setting the duty and limiting temperatures to the values found by the HEN and BE models. With 2 heat exchanger splits, the minimum pressure drop was found to be 262.2 kPa. After further investigation, the minimum pressure drop was found with 3 splits and calculated as 255.4 kPa. This network can be seen in Figure 12. By using the number of splits as a variable in the formulation, the minimum pressure drop was found to be 321.7 kPa. Once again it can be seen that setting the number of splits manually leads to a better solution.

It is clear from Figure 12 that some mathematically feasible choices are poor design choices, for instance the recycle of saturated condensate from heat exchanger 3 to 5, and then from 5 to 3. This could be eliminated to further save on piping costs. The recycle of subcooled condensate is also very complex. A

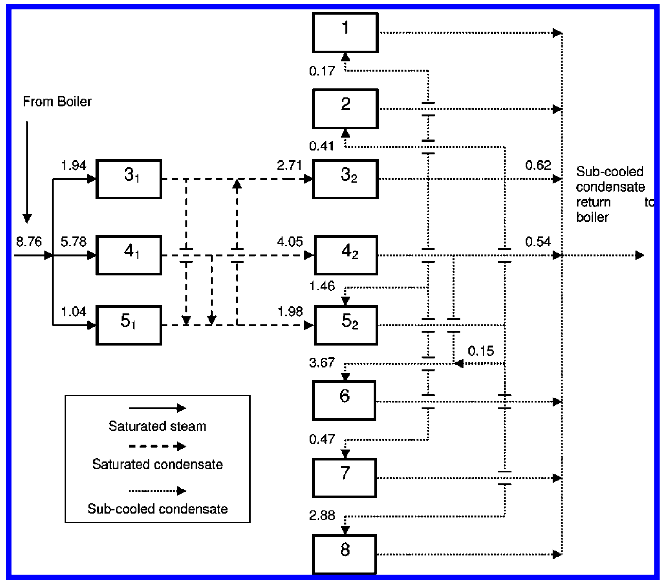

Figure 12. Minimum pressure drop network with dedicated preheater.

cost analysis with the piping could properly determine if this complexity for the sake of reduced pressure drop is necessary.

7. Conclusions and Recommendations

The following conclusions can be drawn from the minimization of pressure drop through the heat exchanger network:

• Multiple networks can exhibit the same minimum flow rate. Thus, the network that exhibits the lowest pressure drop among these can be found.

• Pressure drop is dependent on the network structure as well as stream properties, and thus a means is required to find the pressure drop based on the structure as well as the elements causing pressure drop.

• The CPA can find the largest pressure drop in the network.

• With the largest pressure drop known, it can be minimized using mathematical programming.

From the case study, the following conclusions can be made:

• For the given minimum flow rate, the lowest pressure drop that could be achieved with maintained boiler efficiency using the sensible heat preheater was 344.4 kPa. This was achieved with 1 heat exchanger split.

• For the relaxed flow rate found to maintain the boiler efficiency with the dedicated preheater, the lowest pressure drop that could be achieved was 255.4 kPa. This was achieved with 3 heat exchanger splits.

From this work, the following recommendations and limitations are made:

• A cost analysis would be of great value because maintaining boiler efficiency requires extra heat exchangers and piping.

• An attempt to linearize the nonlinear pressure drop constraints could yield a globally optimal solution.

Appendix A

This appendix is intended to show how the unknown terms relating to the heat exchanger and piping pressure drop correlations have been derived for the pressure drop mathematical model.

A.1. Heat Exchanger Guidelines. Constraint 6 contains the following heat exchanger design-based variables: heat transfer area A, the number of tube passes n_{tp}, the number of tubes N_t, the inside tube diameter d_i, and the outside tube diameter d_o.

For the various heat exchangers in the relevant case study, the heat transfer area is determined using the general equation for heat transfer defined by Sinnot,[5] shown in constraint A.1

$$Q = UA\Delta T_{\mathrm{m}} \tag{A.1}$$

In constraint A.1, Q is the relevant duty for the stream to be heated, U is the overall heat transfer coefficient, A is the heat transfer area, and ΔT_{m} is the log mean temperature difference. The stream duty is known for the case studies. An approximate overall heat transfer coefficient is taken from Sinnot.[5] This value is 1000 $\mathrm{W/m^2 \cdot K}$ for the condensers and 800 $\mathrm{W/m^2 \cdot K}$ for the condensate heat exchangers. The ΔT_{m} for condensers is determined using constraint A.2 as defined by Sinnot.[5]

$$\Delta T_{\mathrm{m}} = \frac{t_2 - t_1}{\ln\left(\dfrac{T_{\mathrm{sat}} - t_1}{T_{\mathrm{sat}} - t_2}\right)} \tag{A.2}$$

In constraint A.2, T_{sat} is the saturated steam temperature, t_2 is the cold stream inlet temperature, and t_1 is the cold stream outlet temperature. The ΔT_{m} for the condensate heat exchangers is assumed to be the minimum global approach temperature, ΔT_{min}, for the case study so as to give a conservative approximation for the area. With the Q, U, and ΔT_{m} known, the heat transfer area can be calculated using constraint A.1.

This area must then be made up by the outside area of the tubes in the heat exchanger. The area is a function of the area of one tube multiplied by the number of tubes, as stated by Nie[9] and shown in constraint A.3:

$$A = N_{\mathrm{t}}\pi d_{\mathrm{o}}L \tag{A.3}$$

In constraint A.3, L is the length of a tube. With A known, the variables N_{t}, L, and d_{o} must be selected. There are several design guidelines given by Sinnot[5] that aid in the selection of these variables. One of the major considerations for heat exchangers is the fluid velocity. Low fluid velocities often lead to fouling, while high fluid velocities increase the pressure drop. Velocity is calculated by constraint A.4, shown by Nie.[9]

$$u = V\frac{4}{\pi d_{\mathrm{i}}^2}\frac{n_{\mathrm{tp}}}{N_{\mathrm{t}}} \tag{A.4}$$

The number of tube passes are assumed to start at 2 for the smaller duties and increase up to 16. Sinnot[5] indicates generally accepted velocity ranges for fluids and gases in pipes, which can be assumed equivalent to heat exchanger tubes. These ranges are shown in Table A.1.

Table A.1. Fluid Velocity Ranges

fluid	velocity (m/s)
liquids	1−3
gases and vapors	15−30

The lower limit is in place to prevent fouling. Since steam and steam condensate contains very little impurities, this lower limit is assumed to be flexible for the approximate heat exchanger design. Since the pressure drop through the steam pipes is assumed to be negligible, the only steam flow rate to be considered is inside the condensers. Thus, the gas and vapor flow rate range is used simply as a guideline.

The relation between the inside and outside pipe diameter is assumed to be a ratio of 0.8, since this is the ratio indicated by the most commonly chosen pipe diameters in ref 5. A relationship between the tube length and the number of tubes exists in the form of the bundle diameter to tube length relationship defined by Sinnot.[5] The bundle diameter is calculated using the number of tubes and the tube pitch. A square pitch is assumed for all heat exchangers.

By alternating the inside diameter, number of tubes, tube length, and number of tube passes while maintaining the heat transfer area and bearing the design guidelines in mind, the various heat exchangers are designed for the grassroots case. In the retrofit case, the actual heat exchanger dimensions can be used. It must be emphasized that the objective is not to design an optimal heat exchanger but simply to simulate the pressure drop for a realistic heat exchanger such that the pressure drop can be modeled.

A.2. Piping Guidelines. Constraint 8 is a fairly simple piping pressure drop estimation. Since an economic design guideline has already been used to derive the constraint, the only choice left is the pipe length. In the model, the various types of pipe length are varied. The recycle and reuse pipes are assumed to be longer than the pipes joining the heat exchangers to the central condensate return hub.

Nomenclature

Sets

$I = \{i \text{ or } j | \ i \text{ or } j = 1, 2, ..., I\}$ is the set of heat exchangers

Parameters

A = heat transfer area for heat exchanger
a = regression parameter
BP = large pressure to compensate for nodes that do not exist
b = regression parameter
c_{p} = heat capacity (kJ/kg·k)
D_{i} = pipe inside diameter (m)
d_{i} = inside tube diameter (m)
d_{o} = outside tube diameter (m)
F^{U} = maximum capacity of the boiler (kg/s)
F_{turb} = flow rate of turbine condensate (kg/s)
h_{sup} = enthalpy of superheated HP steam (kJ/kg)
h_{sat} = enthalpy of saturated HP steam (kJ/kg)
L = pipe length (m)
m_{p} = fluid mass flow rate through pipes (kg/s)
m_{t} = fluid mass flow rate through tube side of heat exchanger (kg/s)
n = number of allowable heat exchanger splits
n_{tp} = number of tube passes in heat exchanger
N_{t} = number of tubes in heat exchanger
$N_{\mathrm{p}}^{\mathrm{EX}}$ = pressure drop factor for pipes
N_{t1} = pressure drop factor for friction loss
N_{t2} = pressure drop factor for sudden expansions, contractions, and flow reversals
Q_i = duty of heat exchanger i (kW)
q = latent and superheated sensible heat of HP steam (kJ/kg)
$T_{\mathrm{in}_i}^{\mathrm{L}}$ = limiting utility inlet temperature for heat exchanger i (°C)
$T_{\mathrm{out}_i}^{\mathrm{L}}$ = limiting utility outlet temperature for heat exchanger i (°C)
T_{turb} = temperature of turbine condensate (°C)
ρ = density of fluid ($\mathrm{kg/m^3}$)
μ = viscosity of fluid ($\mathrm{N \cdot s/m^2}$)
λ = latent heat of steam (kJ/kg)
θ = fraction of sensible heat used for preheating

Binary Variables

$x_i = 1$ if heat exchanger i receives heat from condensate, 0 otherwise
$y_{i,l} = 1$ if heat exchanger i receives heat from steam from level l, 0 otherwise
$y_{A,B} = 1$ if connection between node A and B exists, 0 otherwise

Continuous Variables

F_{in_i} = total flow rate entering heat exchanger i (kg/s)
F_{out_i} = total flow rate leaving heat exchanger i (kg/s)
F_R = total return flow to the boiler (kg/s)
F_{R_i} = condensate returning to the boiler form heat exchanger i (kg/s)
$F_{RR_{j,i}}$ = reused/recycled condensate from heat exchanger j to heat exchanger i (kg/s)
F_S = total saturated steam flow rate to the heat exchanger network (kg/s)
$L_{j,i}$ = subcooled condensate reuse from heat exchanger j to heat exchanger i (kg/s)
n_b = boiler efficiency
P_A = pressure of node after condensate heat exchanger (kPa)
P_B = pressure of node before condensate heat exchanger (kPa)
P_C = pressure of node after condenser (kPa)
P_P = flow rate of process pressure steam through preheater (kg/s)
P_R = pressure of final node for boiler return stream (kPa)
P_S = source node from boiler (kPa)
Q_i^L = portion of heat exchanger duty catered for by condensate
Q_i^S = portion of heat exchanger duty catered for by steam
$Q_{preheat}$ = heat added to boiler return stream by various preheaters
$SL_{j,i}$ = saturated condensate reuse/recycle from heat exchanger j to heat exchanger i (kg/s)
SS_i = saturated steam flow rate to heat exchanger i (kg/s)
T_{boil} = temperature of boiler return flow (°C)
T_{proc} = outlet temperature from HEN (°C)
T_{pump} = combined temperature of process and turbine condensate (°C)
V_p = volumetric flow rate through pipes (m^3/s)
V_t = volumetric flow rate through tube side of heat exchanger (m^3/s)
ΔP_p = pressure drop through pipes (kPa)
ΔP_t = pressure drop through the tube side of heat exchanger (kPa)
ΔT_{min} = global minimum approach temperature for the HEN (°C)
ΔT_{sat} = temperature difference between boiler return and saturated HP steam (°C)
$\Delta T_{sat,l}$ = temperature difference between boiler return and saturated HP steam (°C)

Literature Cited

(1) Coetzee, W. A.; Majozi, T. Steam System Network Design Using Process Integration. *Ind. Eng. Chem. Res.* **2008**, *47*, 4405–4413.

(2) Kim, J. K.; Smith, R. Automated retrofit design of cooling-water systems. *Process Syst. Eng.* **2003**, *49* (7), 1712–1730.

(3) Jegede, F. O. Power, capital and energy cost trade-offs in heat exchanger networks. Ph.D. Thesis, UMIST, Manchester, U.K., 1990.

(4) Jegede, F. O.; Polley, G. T. Optimum heat exchanger design. *Trans. Inst. Chem. Eng., Part A* **1992**, *70*, 133–141.

(5) Sinnot, R. K. *Chemical engineering design*; Elsevier Butterworth Heinemann: New York, 2005; *Vol 6*.

(6) Nie, X.; Zhu, X. X. Heat exchanger retrofit considering pressure drop and heat transfer enhancement. *AIChE J.* **1999**, *5*, 1239.

(7) Nie, X.; Zhu, X. X. Pressure drop considerations for heat exchanger grassroot design. *Comput. Chem. Eng.* **2002**, *26*, 1661–1676.

(8) Kim, J. K.; Smith, R. Cooling water system design. *Chem. Eng. Sci.* **2001**, *56*, 3641–3658.

(9) Nie, X. R. Heat exchanger network retrofit considering pressure drop and heat transfer enhancement. Ph.D. Thesis, UMIST, Manchester, 1998.

(10) Kern, D. Q. *Process heat transfer*; Mcgraw Hill: New York, 1950.

(11) Frank, O. Simplified design procedure for tubular exchangers. *Practical aspects of heat transfer, Chem. Eng. Prog. Tech. Manual*; Am. Inst. Chem. Eng: 1978.

(12) Hewit, G. F.; Spires, G. L.; Bott, T. R. *Process Heat Transfer*; CRC Press: Boca Raton, FL, 1994.

(13) Peters, M. S.; Timmerhaus, K. D. *Plant design and economics for chemical engineers*, 4th ed.; Mcgraw-Hill: New York, 1991.

(14) Savelski, M. J.; Bagajewicz, M. J. On the optimality of water utilization systems in process plants with single contaminants. *Chem. Eng. Sci.* **2000**, *55*, 5035–5048.

(15) Gass, S. I. *Linear Programming*, 5th ed.; Mcgraw-Hill: New York, 1985.

Received for review April 11, 2010
Revised manuscript received July 25, 2010
Accepted August 3, 2010

IE1008585