The generation of flat-top beams by complex amplitude modulation with a phase-only spatial light modulator

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ABSTRACT

Phase-only spatial light modulators are now ubiquitous tools in modern optics laboratories, and are often used to generate so-called structured light. In this work we outline the use of a phase-only spatial light modulator to achieve full complex amplitude modulation of the light, i.e., in amplitude and phase. We outline the theoretical concept, and then illustrate its use with the example of the laser beam shaping of Gaussian beams into flat-top beams. We quantify the performance of this approach for the creation of such fields, and compare the results to conventional lossless approaches to flat-top beam generation.

Keywords: Flat-top beams, spatial light modulator, complex amplitude modulation.

1. INTRODUCTION

Laser beam shaping is necessary for the transformation of a beam from one profile to another in order to be suited to a specific application. Beam shaping external to a laser cavity is separated into three broad classes: in the first one may simply use an aperture or spatial filter where a portion of an expanded beam is selected. The critical disadvantage of this technique is the inefficient use of the input power as a minimal portion of the total energy is transformed. The second class is termed field mapping where an electric field is mapped one to one in amplitude and phase from some input plane to an output plane and may be achieved through the use of lossless reflective, diffractive and refractive optical elements. The final class is referred to as beam integrators where the energy in the initiate beam is integrated to produce some energy distribution in the final beam. Often this is done by slitting the initial beam into beamlets by using a lenslet array and thereafter superimposed with a lens in the output plane. Beam integrators are particularly effective in shaping high power sources of modest coherence, e.g., multimode laser beams, by smoothing out the irradiance profiles through the lenslet array which can be reflective, diffractive or refractive. Field mapping and beam integration offer the prospect of efficient flat-top beam transformation from a Gaussian beam. A laser beam that has a spatially flattened profile is essential in applications that require a specific area to be uniformly irradiated, and some of these applications include laser lithography, material processing, laser printing, optical data storage, medical and military applications and holography[1,2].

In transforming a collimated Gaussian beam into a flat top beam, there are four families of functions that are usually considered: super-Gaussian, flattened Gaussian, Fermi-Dirac and super-Lorentzian[3]. These functions have been extensively researched and several techniques in transforming a Gaussian profile into a flat-top profile have been reported which include the use of absorptive filters[4-6], spherically aberrated lenses[7], computer generated holograms (CGH)[8], aspheric lenses[9-12], refractive optical systems[13-16], reflective mirrors[17,18] and phase plates[19,20]. The use of phase-only spatial light modulators (SLM) have been exploited in transforming quasi-Gaussian beams into flat top beams through the realisation of a refractive system by the implementation of two aspheric lens phase profiles[13,16,21,22]. We build on the use of a SLM in transforming a Gaussian beam into a flat top beam, however, through full complex amplitude modulation of the incident light. We report on two theoretical approaches presented in Section 2 that execute

the full complex amplitude into a phase only expression so as to be addressed to the SLM. In Section 3 we outline the experimental procedure and follow with the obtained results and discussion in Section 4.

2. THEORETICAL CONSIDERATIONS

2.1 Phase only field mapping

In converting a circular Gaussian beam into a flat-top beam with a rectangular cross section, the solution exists as the superposition of two one-dimensional solutions. The corresponding one-dimensional solution for the phase only transmission function is given as[1]

$$\phi(\xi_{x,y}) = \frac{\pi}{2}\xi_{x,y}\mathrm{erf}(\xi_{x,y}) + \frac{1}{2}\exp(-\xi_{x,y}^2) - \frac{1}{2}, \tag{1}$$

with

$$\xi_x = \frac{\sqrt{2}x}{w_0} \qquad \text{and} \qquad \xi_y = \frac{\sqrt{2}y}{w_0}, \tag{2}$$

where w_0 is the $1/e^2$ width of the incident Gaussian beam. The phase only transmission function for this rectangular symmetric phase element is given as

$$\phi(x,y) = \beta_x\phi(\xi_x) + \beta_y\phi(\xi_y), \tag{3}$$

with

$$\beta_x = \frac{2\sqrt{2\pi}w_0x_0}{f\lambda} \qquad \text{and} \qquad \beta_y = \frac{2\sqrt{2\pi}w_0y_0}{f\lambda}, \tag{4}$$

where x_0 and y_0 represent half the width of a square or rectangle in the respective dimensions, f is the Fourier transforming lens and λ is the wavelength of the incident Gaussian beam. A solution of this phase only transmission function in circular symmetry in transforming a Gaussian beam into a flat-top beam is expressed as

$$\phi(\xi_r) = \frac{\beta\sqrt{\pi}}{4}\int_0^{\xi_r}\sqrt{1-\exp(-\rho^2)}d\rho, \tag{5}$$

with

$$\xi_r = \frac{\sqrt{2}r}{w_0} \qquad \text{and} \qquad \beta = \frac{2\sqrt{2\pi}w_0r_0}{f\lambda}, \tag{6}$$

where $r^2 = x^2 + y^2$ and r_0 is half the width of the circular spot.

2.2 Complex amplitude and phase mapping

It is well known that two identical light beams within the same amplitude and with opposite phase will interfere destructively, and this fact can be exploited to create amplitude modulation from a phase-only spatial light modulator. Consider for example a desired field

$$E(x,y) = A(x,y)\exp[i\phi(x,y)], \tag{7}$$

with A the real amplitude, and ϕ the phase of the resultant beam. Pixelated phase-only SLMs change the phase of some incident light beam on each pixel; if adjacent pixels are made to oscillate in phase about two values, the required

modulation can be achieved. This concept may be applied over all the pixels in the SLM array to achieve some arbitrary amplitude modulation using a phase-only transmission function given as[23]

$$T(x,y) = \frac{1}{2}\left\{\exp\left[i\phi(x,y) + i\theta(x,y)\right] + \exp\left[i\phi(x,y) - i\theta(x,y)\right]\right\}. \tag{8}$$

The amplitude $A(x,y)$ is then expressed as

$$A(x,y) = \cos\left[\theta(x,y)\right]. \tag{9}$$

2.3 Phase transmittance CGH through a Fourier series expansion

It is pertinent to express a complex field of the form

$$S(x,y) = A(x,y)\exp(i\phi(x,y)), \tag{10}$$

into a phase transmittance CGH where the amplitude $A(x,y) \in [0,1]$ and phase $\phi(x,y) \in [-\pi, \pi]$ are independently specified. The transmittance of such a CGH expressed as a function of amplitude and phase is given by

$$b(x,y) = \exp(i\psi(A,\phi)), \tag{11}$$

where $\psi(A,\phi)$ is the CGH phase modulation. It is necessary to obtain phase functions of the form of that in Eq.(8) so as to perform the necessary encoding. One method of achieving this formulation is based on the representation of $b(x,y)$ and an expansion by the Fourier series in the domain of ϕ is chosen. This leads to a signal encoding condition ($c_q^A = aA$) that is required to be met in order to express the full complex amplitude function as a phase only function. If we consider the function $\psi(A,\phi)$ with odd symmetry we may then choose a CGH phase modulation of the form[24]

$$\psi(\phi,A) = f(A)\sin(\phi), \tag{12}$$

then the phase CGH transmittance is $b(x,y)=\exp[if(A)\sin(\phi)]$. The Fourier series in the variable ϕ can be found through the use of the Jacobi-Anger identity and is expressed as

$$\exp\left[if(A)\sin(\phi)\right] = \sum_{m=-\infty}^{\infty} J_m[f(A)]\exp(im\phi), \tag{13}$$

where J_m is the mth order Bessel function. The signal encoding condition then reads as $c_q^A = J_q[f(A)]$ and is valid if $f(A)$ is inverted from $J_1[f(A)]=aA$. The maximum value of A for which this is true is 0.58 which corresponds to x=1.84 for the Bessel function $J_1(x)$. $f(A)$ can thus be obtained through numerical inversion from $J_1[f(A)]=aA$. This concept of determining the maximum of A and the function $f(A)$ applies similarly to CGH phase modulations of the form

$$\psi(\phi,A) = f(A)\phi, \tag{14}$$

and

$$\psi(\phi,A) = \phi + f(A)\sin(\phi). \tag{15}$$

3. EXPERIMENTAL REALISATION

The transformation of a Gaussian beam into a flat-top beam through Eq. (3) was experimentally performed through the use of a phase-only SLM. We considered a laser source emitting a Gaussian beam at a wavelength of 633 nm (HeNe laser). We expanded and collimated the output beam through the use of a pair of lenses (beam expander) such that the beam incident on the SLM was 4 mm in diameter. The expanded beam was directed onto a phase-only liquid crystal on Silicon SLM operating in reflection mode. Phase only spatial light modulators function through electronically varying the birefringence of the liquid crystal micro display pixels and light incident on a micro display pixel will exhibit a phase modulation proportional to its birefringence and can be represented by a transmission function electronically addressed to the SLM. The plane of the SLM was Fourier transformed with a lens f_1 = 500 mm and at the Fourier plane, the beam images were captured on a CCD camera.

The transformation of a Gaussian beam into a flat top beam through Eq. (8) was implemented with the same experimental system described above, however, the plane of the SLM was relay imaged through an afocal telescope onto a CCD camera (Spiricon Beamgage) with lens f_1 = 500 mm and f_2 = 750 mm with an aperture positioned at the Fourier plane of f_1 to spatially filter the reflected beam in the selection of the first order of diffraction. In much the same setup, the transmission through Eq.(12) considered a 10X Olympus microscope objective and an eye piece lens of focal length, f = 100 mm instead of a beam expanding telescope such that the expanded beam has a flat wavefront with an approximate diameter of 25.4 mm. The SLM screen (Holoeye-Pluto with 1920x1080 pixels of pitch 8μm and calibrated for a 2π phase shift at 633 nm) which has an active area of 15.36x8.64 mm was completely filled with the expanded Gaussian beam such that the beam on the SLM can be approximated as a plane wave over the physical area of the transmission function as illustrated in Fig. 1.
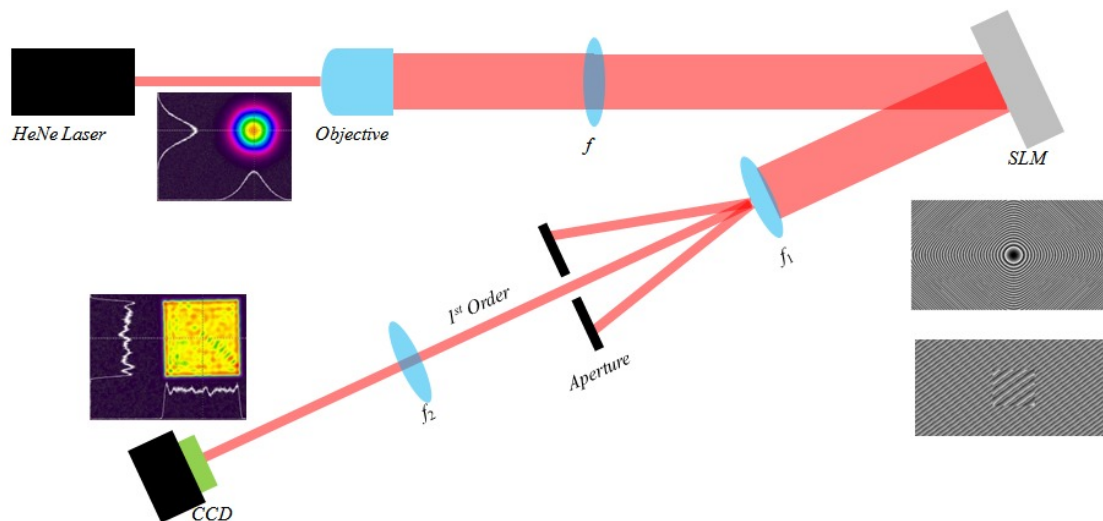

Figure 1: Experimental setup for the transformation of a Gaussian beam into a flat top beam through the use of a phase-only SLM.

4. RESULTS AND DISCUSSION

The phase only transmission function for obtaining a flat-top beam through Eq. (3) can be controlled by selecting the dimensionless β parameter, which depends on the chosen scale values, x_0 and y_0. For an input Gaussian beam of w_0 = 1 mm we selected three values of 25, 75 and 100 for the parameter β and this corresponds to half width sizes of 0.395 mm, 1.182 mm and 1.578 mm, respectively. The transmission functions were addressed to the SLM as gray scale images that varied from 0 to 2π and at the Fourier plane of lens f_1 we recorded the flat–top images as illustrated in Fig. 2. As there is no spatial filtering involved, the conversion efficiency from the Gaussian to the flat–top is high relative to the losses from that of the SLM and the higher diffraction orders.
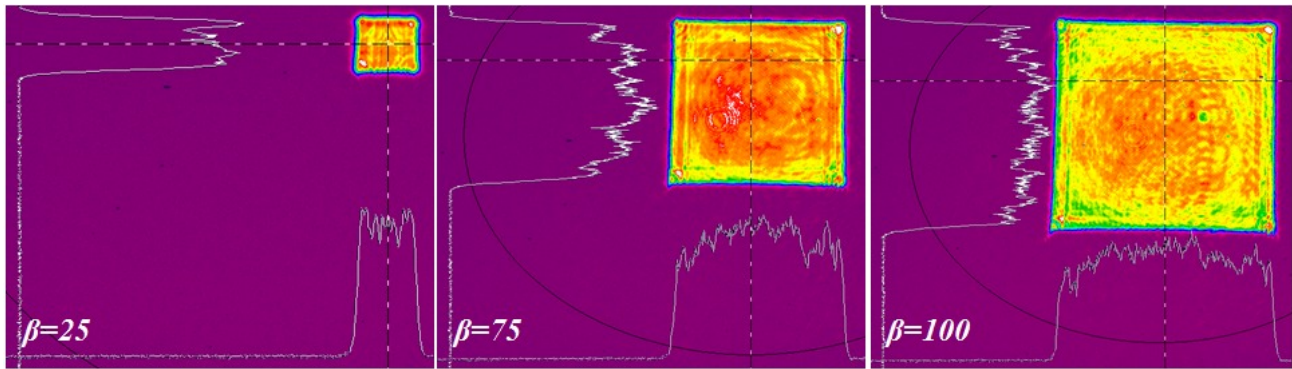
Figure 2: Gaussian to flat-top transformation in rectangular symmetry using a phase-only approach where the parameter β is increased to present an increase in the flat-top size at the Fourier plane of the SLM.

The full complex amplitude and phase transmission function in Eq. (8) describes a constant phase modulation of $\theta_0 = (\theta_A + \theta_B)/2$ and an amplitude modulation of $A = \cos[(\theta_A - \theta_B)/2]$ to some input field. These two expressions describe the entire complex plane. If we consider two points on a unit circle as in Fig. 3a, their phase variation is represented by θ_A and θ_B, respectively, and both have an amplitude of $A = 1$. If we connect these two points with a straight line, as in Fig. 3b, then the amplitude and phase of the mid-point is given by the expression for A and θ_0, respectively. This enables complete field mapping as the two points on the periphery of the unit circle may be varied along the unit circle and consequently the mid-point will also vary as in Fig. 3c.

A calibration test of the response of the amplitude was performed for a Gaussian beam incident on the checkerboard phase pattern (inset of Fig. 4) where the phase modulation θ_0 is fixed. We suitably select values of θ_A and θ_B such that the value of g is varied from 0 to 1 in steps of 0.05 and we measure the output power (P) of the reflected Gaussian beam in the first order of diffraction. To infer the amplitude response, the power of the reflected Gaussian beam is raised to the power -4 (P^{-4}) and the resulting data is normalised to 1 which corresponds to an amplitude response of $A = 1$. We anticipate that the amplitude response of the remaining encoded values of A will follow a linear relationship from 0 to 1 and as illustrated in Fig. 4, the amplitude response is exceedingly accurate to what is predicted.
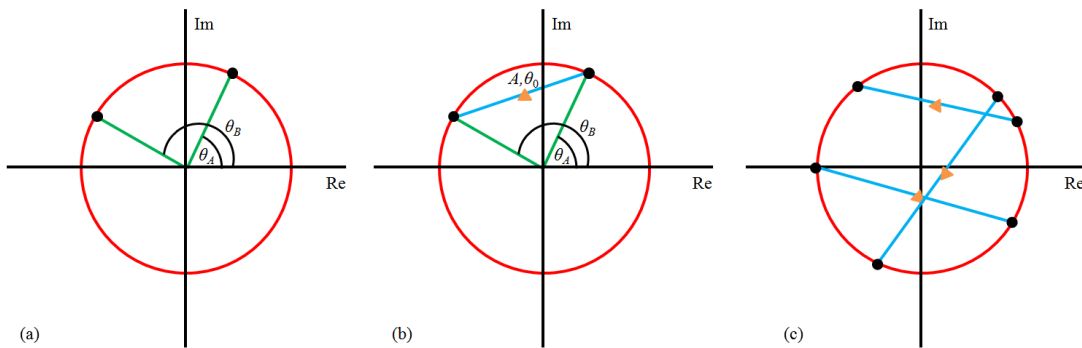

Figure 3: Complex amplitude and phase representation on a unit circle. The midpoint of two arbitrarily connected points is described by A and θ_0 and as the two points are varied along the unit circle, the midpoint also varies, thus mapping the entire complex plane.
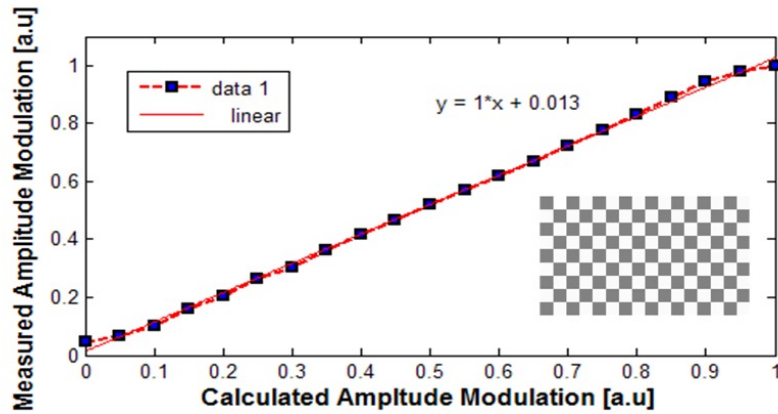
Figure 4: Amplitude response of *A* for a Gaussian incident on the checkerboard (inset) illustrates an expected linear response.

The full power of the Gaussian beam incident on the SLM may not be utilised in the transformation into a flat-top beam due to its transverse profile. We thus have to select an amplitude value of *A* such that for an arbitrary number of pixels, the skirts of the flat top fit within the Gaussian profile and the maximum of the uniformly distributed power of the flat top corresponds to *A*. We select this value of *A* to be 0.4 and this is further illustrated in Fig. 5. We select an arbitrary number of pixels in addressing the transmission function to the SLM and we record the beam images at the plane of the SLM as illustrated in Fig. 6. If the amplitude value is chosen correctly as in Fig. 5, then skirts of the flat-top are steep, however, as illustrated in Fig. 6c, the amplitude value was chosen as 0.5 and thus the steepness of the skirts starts to decrease as compared to that of Fig. 6b for the same pixel dimensions. The checkerboard creates a lot of scattered light however at larger pixel dimensions; the purity of the intensity distribution is significantly improved.
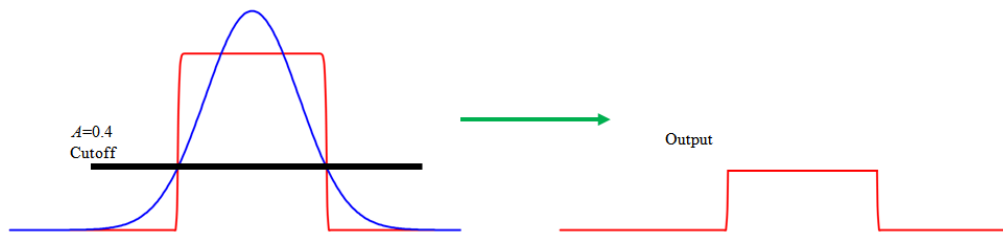

Figure 5: Cut off amplitude such that the flat-top beam fits within the incident Gaussian beam to provide a flat-top beam with steep skirts.
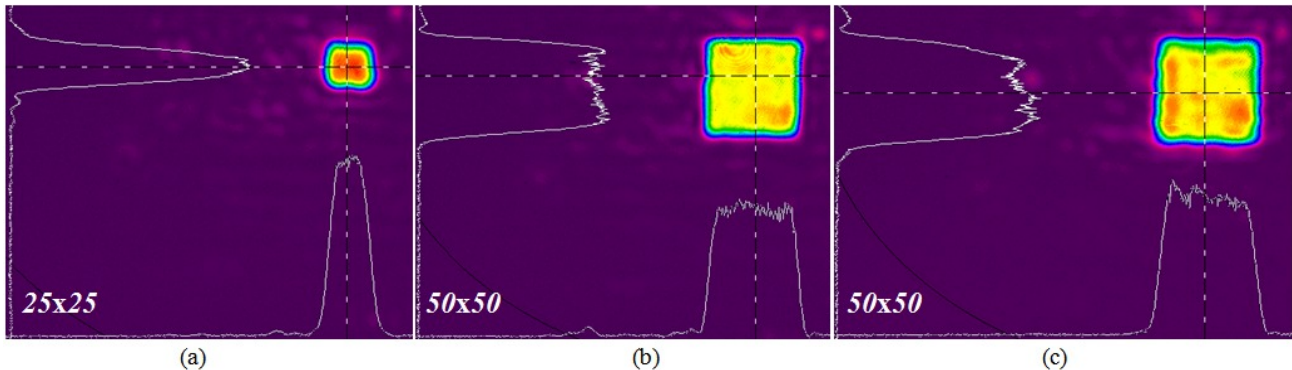

Figure 6: Arbitrary number of pixels in the selection of a flat-top where for a square flat-top we chose (a) 25x25 pixels at an amplitude of 0.4, (b) 50x50 pixels at an amplitude of 0.4 and (c) 50x50 pixels at an amplitude of 0.5.

We considered a circular and square flat-top for the phase transmittance CGH approach. For the case of the circular flat-top, we define a circle with a set number of pixels and all the values within the circle are given the value of 1.9π and all the values outside the circle are given a value of 0. This maximises the energy output in the first order of diffraction from the SLM and this concept is also applied to the square case. We investigated two circles one with a 50 pixel radius and the other with a 100 pixel radius and these values were also used to determine the dimensions of the square. Coupled with defining the area of the flat-top we also consider the complex amplitude function $S(x,y)$ (Eq. (10)) to be super-Gaussian with $w_0 = 0.5$ mm. At the plane of the SLM we find that at higher pixel dimensions for the circle and square, the energy is distributed more evenly over the flat-top area. In the case of a super-Gaussian, the super-Gaussian of order 10 is significantly smoother than that of order 50 as illustrated in Fig. 7. The flat-top beams are demonstrated with high fidelity as the skirts in these beams are very steep and in the case of larger pixels, the intensity distribution of the circle and square is smoother.
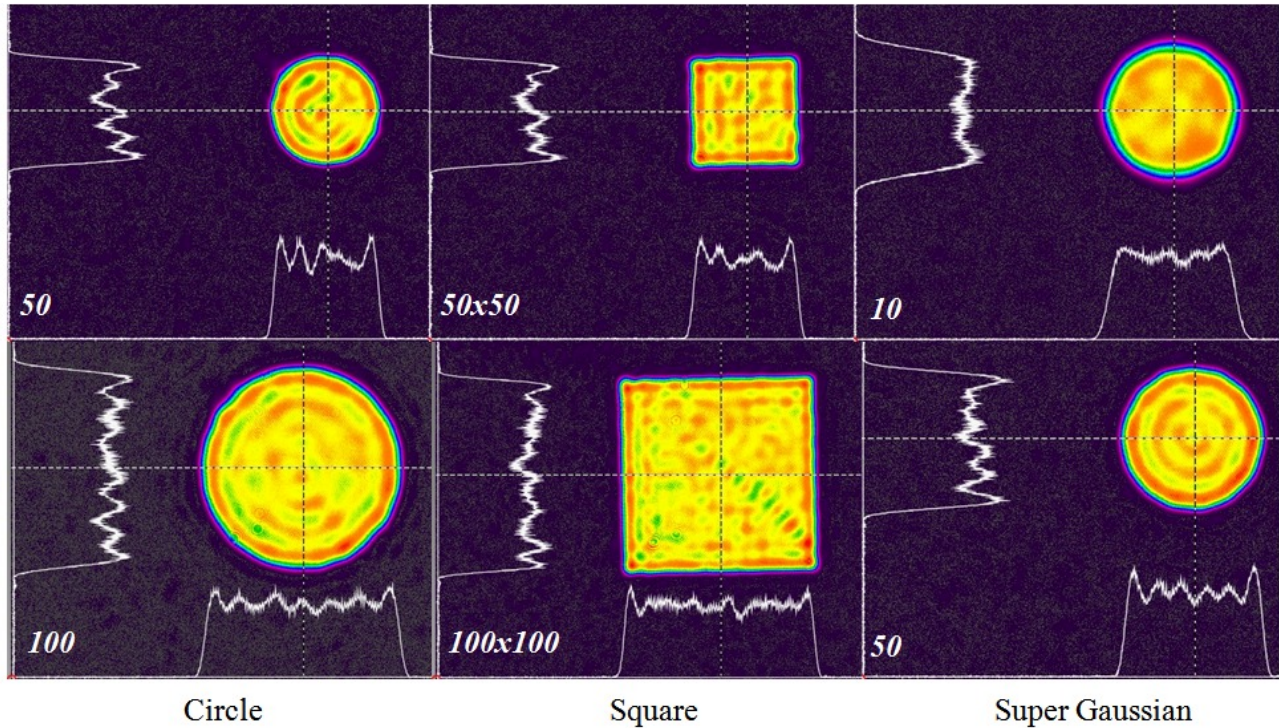

Figure 7: Flat-top beams through the phase transmittance CGH approach where we implemented a circle, square and super-Gaussian.

5. CONCLUSION

We have demonstrated three methods in converting a Gaussian beam into a flat-top beam either circular or rectangular through the use of a phase-only spatial light modulator (SLM) operating in reflection mode. The first approach considers a phase-only definition of the transmission function and the flat-top beams are achievable at the Fourier plane of the SLM. The next two approaches consider a phase-only transmission function that contains information on the full complex amplitude plane. In both of these approaches, the plane of the SLM is imaged onto a CCD camera to determine the fidelity of the flat-top beams. We have demonstrated that all three methods present flat-top beams with steep skirts and with an evenly distributed intensity profile.

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