Intra-cavity decomposition of a dual-directional laser beam

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ABSTRACT

A method of decomposing a dual-directional laser beam into a forward propagating field and a backward propagating field for an apertured plano-concave cavity is presented. An intra-cavity aperture is a simple method of laser beam shaping as higher-order transverse modes are discriminated. Two fundamental resonator theories, namely, Fox-Li and Laguerre-Gaussian decomposition are used in the determination of the respective beam profiles at a specific plane. A preliminary set-up is characterized for Gaussian propagation in an attempt to verify that the cavity is viable. A comparison of experimental data with the theories is presented.

Keywords: Resonators, Fox-Li analysis, Transverse modes.

1. INTRODUCTION

The electromagnetic radiation emitted by optical resonators contains some discrete frequencies and these frequencies are separated from one another by frequency differences which may be associated with the different modes in a resonator. There are two sets of modes present in resonators; the first of which is termed longitudinal modes and have a single distinctive difference from one another, namely their oscillation frequency. The other set of modes are transverse modes; they differ from one another in their field distribution in a plane perpendicular to the direction of propagation in addition to a difference in their oscillation frequency[1]. Hermite-Gaussian (TEM_{mnq}) and Laguerre-Gaussian (TEM_{plq}) functions are utilized in describing the transverse and longitudinal mode structure within a resonator cavity for rectangular and circular symmetry respectively[2]. The indices m, n, p and l represent a particular transverse mode and the index q denotes a longitudinal mode. Comprehensive treatment of these modes is provided by Kogelnik and Li[2] and Hogson and Weber[3].

The spectral and spatial characteristics, which correspond to longitudinal and transverse modes respectively, of most lasers are independent in that multi frequency lasers may often have controlled transverse properties and can oscillate in an ideal single transverse mode[4]. Line width and coherence length are primarily determined by the spectral characteristics of a mode whereas beam divergence, beam diameter and energy distribution are properties of the spatial characteristics. The oscillation within a resonator will generally occur only in a few longitudinal and transverse modes depending on the size of the apertures within the resonator cavity and the mode with the lowest diffraction loss propagates as the laser beam, usually a Gaussian beam. The pioneering work on resonators was done by Fox and Li[5] and this is the basis of the development of resonator theory. In the selection of fundamental low order single transverse modes, an intra-cavity aperture may be used to attenuate undesired modes as different transverse modes have different spatial distributions. It is possible to also select single longitudinal modes; the first approach is to increase the loss sufficiently such that the mode having the largest gain oscillates. The second approach is to decrease the cavity length hence increasing the longitudinal mode spacing[6].

Modifying a laser fundamental mode is essential so as to generate a laser mode which is matched to the laser application. In applications where lasers are used, such as ablation, a beam of uniform intensity distribution is preferable. A method of achieving such modification with minimal diffraction loss is intra-cavity beam shaping. Bélanger *et al*[7,8,9] propose the use of a low loss stable graded-phase mirror (GPM) resonator with a very large discrimination of higher-order modes. This resonator is used to produce a super-Gaussian output beam with a super-Gaussian input beam from a technique referred to as reverse propagation. The input beam is propagated from the output coupler to the GPM and from diffraction the wavefront of the beam on this mirror is used to select an appropriate phase profile on the mirror such that the conjugate of this field propagates to the output coupler. This essentially allows for the reflected field to be reconstructed into the desired output field. This mirror has also been utilized in the generation of elegant Hermite-Gaussian mode of high power[10].

Building on the GPM resonator, a diffractive mode selecting mirror and an intra-cavity diffractive phase plate are used as part of this resonator in the generation of a low loss fundamental mode with large mode discrimination of higher-order modes[11,12]. The hallmark of such a resonator is that the input field is established as a mode of the cavity. The diffractive mode selecting mirror is also used as both resonator mirrors where they are designed such that a specific fundamental mode is achieved by reflecting the conjugate field, particularly a square flat-top mode[13]. Specialised resonator designs using an annular mirror and aspheric radially phase-conjugating mirrors are used in the generation of Bessel-Gauss beam[14,15]. Diffractive optical elements (DOE) which employ phase delays have been used as intra-cavity elements in generating a Gaussian distribution by metamorphosis of a Gaussian beam into a flat-top distribution on opposing mirrors[16]. The combination of the transmission DOE and a resonator mirror mimics a GPM. DOEs have also been utilized as a dual functional resonator mirror where the DOE operates as a mode selection element in reflection and as a beam shaping element in transmission[17]. Intra-cavity spatial filters have been employed to create a uniphase beam with a flat spatial profile[18]. The simplest technique in intra-cavity beam shaping is aperturing the cavity. Although this is extremely lossy, it allows for higher-order mode discrimination[19] and this is the case that we consider.

In this paper we present a comparative analysis of two fundamental theories, Fox-Li[5] analysis and Laguerre-Gaussian decomposition[20] on a simple plano-concave apertured cavity. The apertured cavity is used to express the variety of intra-cavity beam shaping techniques by decomposing a dual directional beam. We consider that dual directional beam is comprised of a forward component (plane to concave propagation) and a backward component (concave to plane propagation). At a given plane the Fox-Li analysis suggests that the beam profile in the respective directions are identical; however the Laguerre-Gaussian decomposition analysis suggests that the profiles differ significantly. We characterize this laser cavity for Gaussian propagation in an unperturbed laser cavity allowing for further measurements to be performed. A summary of the two fundamental theories is presented in section (2) and section (3) respectively and the experimental procedure and data analysis is presented in section (4).

2. FOX-LI APPROACH TO MODE ANALYSIS

The central idea in the so–called Fox-Li approach is to consider an arbitrary starting field (usually random noise to approximate spontaneous emission), and to calculate the propagation of this field as it traverses the length of the resonator and then returns again[5]. In a stable resonator, the losses after each pass result in an exponential decay in amplitude to the extent that a steady state is reached where a field with the lowest loss emerges. This lowest loss field is considered a normal mode of the resonator, the so-called fundamental mode.

To formulate this approach mathematically, one starts with some field u_1 on mirror M_1 and propagate the field using the Huygens-Fresnel diffraction integral for free space a distance z = L to mirror M_2. The field at this mirror is then given by[5]:

$$\gamma_2 u_2(r_2) = \int_0^{X_1} K_1(r_1, r_2) u_1(r_1) r_1 dr_1 \tag{1}$$

The field at mirror M_1 is then found from:

$$\gamma_1 u_1(r_1) = \int_0^{X_2} K_2(r_1, r_2) u_2(r_2) r_2 dr_2 \tag{2}$$

Here γ_i is the eigenvalue corresponding to field u_i, and the kernels of the diffraction integrals are given by:

$$K_1 = \frac{i}{L} J_0\left(k\frac{r_1 r_2}{L}\right) \exp\left(\frac{ik}{2L}(r_1^2 + r_2^2)\right) \tag{3}$$

$$K_2 = \frac{i}{L} J_0\left(k\frac{r_1 r_2}{L}\right) \exp\left(\frac{ik}{2L}(r_1^2 + r_2^2)\right) \exp\left(-\frac{ikr_2^2}{R}\right) \tag{4}$$

where the last term of Eq. (4) takes care of the curvature of the mirror M_2.

It useful to note that only the field of the two propagation integrals (one for each direction) is changing on each pass of the resonator, and not the kernels themselves. Therefore, if the transformation of a field on passing through the resonator could be expressed as the product of two matrices – one representing the starting field and the other the transformation of that field – only the former would have to be calculated on each pass, and not the latter[21]. For example, since the integrant in Eq. (2) does not change with the changing field, we may express Eq. (2) in matrix form as

$$\vec{u}_1 = T\,\vec{u}_2 \tag{5}$$

If we write the matrices for the forward and backward propagation directions inside the resonator as T_1 and T_2 respectively, then the characteristic integral equation for any resonator system can be presented in the terms of the matrix method as:

$$\lambda \vec{u}_1 = T_1 T_2 \vec{u}_1. \tag{6}$$

Equation (6) has solutions if the determinant of $I\lambda - T_1T_2$ is zero; consequently all eigenvectors of T_1T_2 represent the possible resonator modes, while all eigenvalues represent the losses with phase shift for these corresponding modes.

3. LAGUERRE-GAUSSIAN EXPANSION ANALYSIS

The numerical calculation of the resonant field in the resonator of length L including a circular aperture is based on its expansion on the basis of the eigenfunctions of the bare cavity that is without any diffracting objects. The eigenfunctions are the Laguerre-Gauss functions written for the cylindrical symmetry. The origin of the longitudinal coordinate is set on the plane mirror so that the concave mirror is set at z=L. The study of the resonant field involves a decomposition into its two progressive components: a forward beam, propagating in the direction z>0, and a backward beam in the direction z<0. The Laguerre-Gauss functions make up an orthonormalised basis which are written, for the forward beam as

$$G_{fp}(\rho, z) = \left(\frac{2}{\pi}\right)^{1/2} \frac{1}{W(z)} L_p\left(\frac{2\rho^2}{W^2}\right) \exp\left(-\frac{\rho^2}{W^2}\right) \exp\left\{+i\left[\frac{k\rho^2}{2R_c(z)} - (2p+1)\phi(z)\right]\right\}, \tag{7}$$

and for the backward beam as

$$G_{bp}(\rho, z) = \left(\frac{2}{\pi}\right)^{1/2} \frac{1}{W(z)} L_p\left(\frac{2\rho^2}{W^2}\right) \exp\left(-\frac{\rho^2}{W^2}\right) \exp\left\{-i\left[\frac{k\rho^2}{2R_c(z)} - (2p+1)\phi(z)\right]\right\}, \tag{8}$$

where $k = 2\pi/\lambda$. Hereafter, the subscripts f and b denote, respectively, forward and backward quantities,. The Gaussian mode of the nonapertured cavity is characterized by its beam diameter $2W(z)$ and its radius of curvature R_c at point z. These quantities of reference, as well as the Gouy phase shift ϕ, are z dependent and obey the well known formulas :

$$W^2(z) = W_0^2\left[1+(z/z_0)^2\right], \qquad R_c(z) = z\left[1+(z_0/z)^2\right], \qquad \phi(z) = \arctan(z/z_0), \tag{9}$$

where $z_0 = \pi W_0^2/\lambda$ is the Rayleigh range and W_0 is the beam-waist radius expressed by $W_0^2 = (\lambda d/\pi)\sqrt{g/(1-g)}$ for our plano-concave cavity. $L_p(X)$ is the Laguerre polynomial of order *p*.

The forward and backward fields are assumed to be linearly polarized and are expressed as linear combinations of the basis functions :

$$E_f(\rho,z) = \exp\left[i(kz-\omega t)\right]\sum_p f_p G_{fp}(\rho,z), \tag{10}$$

$$E_b(\rho,z) = \exp\left\{i\left[k(2d-z)-\omega t\right]\right\}\sum_p b_p G_{bp}(\rho,z). \tag{11}$$

The objective of the calculation is to find the coefficients f_p and b_p which are ρ and *z* independent. The determination of these coefficients involves a matrix ***M*** which represents the round-trip operator and expresses the change of the forward coefficients after a round-trip in the cavity:

$$f_p^{'} = \sum_m M_{pm}.f_m, \tag{12}$$

where M_{pm} is the typical element of a matrix ***M***. The calculation of matrix ***M*** can be found in[20] for the cavity including a hard aperture of radius ρ_0 set against the concave mirror on which the Gaussian beam has a width W_c:

$$M_{pm} = -r_p r_c \exp\left\{2i\left[kL-(p+m+1)\phi(L)\right]\right\}C_{pm}, \tag{13}$$

where r_p (r_c) is the reflectance (field ratio) of the plane (concave) mirror. The overlapping integral is given by

$$C_{pm} = \int_0^{2Y^2} \exp[-X]L_p(X)L_m(X)dX, \tag{14}$$

where $X = 2\rho^2/W_c^2$ and $Y = \rho_0/W_c$. The resonance condition states that, after a round trip, the relation $f_p^{'} = \Gamma.f_p$ holds for all *p*. This condition means that the eigenmodes of the apertured resonator are represented by the eigenvectors ***u*** of the matrix ***M***. Each of them is characterised by a complex eigenvalue Γ such that ***Mu***=Γ**u**. The eigenvector of ***M*** having the largest eigenvalue $|\Gamma_0|^2$ corresponds to the fundamental mode, i.e. the first mode to reach the threshold. Determination of eigenvectors of ***M*** of size (80×80) is done numerically using a FORTRAN 77 routine based on *International Mathematics and Statistics Library* (IMSL) subroutines. It is important to note that the method of Laguerre-Gauss functions is powerful since the coefficients f_p and b_p which are ρ and *z* independent. This makes possible the complete mapping of the spatial distribution of the resonant field with a very fast computing time since it involves only summations when applying Eqs. (10) and (11).

4. EXPERIMENTAL SET-UP AND RESULTS

We consider an apertured plano-concave cavity where the aperture is located on the concave mirror (output coupler). The gain medium, Nd:YAG (30mm x 4 mm rod), was end-pumped with a Jenoptik (JOLD-75-CPXF-2P W) 75 W multi-mode fiber-coupled diode. The resonator is designed in an L-shaped cavity so as to separate the forward and backward components of the field within the cavity. A 45 degree mirror which is 99.8% reflective is placed at the apex of the L and the forward and backward beams incident on the reflective surface of the mirror; propagate outside the cavity. The *g* parameter of the resonator was chosen to be 0.4 which corresponds to the output coupler having a curvature of 400 mm. The Gaussian beam width on the concave mirror in an unperturbed cavity was calculated to be 407.3 µm and the radius of the aperture was chosen to be 400 µm.

These parameters were selected so as to theoretically predict, by Laguerre-Gaussian decomposition, a significant difference in the beam profiles in a plane for the respective directions. This is in accordance with this theory as the ratio of the aperture radius to the Gaussian beam width on the concave mirror is required to be unity for the greatest difference. Figs. (1a) and (1b) illustrate the experimental set-up for the forward beam and backward beam respectively.

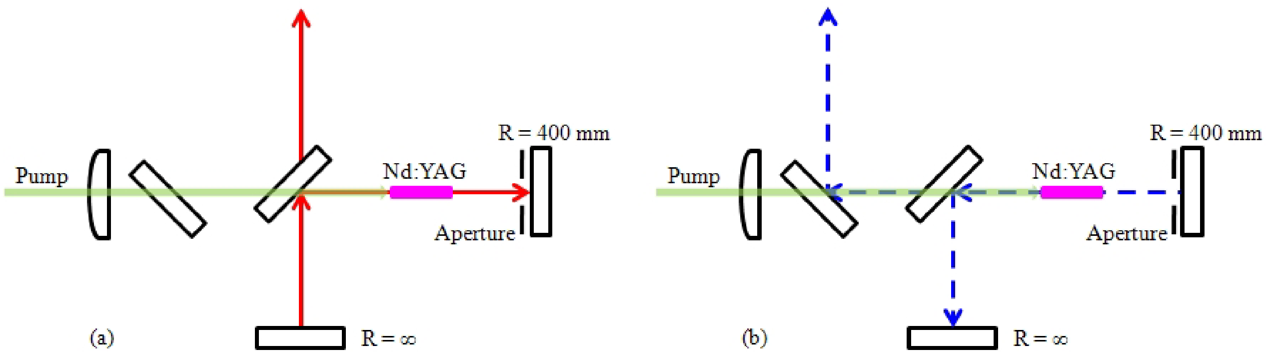

Figure 1: Experimental set-up where the dual-directional beam is decomposed as (a) Forward beam and (b) Backward beam in an aperture plano-concave cavity with the output coupler having a radius of curvature of 400 mm.

The forward and backward beams were individually relay imaged by use of afocal telescopes from the plane and concave mirrors respectively to independent positions outside the cavity as presented in Fig. (2). We were thus able to scan the beam profile external to the cavity for the propagation from the plane to the concave mirror and vice versa with a CCD device (Spiricon LBA-USB L130). To verify that the imaging systems were correctly assembled, a test on an unperturbed cavity for Gaussian propagation was done. The beam width at respective positions along the propagation axis was measured for each respective beam and compared to a theoretical prediction where a propagation distance of zero corresponds to the plane mirror and a propagation distance of 240 mm corresponds to the concave mirror as presented in Fig. (3).

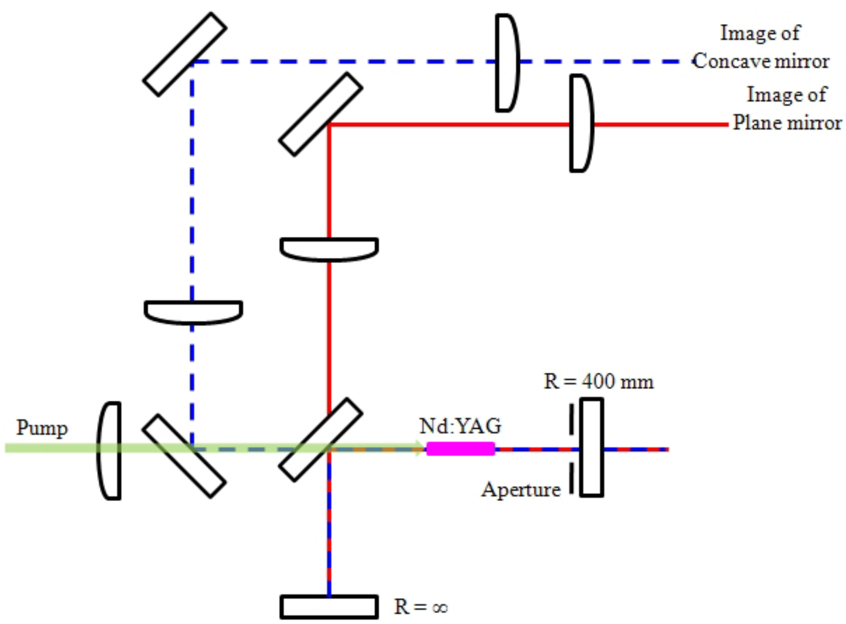

Figure 2: Relay imaging of the respective beams independently, allowing external to the cavity determination of the beam profile within the cavity.

The results in Fig. (3) coincide exceptionally and we are confident that the cavity is assembled correctly. The plane of significant interest is positioned at 176 mm from the plane mirror as this position indicates the largest difference in beam profile with regard to the Laguerre-Gaussian decomposition analysis. The intensity distribution of the forward and backward beams as per the Fox-Li and Laguerre-Gaussian decomposition analysis is presented in Fig. (4) and (5) respectively.

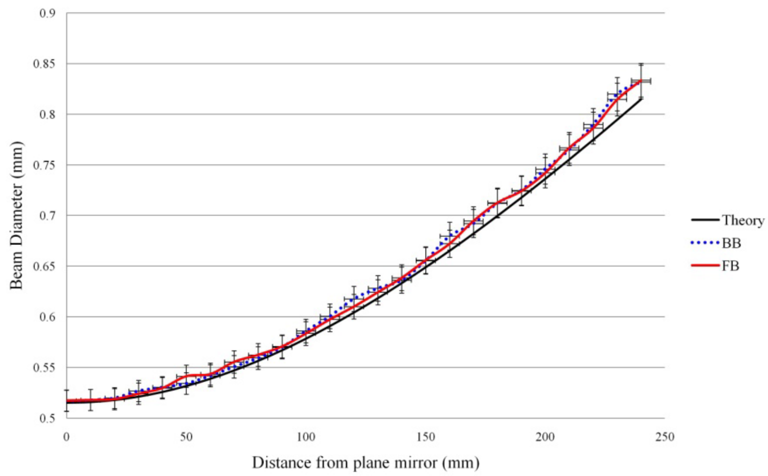

Figure 3: Gaussian beam width profile within the cavity for the forward (FB) and backward (BB) beams.

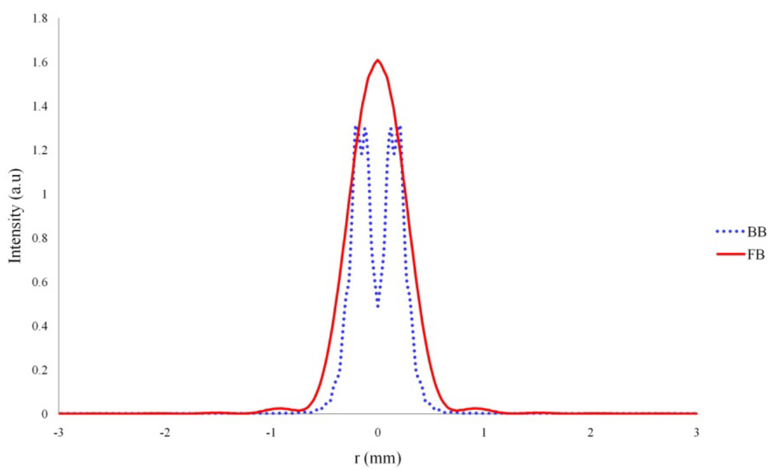

Figure 4: Intensity profile of forward (FB) and backward (BB) beams at 176 mm from plane mirror generated by Fox-Li analysis.

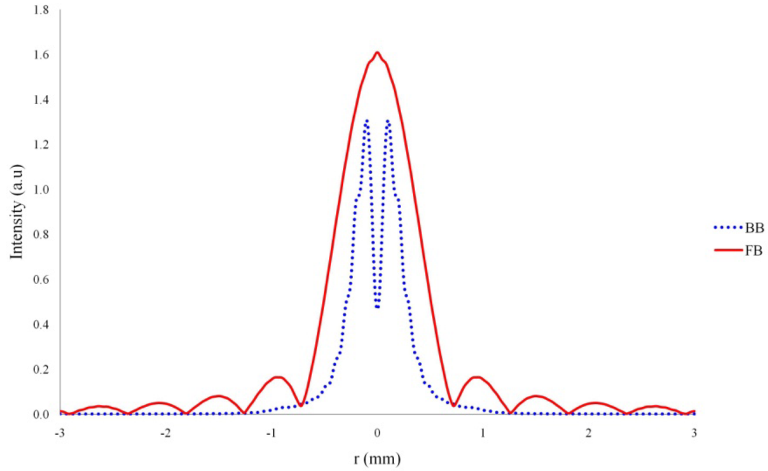

Figure 5: Intensity profile of forward (FB) and backward (BB) beams at 176 mm from plane mirror generated using Laguerre-Gaussian decomposition analysis.

As seen in Figs. (4) and (5) there is a significant difference between the forward and backward beam profiles. The Fox-Li analysis presented in Fig. (4) is remarkably different from what we expected. We anticipated that the intensity profiles in the respective directions are identical in their profile, with a standard difference, due to diffraction loss; the peak intensity would be lower for the backward propagating field. Experimentally the respective beam widths were captured at 172 mm from the plane mirror as experimental error was factored into the measurement process and Fig. (6) presents the experimental data on the intensity profiles captured on CCD device.

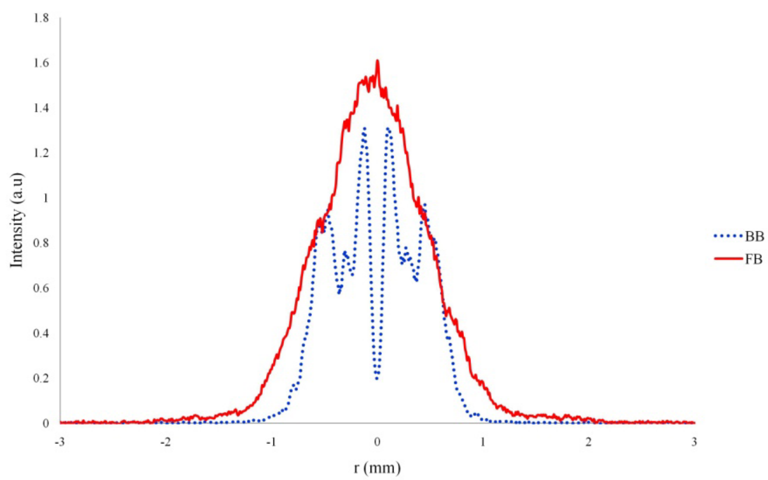

Figure 6: Experimental intensity profile of forward (FB) and backward (BB) beams at 172 mm from plane mirror.

The beam intensity profiles in Figs. (4) – (6) are normalised consistently for the forward and backward profiles so as to achieve uniformity. This result currently does not provide clarity on which resonator analysis is more accurate. However further analysis is required to find a correlation between theoretical predictions and experimental data.

5. CONCLUSION

We have successfully demonstrated a method in which to decompose an intra-cavity dual-directional laser beam into two components in an apertured plano-concave resonator cavity. The beam which is shaped by an intra-cavity aperture is decomposed into a forward propagating field and a backward propagating field which correspond to propagation from the plane to concave mirror and the concave to the plane mirror respectively. The forward and backward beam intensity profiles at a plane situated 176 mm from the plane mirror are determined by two fundamental resonator theories, namely Fox-Li and Laguerre-Gaussian decomposition. The beam intensity profiles for the different directions show significant difference in both theoretical approximations. Experimentally we have successfully characterized the resonator by measuring the Gaussian beam widths in an unperturbed cavity from the plane mirror to the concave mirror for the respective directions and this compares remarkably well with a theoretical prediction. We thus have a cavity with which further analysis will allow for a comparison of the theoretical predictions to experimental data.

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