A FRAMEWORK FOR T-TAIL FLUTTER ANALYSIS

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Abstract: The flutter analysis of T-tail aircraft poses challenges that are unique to this configuration, including the fact that the unsteady air loads are dependent on the steady load distribution and static deformation of the aircraft. In particular, the trim load on the horizontal stabilizer and the static deformation of the horizontal stabilizer, an induced dihedral effect, are significant. These effects are now well understood and accurate analyses can be made for a given set of conditions of incidence angle, elevator deflection and deformation. This paper considers the process required to perform the flutter analyses of a T-tailed aircraft.

1 INTRODUCTION

From the considerations mentioned above it is clear that the mass and balance of a T-tailed aircraft has significant flutter implications, apart from its effect on the natural modes of the structure, as it affects the trim load on and static deformation of the horizontal stabilizer. The permissible loading range of the aircraft also needs to be explored in the flutter analysis. It is no longer sufficient to use tabulated unsteady generalized aerodynamic forces (GAFs) for a given set of mode shapes, reduced frequencies and Mach numbers in the flutter solution - a static aeroelastic trim analysis needs to be performed for each flight condition before the GAFs for that condition can be calculated.

The present paper presents a procedure for performing a flutter analysis of a T-tailed aircraft in a given loading condition over a range of speeds. The first step at each speed is the aeroelastic trim analysis. Flutter analyses usually start at speeds below the minimum flight speed to facilitate the tracking of modes. A trim analysis below the minimum flight speed would however be meaningless. To accommodate this practice, a minimum flight speed is specified. For analyses at speeds below the minimum flight speed, the trim analysis calculates the trim conditions (i.e. angle of attack, control deflection and deformation) for the minimum flight speed. Only the change in orientation of the panels in the aerodynamic model due to the rigid motion and static deformation is considered.

The calculation of the GAFs is performed for the actual Mach number at the particular flight condition and over a range of reduced frequencies. The quadratic mode shape components and static pressure distribution is also taken into account in the calculation of the GAFs.

The T-tail flutter solver is derived from a standard p-k solver that interpolates between tabulated GAFs, first on Mach number and then on reduced frequency. The interpolation on Mach number is replaced by a call to the aerodynamics routine to perform the trim analysis, followed by the calculation of GAFs at a number of reduced frequencies. The flutter solver then interpolates on reduced frequency in the usual manner.

2 BUILDING BLOCKS

The suggested procedure builds on the following capabilities:

- A matched-point p-k flutter solver. The solver in its original form interpolates on both Mach number and reduced frequency during the solution.
- A static aeroelastic trim analysis. The procedure used in the present study is detailed below.
- An enhanced DLM that calculates all the significant unsteady aerodynamic loads that are required for a T-tail flutter analysis. Details of this method are presented in [1].

The trim equation is essentially the flutter equation with the unsteady terms omitted. Starting with the set of elastic airframe modes, we prepend the six rigid body modes, viz.:

1. Streamwise displacement, x_r (along the velocity vector)
2. Vertical displacement, z_r (normal to the velocity vector and in a plane normal to the x-y plane)
3. Pitch, α (about an axis normal to the velocity vector and parallel to the x-y plane)
4. Lateral displacement, y_r (normal to the velocity vector and parallel to the x-y plane)
5. Yaw, β (about an axis normal to the velocity vector and in a plane normal to the x-y plane)
6. Roll, γ (about the velocity vector)

and append the basic control modes (if they are not already included in the set of elastic airframe modes), viz.:

1. Elevator deflection, δ_e (or any control mode that primarily controls pitch)
2. Rudder deflection, δ_r (or any control mode that primarily controls yaw)
3. Aileron deflection, δ_a (or any control mode that primarily controls roll)
4. Thrust, T

The thrust mode does not have any aerodynamic effect in the present model. The contribution of thrust to the generalized forces depends on the total thrust required, which needs to be solved. For multi-engined aircraft this is best handled through displacement tables. Each table gives the modal displacements of the point where the thrust acts. The table header gives the thrust vector and the fraction of total thrust. The thrust contribution to the generalised forces ends up in the last column of the stiffness matrix.

In addition, the contribution of gravity must be added. The gravity contribution is constant, representing the weight of the aircraft, and ends up in the right hand side of the trim equation, Eq. (1). The gravity vector is specified in the wind axis system.

Of the six rigid body modes, four do not generate any aerodynamic forces and can be eliminated from the unknowns. The corresponding equilibrium equations must however be retained. On the other hand, the control mode displacements are not governed by the equilibrium equation and the corresponding equations can be omitted. This leaves us with as many equations as unknowns so that the pitch and yaw angles, control deflections and elastic deformation can easily be solved. The equation is solved by eliminating the four equations (rows) corresponding to the control deflections and thrust, and the columns of the coefficient matrix corresponding to the four rigid body modes listed above. The resulting set of equations is shown in Eq. (2).

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\left[\begin{array}{cccccc|ccc|cccc}
0 &&&&&&&&&&&& \\
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&&&&&& K_1 &&&&&& \\
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&&&&&&&& K_n &&&& \\
\hline
&&&&&&&&& K_e &&& \\
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&&&&&&&&&&& K_a & \\
&&&&&&&&&&&& 0
\end{array}\right]
\left\{\begin{array}{c} x_r \\ z_r \\ \alpha \\ y_r \\ \beta \\ \gamma \\ \hline q_1 \\ \vdots \\ q_n \\ \hline \delta_e \\ \delta_r \\ \delta_a \\ T \end{array}\right\} =
$$

$$
\left\{\begin{array}{c} Q_x^{(0)} + mg_x \\ Q_z^{(0)} + mg_z \\ Q_\alpha^{(0)} \\ Q_y^{(0)} + mg_y \\ Q_\beta^{(0)} \\ Q_\gamma^{(0)} \\ \hline Q_1^{(0)} \\ \vdots \\ Q_n^{(0)} \\ \hline Q_e^{(0)} \\ Q_r^{(0)} \\ Q_a^{(0)} \\ 0 \end{array}\right\} +
\left[\begin{array}{cccccc|ccc|cccc}
0 & 0 & Q_{x,\alpha} & 0 & Q_{x,\beta} & 0 & Q_{x,1} & \cdots & Q_{x,n} & Q_{x,e} & Q_{x,r} & Q_{x,a} & t_x \\
0 & 0 & Q_{z,\alpha} & 0 & Q_{z,\beta} & 0 & Q_{z,1} & \cdots & Q_{z,n} & Q_{z,e} & Q_{z,r} & Q_{z,a} & t_z \\
0 & 0 & Q_{\alpha,\alpha} & 0 & Q_{\alpha,\beta} & 0 & Q_{\alpha,1} & \cdots & Q_{\alpha,n} & Q_{\alpha,e} & Q_{\alpha,r} & Q_{\alpha,a} & t_\alpha \\
0 & 0 & Q_{y,\alpha} & 0 & Q_{y,\beta} & 0 & Q_{y,1} & \cdots & Q_{y,n} & Q_{y,e} & Q_{y,r} & Q_{y,a} & t_y \\
0 & 0 & Q_{\beta,\alpha} & 0 & Q_{\beta,\beta} & 0 & Q_{\beta,1} & \cdots & Q_{\beta,n} & Q_{\beta,e} & Q_{\beta,r} & Q_{\beta,a} & t_\beta \\
0 & 0 & Q_{\gamma,\alpha} & 0 & Q_{\gamma,\beta} & 0 & Q_{\gamma,1} & \cdots & Q_{\gamma,n} & Q_{\gamma,e} & Q_{\gamma,r} & Q_{\gamma,a} & t_\gamma \\
\hline
0 & 0 & Q_{1,\alpha} & 0 & Q_{1,\beta} & 0 & Q_{1,1} & \cdots & Q_{1,n} & Q_{1,e} & Q_{1,r} & Q_{1,a} & t_1 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & Q_{n,\alpha} & 0 & Q_{n,\beta} & 0 & Q_{n,1} & \cdots & Q_{n,n} & Q_{n,e} & Q_{n,r} & Q_{n,a} & t_n \\
\hline
0 & 0 & Q_{e,\alpha} & 0 & Q_{e,\beta} & 0 & Q_{e,1} & \cdots & Q_{e,n} & Q_{e,e} & Q_{e,r} & Q_{e,a} & 0 \\
0 & 0 & Q_{r,\alpha} & 0 & Q_{r,\beta} & 0 & Q_{r,1} & \cdots & Q_{r,n} & Q_{r,e} & Q_{r,r} & Q_{r,a} & 0 \\
0 & 0 & Q_{a,\alpha} & 0 & Q_{a,\beta} & 0 & Q_{a,1} & \cdots & Q_{a,n} & Q_{a,e} & Q_{a,r} & Q_{a,a} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\left\{\begin{array}{c} x_r \\ z_r \\ \alpha \\ y_r \\ \beta \\ \gamma \\ \hline q_1 \\ \vdots \\ q_n \\ \hline \delta_e \\ \delta_r \\ \delta_a \\ T \end{array}\right\}
\tag{1}
$$

$$
\left[\begin{array}{cc|ccc|cccc}
-Q_{x,\alpha} & -Q_{x,\beta} & -Q_{x,1} & \cdots & -Q_{x,n} & -Q_{x,e} & -Q_{x,r} & -Q_{x,a} & -t_x \\
-Q_{z,\alpha} & -Q_{z,\beta} & -Q_{z,1} & \cdots & -Q_{z,n} & -Q_{z,e} & -Q_{z,r} & -Q_{z,a} & -t_z \\
-Q_{\alpha,\alpha} & -Q_{\alpha,\beta} & -Q_{\alpha,1} & \cdots & -Q_{\alpha,n} & -Q_{\alpha,e} & -Q_{\alpha,r} & -Q_{\alpha,a} & -t_\alpha \\
-Q_{y,\alpha} & -Q_{y,\beta} & -Q_{y,1} & \cdots & -Q_{y,n} & -Q_{y,e} & -Q_{y,r} & -Q_{y,a} & -t_y \\
-Q_{\beta,\alpha} & -Q_{\beta,\beta} & -Q_{\beta,1} & \cdots & -Q_{\beta,n} & -Q_{\beta,e} & -Q_{\beta,r} & -Q_{\beta,a} & -t_\beta \\
-Q_{\gamma,\alpha} & -Q_{\gamma,\beta} & -Q_{\gamma,1} & \cdots & -Q_{\gamma,n} & -Q_{\gamma,e} & -Q_{\gamma,r} & -Q_{\gamma,a} & -t_\gamma \\
\hline
-Q_{1,\alpha} & -Q_{1,\beta} & K_1 - Q_{1,1} & \cdots & -Q_{1,n} & -Q_{1,e} & -Q_{1,r} & -Q_{1,a} & -t_1 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
-Q_{n,\alpha} & -Q_{n,\beta} & -Q_{n,1} & \cdots & K_n - Q_{n,n} & -Q_{n,e} & -Q_{n,r} & -Q_{n,a} & -t_n
\end{array}\right]
\left\{\begin{array}{c} \alpha \\ \beta \\ \hline q_1 \\ \vdots \\ q_n \\ \hline \delta_e \\ \delta_r \\ \delta_a \\ T \end{array}\right\} =
\left\{\begin{array}{c} Q_x^{(0)} + mg_x \\ Q_z^{(0)} + mg_z \\ Q_\alpha^{(0)} \\ Q_y^{(0)} + mg_y \\ Q_\beta^{(0)} \\ Q_\gamma^{(0)} \\ \hline Q_1^{(0)} \\ \vdots \\ Q_n^{(0)} \end{array}\right\}
\tag{2}
$$

The trim analysis was implemented as part of the p-k flutter solver. The rigid body modes and the control modes (excluding the thrust mode) are included in the input set of modes. The rigid body modes define the centre of mass position. The vector of steady generalised forces $Q^{(0)}$ is calculated as part of the steady solution. An unsteady analysis at zero frequency is performed to calculate the matrix $Q^{(1)}$. The dynamic pressure, gravity vector and thrust tables are required as additional input. The thrust tables consist of the fraction of total thrust, thrust vector and displacement of the point of application in each mode, for each propulsion unit. With this information, Eq. (2) can be constructed and solved. An iterative solution would generally be required because the induced drag does not vary linearly with angle of attack.

The method was implemented in a sufficiently general way that not all rigid body degrees of freedom or control modes need to be used, provided that the trim problem remains well-posed. For rigidly mounted structures no rigid or control modes need to be specified at all. Trim tab modes can also be used instead of primary control surface modes. The difference is illustrated in Fig. 1 and Fig. 2. The trim tab in this example is rigged as an anti-servo tab and deflects up with the elevator when the elevator is used for pitch control. When the trim tab is used for pitch control, the trim tab deflects down and the elevator up, as expected.

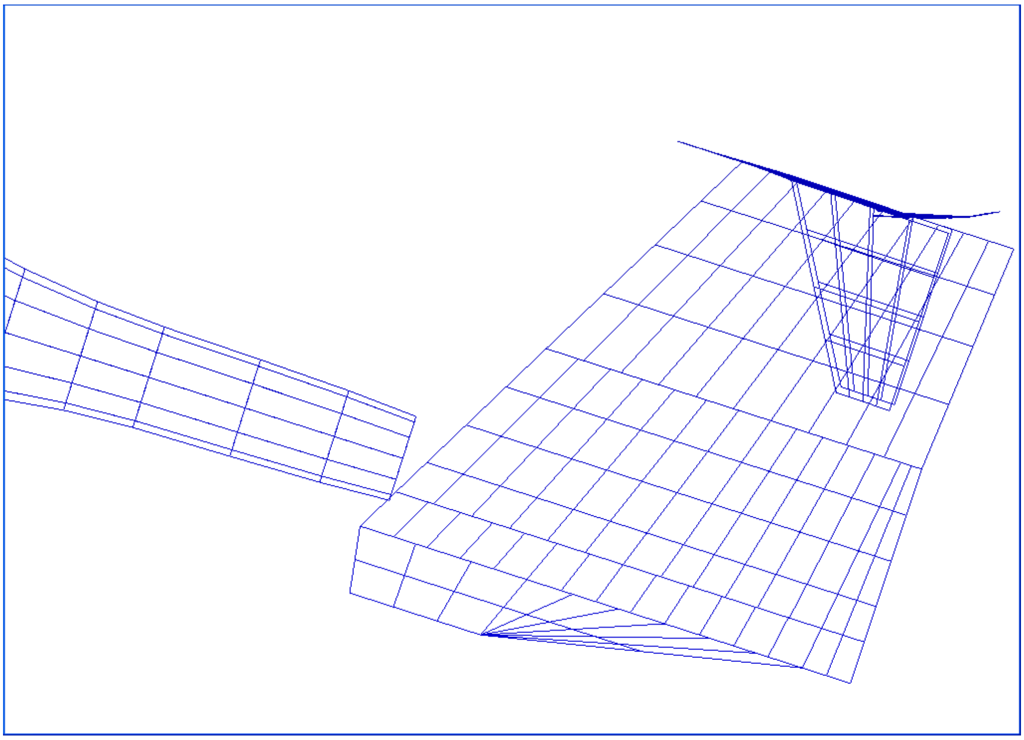

Fig. 1: Trim analysis using elevator for pitch control (displacements exaggerated 5×)

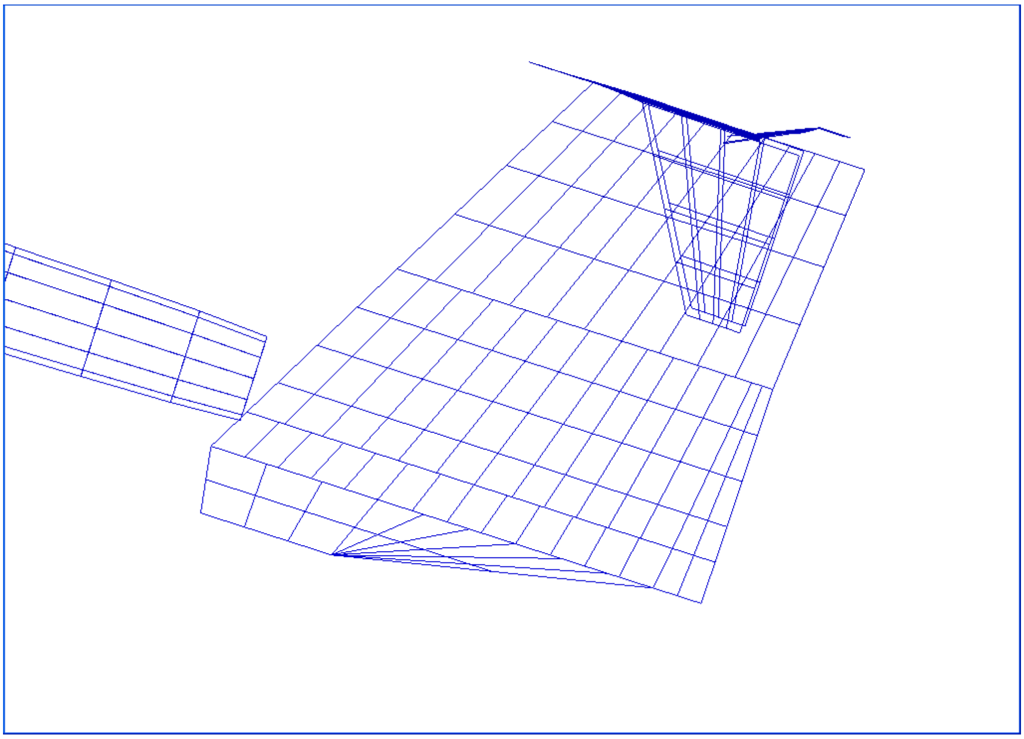

Fig. 2: Trim analysis using elevator trim tab for pitch control (displacements exaggerated 5×)

3 IMPLEMENTATION

The DLM code was written as a subroutine that could be called to perform different tasks, including

- Reading and processing geometry input
- Modifying the geometry, i.e. rotating the normal vectors, according to the modal deflection
- Calculating the steady load distribution for a given Mach number and deflected shape
- Calculating unsteady generalized aerodynamic forces for a given Mach number, reduced frequency, steady load distribution and deflected shape

The p-k flutter solver was modified to

- Call the DLM routine once to read the geometry
- At each speed increment:
  1) Call the DLM routine to calculate the steady load in the un-trimmed state and the unsteady generalized aerodynamic forces at zero frequency.
  2) Perform the static aeroelastic trim analysis
  3) Call the DLM routine to deform the geometry
  4) Call the DLM routine to calculate the steady load in the trimmed state
  5) Call the DLM routine to calculate GAFs for a set of reduced frequencies

The input consists of the aircraft geometry, the mode shapes and the modal properties, and the trim data. The trim data consists mainly of the weight of the aircraft, gravity vector and the identification of rigid and control modes.

4 APPLICATION

An illustration case was constructed by adding a hypothetical aircraft fuselage and wings to a T-tail wind-tunnel model that had previously been tested and which has shown a strong dependence of flutter speed on stabilizer setting angle. The wind-tunnel model is shown in Fig. 3. The predicted and measured flutter speed of the model, as a function of stabiliser setting angle, is shown in Fig. 4. A general description of the wind-tunnel model is given in [1] and more details are presented in the Appendix.

The aerodynamic model of the hypothetical aircraft model is shown in Fig. 5. Any similarity to any actual aircraft is not purely coincidental, but this work was not sponsored or sanctioned by any aircraft manufacturer. The mass of the model was chosen to result in a lift coefficient of 0.3 at the design flutter equivalent air speed of 55 m/s. Two centre of mass positions were considered: At the calculated neutral point and one mean chord length ahead of the neutral point (See Fig. 6).

The three elastic modes of the wind-tunnel were used in the analysis, together with a pitch control mode consisting of the rigid rotation of the stabilisers and rigid vertical plunge and pitch modes of the complete aircraft model.

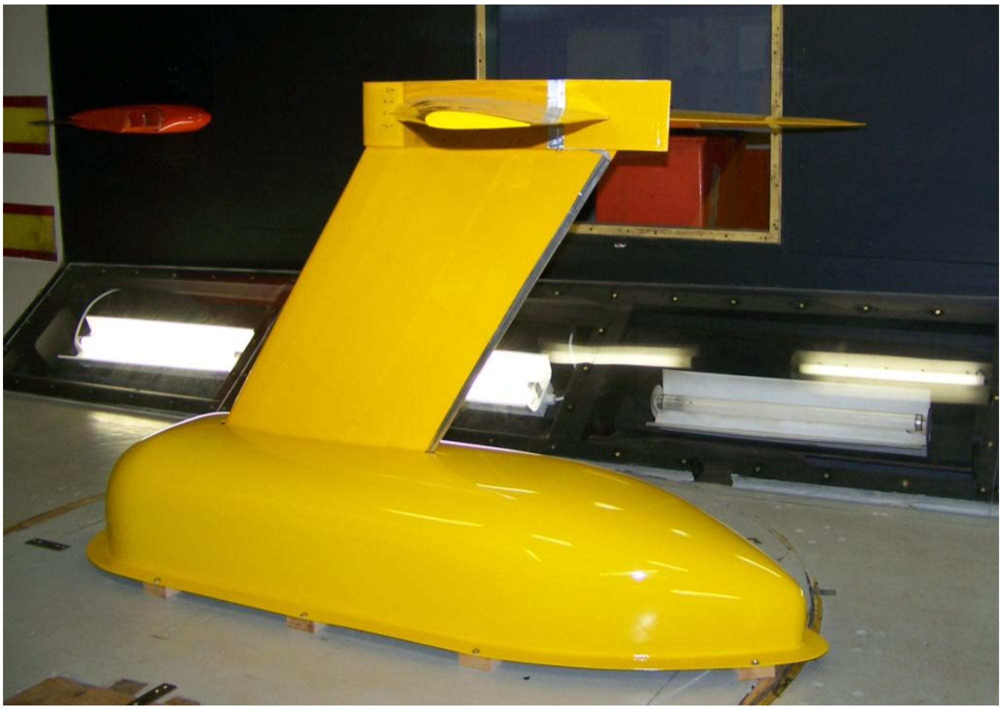

Fig. 3: T-tail model installation in the wind-tunnel

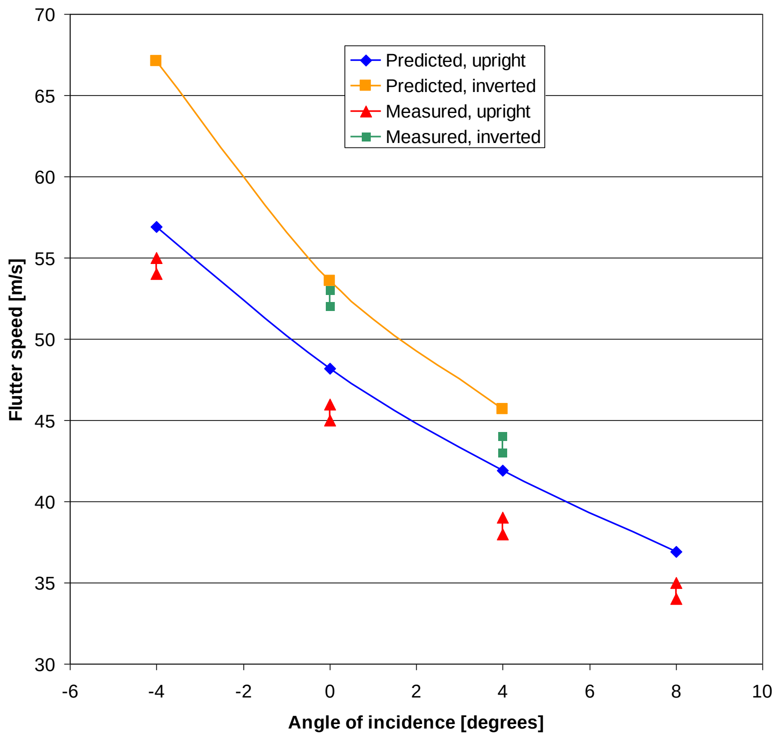

Fig. 4: T-tail model flutter speed vs. HTP incidence

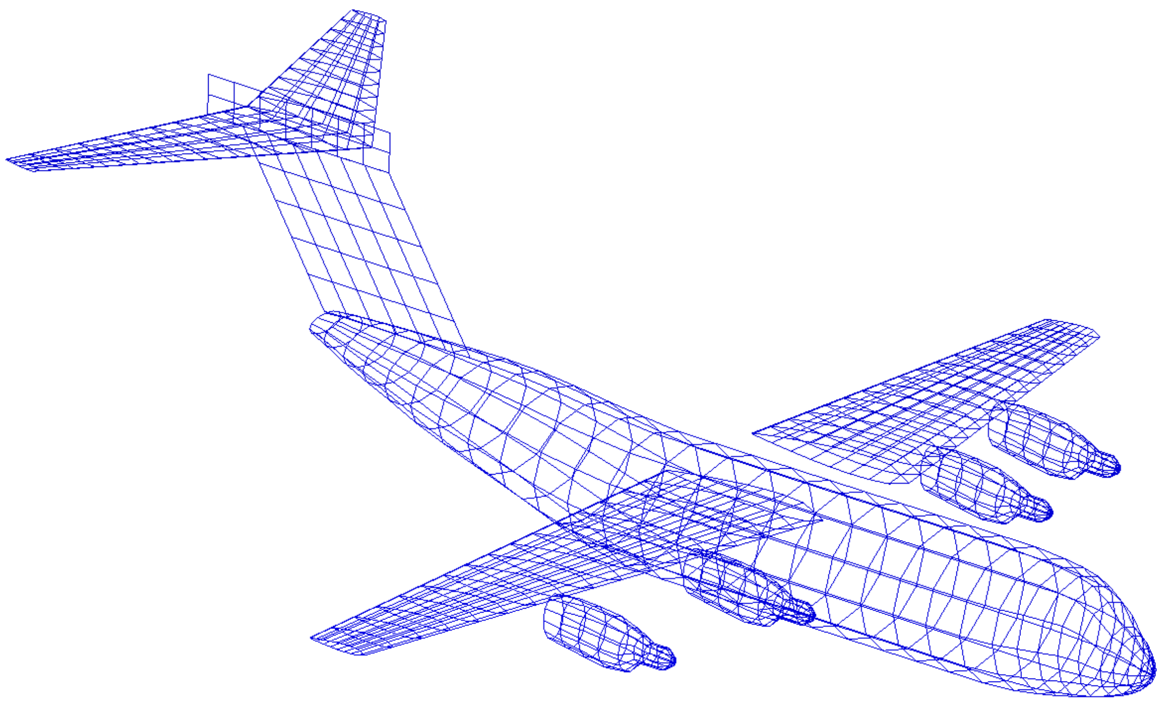

Fig. 5: Aerodynamic model of the hypothetical aircraft model

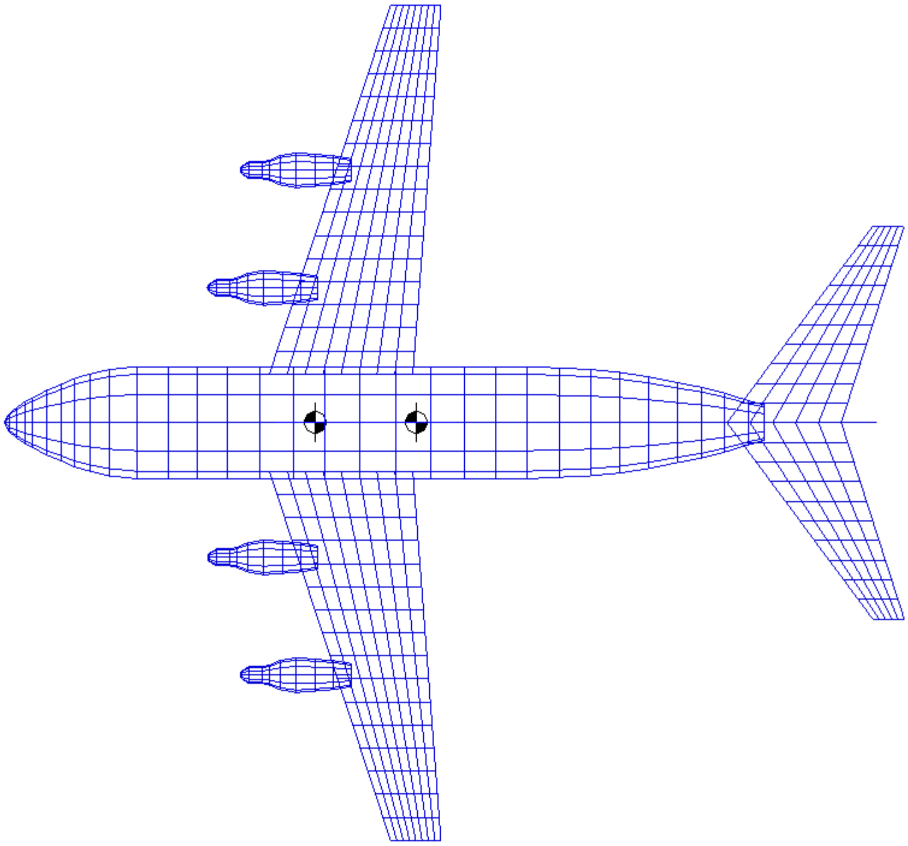

Fig. 6: Centre of mass positions considered

The aircraft model, analysed in its un-trimmed state, had a flutter equivalent air speed of 55.64 m/s, irrespective of the centre of mass position. In this state the wing generated a small upward lift force and the stabilizer a downward lift force of 57.4 N because of the upright and inverted NACA23015 wing profile of the wing and stabilizer, respectively.

Next, flutter analyses were performed with the aircraft trimmed for the design flutter equivalent air speed. The trim conditions, in terms of angle of attack and stabilizer setting angle, are shown in Fig. 7 and Fig. 8.

For the rearward (neutral) centre of mass position the flutter speed was 48.36 m/s. For the forward position the flutter speed was 54.31 m/s. The flutter speeds and corresponding trim loads are summarized in Table 1. The flutter speed follows the expected trend of increasing with increasing downward trim load.

In the final analysis, the trim state of the aircraft model was determined at each speed increment, before the unsteady GAFs were calculated. The minimum flight speed was determined to limit the lift coefficient to be less than 1. The results of the analyses for the two centre of mass positions are shown in Fig. 9 to Fig. 12. In the case of the forward centre of mass position, there is little change in the predicted flutter speed compared to the previous analysis, conducted at the trim conditions for 55 m/s. In the case of the rearward centre of mass position there is a further reduction in flutter speed compared to the analysis at the trim conditions for 55 m/s.

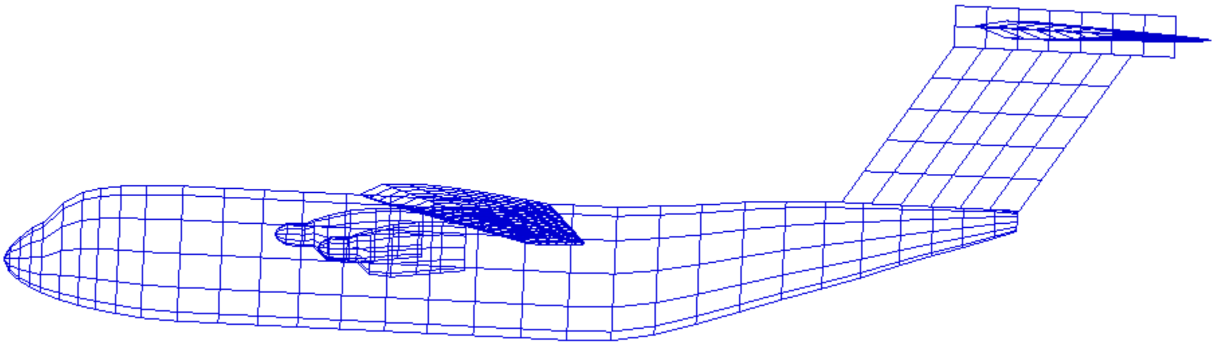

Fig. 7: Trim state for centre of mass at the neutral point

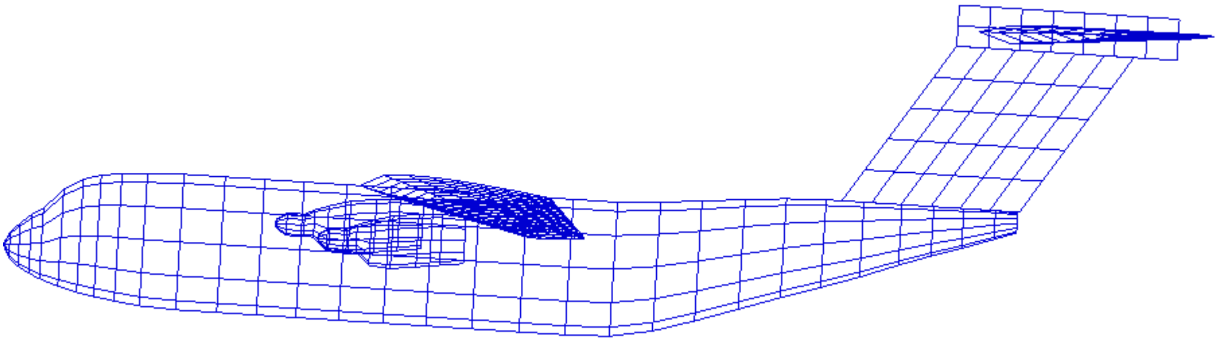

Fig. 8: Trim state for centre of mass one mean chord length ahead of the neutral point

Table 1: Summary of trim loads and flutter speeds

Centre of mass	Wing lift [N]	Stabiliser lift [N]	Total lift [N]	Flutter speed [m/s]
Un-trimmed	85.5	-57.4	28.0	55.64
Forward, trimmed for 55 m/s	532.6	-50.5	482.1	54.31
Neutral, trimmed for 55 m/s	427.2	50.2	477.4	48.36
Forward, trimmed				54.30
Neutral, trimmed				46.13

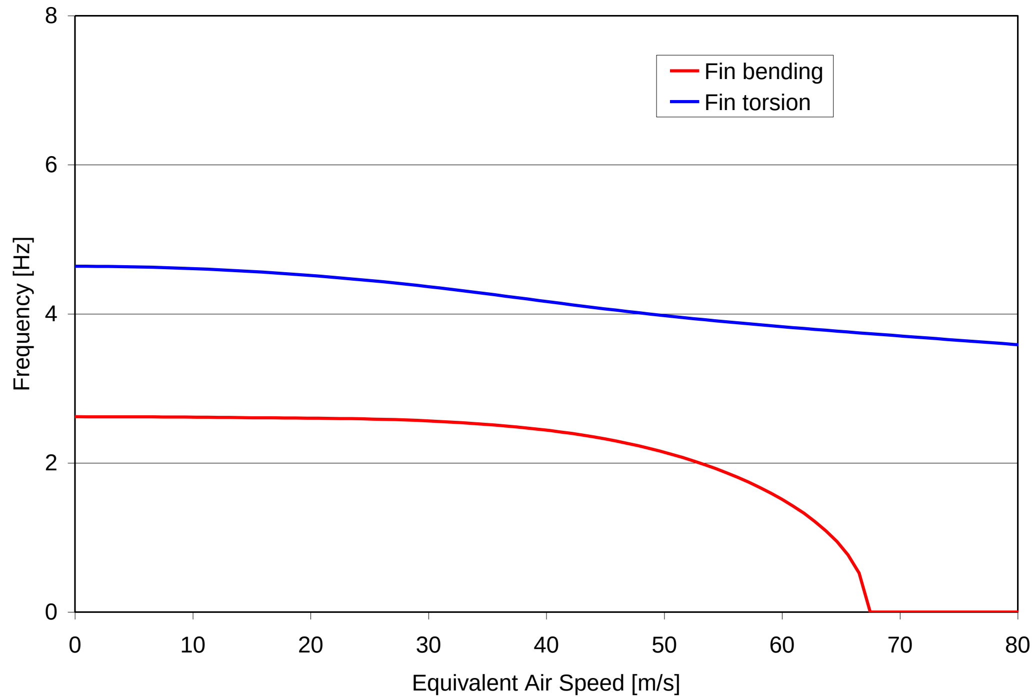

Fig. 9: Frequency vs. speed for rearward centre of mass

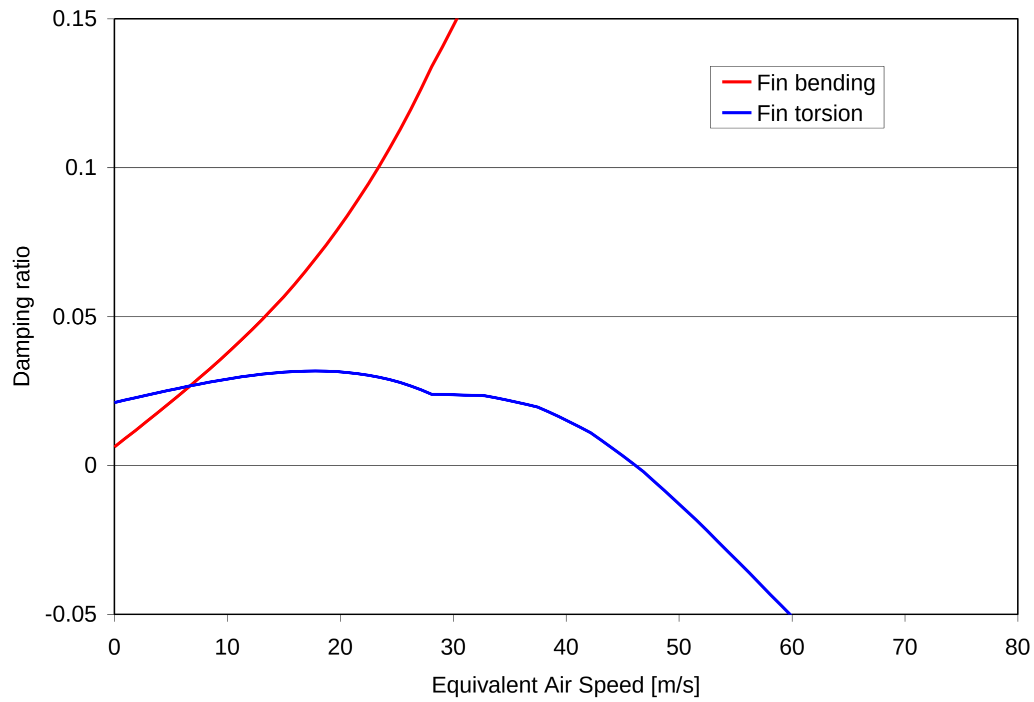

Fig. 10 : Damping vs. speed for rearward centre of mass

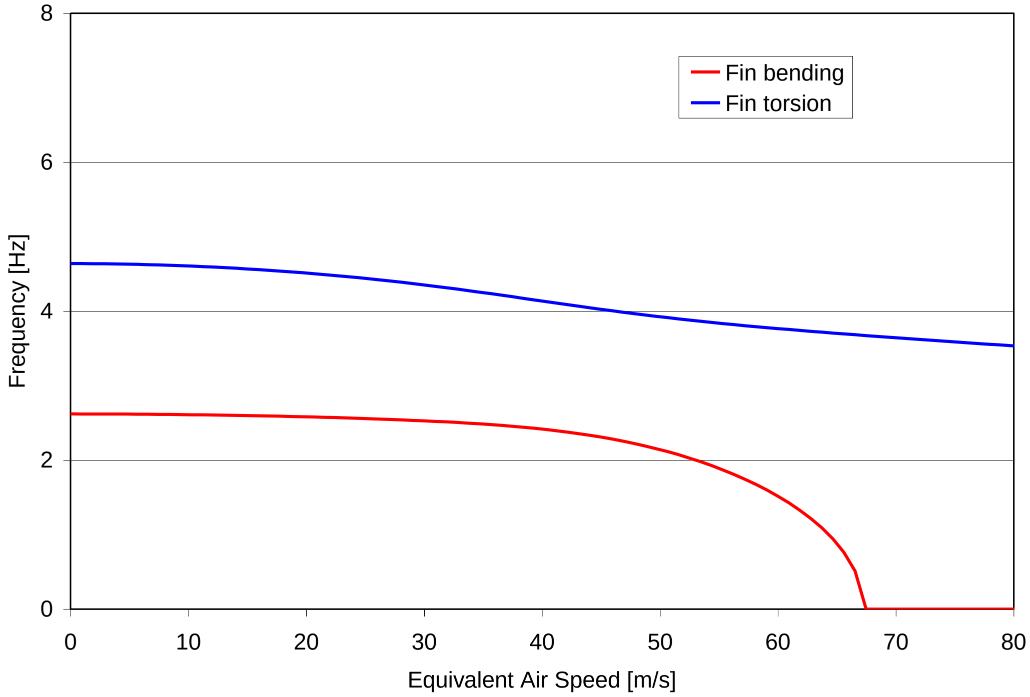

Fig. 11: Frequency vs. speed for forward centre of mass

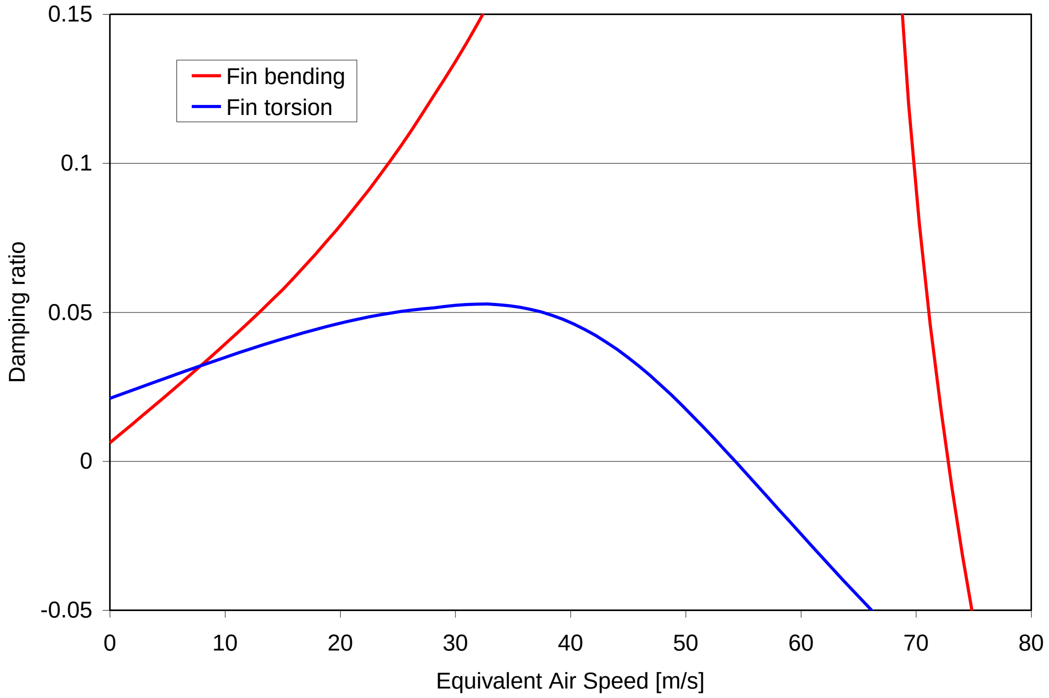

Fig. 12: Damping vs. speed for forward centre of mass

5 CONCLUSION

A procedure for the flutter analysis of T-tailed aircraft has been presented that is only slightly more labour-intensive than the flutter analysis of conventional aircraft. In addition to the geometry and mode shape information that is required for conventional aircraft, the aircraft mass and gravity/acceleration vector must be specified. In addition, the rigid body modes must accurately reflect the centre of mass position and the fidelity of the aerodynamic model must be sufficient to allow for a meaningful trim analysis to be done. To this end, the T-tail DLM used in this procedure allows for the inclusion of wing profiles (which is used to derive the camber of the lifting surfaces) and uses a surface panel body model. On the other hand, the computational effort is 10-fold to 100-fold that of the flutter analysis of a conventional aircraft. This increase in computational effort is largely mitigated by advances in computing power and the prospect of parallelising the present code.

6 REFERENCES

[1] VAN ZYL, LOUW H., MATHEWS, EDWARD H., *Aeroelastic Analysis of T-tails using an Enhanced Doublet Lattice Method*, Journal of Aircraft, May-June 2011, to be published.

APPENDIX

T-TAIL FLUTTER MODEL GEOMETRY AND MODAL PROPERTIES

The geometry of the T-tail flutter model is defined in terms of chord line leading edge coordinates and chord lengths in Table 2. The z=0 reference plane was the tunnel floor and the y=0 reference plane coincided with the vertical tunnel centre plane. The inboard and outboard camber lines of the stabilisers are listed in Table 3.

Table 2: T-tail flutter model chord lines

Chord line	***x* [m]**	***y* [m]**	***z* [m]**	**Chord [m]**
Fin root	0.000	0.000	0.217	0.425
Fin tip	0.324	0.000	0.714	0.425
Fin tip fairing root	0.324	0.000	0.714	0.528
Fin tip fairing tip	0.324	0.000	0.812	0.528
Stabilizer root	0.375	0.000	0.763	0.363
Stabilizer tip	0.838	±0.625	0.763	0.100

Table 3: Camber lines of the T-tail flutter model stabilisers

x/c	$(z/c)_{root}$	$(z/c)_{tip}$
0.000	0.0000	0.0000
0.025	0.0044	0.0043
0.050	0.0085	0.0084
0.100	0.0129	0.0128
0.200	0.0131	0.0130
0.300	0.0109	0.0107
0.400	0.0088	0.0086
0.500	0.0066	0.0064
0.600	0.0044	0.0042
0.700	0.0022	0.0020
0.800	-0.0001	-0.0002
0.900	-0.0023	-0.0024
1.000	-0.0045	-0.0047

The first three mode shapes and corresponding modal properties of the model were measured using a sine-dwell test technique. The modal properties are listed in Table 4.

Table 4: Modal properties of the T-tail flutter model

Mode No.	Description	Frequency [Hz]	Damping ratio	Modal mass [kg]
1	First fin bending	2.621	0.0062	3.947
2	Fin torsion	4.641	0.0211	3.589
3	Second fin bending	13.695	0.0345	3.366

The mode shapes were approximated by polynomials expressed in a local coordinate system for each element. The origin of the local coordinate system is at the root leading edge; the chordwise coordinate is normalized by the root chord and the spanwise coordinate is normalized by the span of the element. The polynomial approximations to the linear mode shape components are given below. The displacements are in the global coordinate system and subscripts denote the mode number corresponding to Table 4.

Fin

$$\eta = \left(- 0.217 \right) 0.497$$

$$y_1 = -0.106510 - 0.558676\,\eta + 0.079981\,\xi\eta - 0.509418\,\eta^2$$

$$y_2 = 0.002565 + 0.575753\,\eta - 1.409752\,\xi\eta + 1.265927\,\eta^2$$

$$y_3 = 0.049624 + 0.856267\,\eta - 1.075774\,\xi\eta + 1.267945\,\eta^2$$

Fin tip fairing

$$\xi = \left(- 0.324 \right) 0.528$$

$$\eta = \left(- 0.714 \right) 0.098$$

$$y_1 = -1.113630 + 0.099365\,\xi - 0.180162\,\eta$$

$$y_2 = 0.769517 - 1.751409\,\xi + 0.227724\,\eta$$

$$y_3 = 1.353715 - 1.336491\,\xi - 0.329526\,\eta$$

Left stabiliser

$$\xi = \left(- 0.375 \right) 0.363$$

$$\eta = -y / 0.625$$

$$x_1 = 0.117620\,\eta$$

$$y_1 = -1.194113 + 0.068313\,\xi$$

$$z_1 = -1.148991\,\eta$$

$$x_2 = -2.073164\,\eta$$

$$y_2 = 0.714209 - 1.204094\,\xi$$

$$z_2 = 1.452323\,\eta$$

$$x_3 = -1.582021\,\eta$$

$$y_3 = 1.059859 - 0.918838\,\xi$$

$$z_3 = -2.101571\,\eta$$

Right stabiliser

$$\xi = \left(- 0.375 \right) 0.363$$

$$\eta = y / 0.625$$

$$x_1 = -0.117620\,\eta$$

$$y_1 = -1.194113 + 0.068313\,\xi$$

$$z_1 = 1.148991\,\eta$$

$$x_2 = 2.073164\,\eta$$

$$y_2 = 0.714209 - 1.204094\,\xi$$

$$z_2 = -1.452323\,\eta$$

$$x_3 = 1.582021\,\eta$$

$$y_3 = 1.059859 - 0.918838\,\xi$$

$$z_3 = 2.101571\,\eta$$