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A THEORY OF LONGITUDINALLY POLARISED PIEZOCOMPOSITE ROD BASED ON MINDLIN-HERRMANN MODEL

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Abstract. *The conventional theory of the piezoelectric rod is based on an assumption that its lateral vibrations are negligible. In this case the rod, vibration could be described in terms of one-dimensional wave equation and a set of mechanical and electric boundary conditions. However, the main assumption of the model is only valid in the case of long and relatively thin rods. As a rule, the linear dimensions of piezoelectric components used in real transducers are comparable with the characteristic dimensions of their cross-sections, and hence, their lateral displacements have to be taken into consideration. In the present paper, the theory of the piezoelectric rod is developed on the basis of the Mindlin-Herrmann model. In the frame of this theory, the longitudinal and lateral displacements are described by two independent functions and vibration of the rod is obtained in terms of a system of two partial differential equations. The Hamilton variational principle is used for derivation of the system of equations of motion and for obtaining the mechanical and electric boundary conditions. On the basis of the formulated Mindlin-Herrmann model, the electric impedance of the piezoelectric rod is calculated. Possible generalizations of the proposed approach are considered and conclusions are formulated. An example of the application of the piezoelectric rod based on the Mindlin-Herrmann theory is given.*

Keywords*: lateral displacement, Mindlin-Herrmann model, piezoelectric, thick rod.*

1. INTRODUCTION

The piezoelectric transducers generate and detect ultrasonic waves in continuous media such as fluids, solids, etc (Risstic, 1983). They have been developed for many industrial applications. The conventional theory of the thickness vibrator transducer is based on an assumption that its lateral vibrations are negligible. In this case, the transducers' dynamics could be described in terms of one-dimensional wave equation and a set of mechanical and electric boundary conditions. However, the main assumption of the model is only valid in the case of long and relatively thin rods. As a rule, the linear dimensions of thickness vibrators are comparable with the characteristic dimensions of their cross-sections and hence, it is necessary to take into consideration the lateral displacements of these transducers. Theory of relatively thick transducers was developed (Shatalov *et al.*, 2009b). In the present paper, the theory of the relatively thick piezocomposite rod is developed on the basis of the Mindlin-Herrmann model of longitudinal vibrations of rods. In the frame of this theory, it is supposed that the lateral displacements are proportional to the product of an independent function which is subjected to determination and the distance from the neutral line of the transducers' cross-section (Shatalov *et al.*, 2009a). The Hamilton variational principle is used for derivation of the system of equation of motion and for obtaining the mechanical and electric boundary conditions. On the basis of the obtained Mindlin-Herrmann model, the electric impedance of the thickness vibrator is calculated. Possible generalizations of the proposed approach are considered and conclusions are formulated. General solution of the problem is obtained from the variational principle using two orthoganalities conditions of the eigenfunctions (Fedotov *et al.*, 2009).

2. MINDLIN-HERRMANN MODEL FOR THICKNESS VIBRATOR

Suppose that $Oz\left(03\right)$ is the axis of polarization of a piezoelectric rod. According to the Rayleigh-Love and Bishop theories displacements in $Ox\left(01\right), Oy\left(02\right)$, Oz - directions are correspondingly $u, v,$ and w :

$$\begin{aligned} u &= u(x,z,t) = x\varphi(z,t) \\ v &= v(y,z,t) = y\varphi(z,t) \\ w &= w(z,t) \end{aligned} \tag{1}$$

where $\varphi = \varphi(z,t)$ represents the lateral or transverse contraction about the z -axis and x , y - displacements from the neutral axis of any symmetric cross-section of the bar. Note that the transverse displacements u and v are both connected by the function φ and are independent of the longitudinal displacement, w . This proves the independence of the shearing deformation thus, the entire motion can be described by two independent functions: w and φ .

Linear strain are:

Normal strain: $$S_1 = \frac{\partial u}{\partial x} = \varphi \qquad S_2 = \frac{\partial v}{\partial y} = \varphi \qquad S_3 = \frac{\partial w}{\partial x} = w'$$

and shear strain: $$S_4 = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial x} = y\varphi' \qquad S_5 = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial y} = x\varphi' \qquad S_6 = 0$$

Linear stress-strain relations are:

Normal stress: $$\begin{aligned} T_1 &= c_{11}^D S_1 + c_{12}^D S_2 + c_{13}^D S_3 - h_{31} D_3 \\ T_2 &= c_{12}^D S_1 + c_{11}^D S_2 + c_{13}^D S_3 - h_{31} D_3 \\ T_3 &= c_{13}^D (S_1 + S_2) + c_{33}^D S_3 - h_{33} D_3 \end{aligned}$$

and shear stress: $$\begin{aligned} T_4 &= c_{44}^D S_4 \\ T_5 &= c_{44}^D S_5 \\ T_6 &= 0 \end{aligned} \tag{2}$$

Where c_{ij}^D - elastic constants at constant electric charge density and $D_1 = D_2 = 0$, $D_3 = D_{30} e^{i\omega t}$ are the electric displacements in which $D_{30} = const$ and $\omega = 2\pi f$ - frequency of excitation.

Expression for kinetic energy:

$$T = \frac{\rho}{2}\int_0^l \left[A(z)\dot{w}^2 + I_p \dot{\varphi}^2 \right] dz \tag{3}$$

where l - thickness of the transducer, $A = A(z)$ is the cross-section area and $I_p = \int_{(A)} \left(x^2 + y^2\right) dA$ is the polar moment of inertia

Strain energy is as follows:

$$\begin{aligned} P &= \frac{1}{2}\int_0^l \int_{(A)} \left(T_1 S_1 + T_2 S_2 + T_3 S_3 + T_4 S_4 + T_5 S_5 + T_6 S_6 \right) dA dz \\ &= \frac{1}{2}\int_0^l \left\{ \left[2\left(c_{11}^D + c_{12}^D\right)\varphi^2 + c_{33}^D w'^2 + 4c_{13}^D w'\varphi \right] A + I_p c_{44}^D \varphi'^2 - \left(2h_{31}\varphi + h_{33} w'\right) A D_3 \right\} dz \end{aligned} \tag{4}$$

where h_{ij} is the piezoelectric coefficient and $A = \int_{(A)} dA$

Electric energy is:

$$\begin{aligned} W &= \frac{1}{2}\int_0^l \int_{(A)} \left(E_1 D_1 + E_2 D_2 + E_3 D_3 \right) dA dz \\ &= \int_0^l \int_{(A)} \left\{ -h_{31} S_1 D_3 - \frac{1}{2} h_{33} S_3 D_3 + \frac{1}{2}\beta_3^S D_3^2 \right\} dA dz \\ &= \frac{1}{2}\int_0^l \left(\beta_3^S A D_3^2 - 2h_{31} A D_3 \varphi - h_{33} A D_3 w' \right) dz \end{aligned} \tag{5}$$

where β_3^s is the dielectric constant

It is also necessary to keep in mind the following electric boundary condition:

$$\int_0^l E_3 \, dz = V = V(t) \tag{6}$$

where $V(t) = V_0 e^{i\omega t}$ $\left(V_0 = const\right)$ - excitation voltage applied at the edge of the thickness vibrator and $E_3 = -2h_{31}\varphi - h_{33} w' + \beta_3^s D_3$ is the electric field on the z -axis.

Total Lagrangian of the system:

$$L = K - P + W + \lambda \cdot \left[\int_0^l E_3\, dz - V(t) \right]$$
$$= \frac{1}{2}\int_0^l \left\{ \left[A\dot{w}^2 + I_p \dot{\varphi}^2 \right]\rho - \left[2\left(c_{11}^D + c_{12}^D\right)\varphi^2 + c_{33}^D w'^2 + 4c_{13}^D w'\varphi \right] A - I_p c_{44}^D \varphi'^2 + \beta_3^S A D_3^2 \right\} dz - 2\lambda h_{31}\int_0^l \varphi dz - \lambda h_{33}\left(w(l) - w(0)\right) + \beta_3^s \lambda l D_3 \tag{7}$$

where λ is the Lagrange multiplier
We can rewrite the pervious expression in the compact form as follow:

$$L = L\left(\dot{w}, w', \varphi, \dot{\varphi}, \varphi', D_3, \lambda\right) = \int_0^l \Lambda\left(\dot{w}, w', \varphi, \dot{\varphi}, \varphi', D_3, \lambda\right) dz + \Lambda_0\left(D_3, \lambda, w(l), w(0)\right) \tag{7a}$$

where

$$\Lambda = \Lambda\left(\dot{w}, w', \varphi, \dot{\varphi}, \varphi', D_3, \lambda\right)$$
$$= \frac{1}{2}\left\{ \left[A\dot{w}^2 + I_p \dot{\varphi}^2 \right]\rho - \left[2\left(c_{11}^D + c_{12}^D\right)\varphi^2 + c_{33}^D w'^2 + 4c_{13}^D w'\varphi \right] A - I_p c_{44}^D \varphi'^2 + \beta_3^S A D_3^2 - 4\lambda h_{31}\varphi \right\} \tag{8}$$

is the Lagrange density and

$$\Lambda_0 = \Lambda_0\left(D_3, \lambda, w(l), w(0)\right) = \beta_3^S \lambda l D_3 - \lambda h_{33}\left(w(l) - w(0)\right) \tag{9}$$

Applying the Hamiltonian variational principle to Lagrangian (7a) we obtain the system of equation of motion in the compact form:

$$\begin{cases} \dfrac{\partial}{\partial t}\left(\dfrac{\partial \Lambda}{\partial \dot{w}}\right) + \dfrac{\partial}{\partial z}\left(\dfrac{\partial \Lambda}{\partial w'}\right) = 0 \\ \dfrac{\partial}{\partial t}\left(\dfrac{\partial \Lambda}{\partial \dot{\varphi}}\right) + \dfrac{\partial}{\partial z}\left(\dfrac{\partial \Lambda}{\partial \varphi'}\right) - \dfrac{\partial \Lambda}{\partial \varphi} = 0 \end{cases}$$

with the mechanical boundary conditions

$$\left. -\frac{\partial \Lambda}{\partial w'}\right|_{x=0} + \frac{\partial \Lambda_0}{\partial w(0)} = 0 \qquad \text{or} \qquad w\big|_{x=0,l} = 0$$
$$\left. \frac{\partial \Lambda}{\partial w'}\right|_{x=l} + \frac{\partial \Lambda_0}{\partial w(l)} = 0$$

and

$$\varphi'\big|_{x=0,l} = 0 \qquad \text{or} \qquad \varphi\big|_{x=0,l} = 0$$

and the electrical boundary conditions:

$$\frac{\partial \Lambda}{\partial D_3} + \frac{\partial \Lambda_0}{\partial D_3} = 0$$
$$\frac{\partial \Lambda}{\partial \lambda} + \frac{\partial \Lambda_0}{\partial \lambda} = 0$$

The explicit form of the Mindlin-Herrmann model for the polarised piezocomposite thick rod is given as follow.

$$\begin{cases} \rho A \dfrac{\partial^2 w}{\partial t^2} - c_{33}^D A \dfrac{\partial^2 w}{\partial z^2} - 2c_{13}^D A \dfrac{\partial \varphi}{\partial z} = 0 \\ \rho I_p \dfrac{\partial^2 \varphi}{\partial t^2} - c_{44}^D I_p \dfrac{\partial^2 \varphi}{\partial z^2} + 2A\left(c_{11}^D + c_{12}^D\right)\varphi + 2c_{13}^D A \dfrac{\partial w}{\partial z} + 2\lambda h_{31} = 0 \end{cases} \tag{10}$$

associated boundary conditions

$$\left. c_{33}^D A w' + 2c_{13}^D A\varphi \right|_{x=0,l} + \lambda h_{33} = 0 \quad (11) \qquad \text{or} \qquad w\big|_{x=0,l} = 0 \quad (12)$$

and

$$\varphi'\big|_{x=0,l} = 0 \quad (13) \qquad \text{or} \qquad \varphi\big|_{x=0,l} = 0 \quad (14)$$

and the electric boundary condition

$$A D_3 + \lambda l = 0 \tag{15}$$
$$2h_{31}\varphi + h_{33}\left(w(l) - w(0)\right) - \beta_3^S l D_3 = 0 \tag{16}$$

From equation (15), we have $\lambda = -\dfrac{A D_3}{l}$, and substituting it into (10) and (11), this leads to:

$$\begin{cases} \rho A \dfrac{\partial^2 w}{\partial t^2} - c_{33}^D A \dfrac{\partial^2 w}{\partial z^2} - 2c_{13}^D A \dfrac{\partial \varphi}{\partial z} = 0 \\ \rho I_p \dfrac{\partial^2 \varphi}{\partial t^2} - c_{44}^D I_p \dfrac{\partial^2 \varphi}{\partial z^2} + 2A\left(c_{11}^D + c_{12}^D\right)\varphi + 2c_{13}^D A \dfrac{\partial w}{\partial z} = \dfrac{2}{l} A h_{31} D_3 \end{cases} \tag{17}$$

with the electro-mechanic boundary conditions (here we choose these boundary conditions without loosing the generality stated in our main objective):

$$c_{33}^D A w' + 2c_{13}^D A \varphi\Big|_{x=0,l} = \frac{h_{33}}{l} D_3 \quad (18) \qquad \text{and} \qquad \varphi\big|_{x=0,l} = 0 \quad (19)$$

The main problem is entirely defined by considering the initial conditions:

$$w(z,t)\big|_{t=0} = g(z),\ \dot{w}(z,t)\big|_{t=0} = h(z) \qquad \text{and} \qquad \varphi(z,t)\big|_{t=0} = \phi(z),\ \dot{\varphi}(z,t)\big|_{t=0} = q(z) \tag{20}$$

3. FREE VIBRATION PROBLEM

In this section, we are dealing with the free vibration problem, that means $D_3 = 0$ (no electric displacement). Thus we apply the Fourier method to the obtained homogeneous problem. Let us assume that $w(z,t) = Z(z)e^{i\omega t}$ and $\varphi(z,t) = \Phi(z)e^{i\omega t}$ where i^2 and ω is the circular frequency. This leads to the Sturm-Liouville problem:

$$\begin{cases} c_{33} Z'' + 2c_{13}\Phi' = -\omega^2 \rho Z \\ c_{44} I_p \Phi'' - 2A(c_{11} + c_{12})\Phi - 2c_{13} A Z' = -\omega^2 \rho I_p \Phi \end{cases} \tag{21}$$

With the associated boundary conditions

$$c_{33} Z' + 2c_{13}\Phi\big|_{z=0,l} = 0 \ \text{ and } \ \Phi\big|_{z=0,l} = 0 \tag{22}$$

The above Sturm-liouville problem is unusual (two-dimension). So, to compliment the lack of theory in this particular problem, we trade the difficulty by considering the case of a travelling wave. Thus we can write:

$$Z(z) = Z_0 e^{-ikz} \ \text{ and } \ \Phi(z) = \Phi_0 e^{-ikz} \tag{23}$$

where Z_0 and Φ_0 are respectively the longitudinal and lateral non zero amplitude (unknowns) and k is the wave number and need to be determined.
Hence system (21) becomes:

$$\begin{cases} \left(\omega^2 \rho - c_{33} k^2\right) Z_0 - 2ikc_{13}\Phi_0 = 0 \\ 2ikAc_{13} Z_0 + \left[\omega^2 \rho I_p - c_{44} I_p k^2 - 2A(c_{11} + c_{12})\right]\Phi_0 = 0 \end{cases} \tag{24}$$

Solving the characteristic equation of the determinant of system (21) for k, we obtain:

$$k_{1,2}^2 = k_{1,2}^2(\omega) = \frac{\beta \pm \sqrt{\beta^2 + 4\alpha\gamma}}{2\alpha}$$

where $\alpha = c_{33}^D c_{44}^D I_p$, $\beta = \beta(\omega) = \left[2A(c_{11}^D + c_{12}^D)c_{33}^D - 4(c_{13}^D)^2 - \omega^2 \rho (c_{44}^D + I_p c_{33}^D)\right]$ and
$\gamma = \gamma(\omega) = \omega^2 \rho \left[\omega^2 \rho I_p - 2A(c_{11}^D + c_{12}^D)\right]$

Using formula (23) we can express $Z(z)$ as follows:

$$Z = Z(z,\omega) = a_1 Z_1(k_1 z) + a_2 Z_2(k_1 z) + a_3 Z_3(k_2 z) + a_4 Z_4(k_2 z)$$

where $Z_{1,3}(k_j z) = \begin{cases} \cos(k_j z) \text{ if } \operatorname{Im}\left[k_j\right] = 0 \\ \cosh(k_j z) \text{ orthewise} \end{cases}$ and $Z_{2,4}(k_j z) = \begin{cases} \sin(k_j z) \text{ if } \operatorname{Im}\left[k_j\right] = 0 \\ \sinh(k_j z) \text{ orthewise} \end{cases}$ ($j = 1,2$)

and the constants a_1, a_2, a_3 and a_4 are not all equal to zero.
Without loss the generality, the case $\operatorname{Im}\left[k_1\right] = 0$ is considered in the discussion that follows. Hence

$$Z = Z(z,\omega) = a_1 \cos(k_1 z) + a_2 \sin(k_1 z) + a_3 \cosh(k_2 z) + a_4 \sinh(k_2 z) \tag{25}$$

Substituting (25) into the first equation of system (21) and solving the obtained equation for Φ :

$$\Phi = \Phi(z,\omega) = -\frac{1}{2c_{13}^D}\left[a_1\left(\frac{\omega^2\rho}{k_1} - k_1 c_{33}^D\right)\sin(k_1 z) - a_2\left(\frac{\omega^2\rho}{k_1} - k_1 c_{33}^D\right)\cos(k_1 z)\right] - -\frac{1}{2c_{13}^D}\left[+a_3\left(\frac{\omega^2\rho}{k_2} + k_2 c_{33}^D\right)\sinh(k_2 z) + a_4\left(\frac{\omega^2\rho}{k_2} + k_2 c_{33}^D\right)\cosh(k_2 z)\right] \tag{26}$$

Substituting Eq(25) and Eq(26) into the boundary conditions (22), yields a system of four equations with four unknowns: a_1, a_2, a_3 and a_4. Solving the characteristic equation of the determinant for ω using the method developed by Fedotov *et al.* (2008) for solving transcendental equation, we obtain many positive roots ω_n, $n = 1, 2, \ldots$, so called eigenvalues corresponding to the couple of eigenfunction $\left(Z_n(z,\omega_n), \Phi_n(z,\omega_n)\right)$.

4. SOLUTION OF THE FORCED VIBRATION PROBLEM

4.1. Orthogonalities of the eigenfunctions

Using system (21) and boundary conditions (22) we can prove two kind of orthogonality condition of the eigenfunctions:

First orthogonality

$$\left\|(Z_n,\Phi_n),(Z_m,\Phi_m)\right\|_1 = \int_0^l \left\{ I_p Z_n Z_m + A\Phi_n\Phi_m \right\} dz = \left\|(Z_n,\Phi_n)\right\|_1^2 \delta_{nm}$$

where δ_{nm} is the Kroneker's symbol and $\left\|(Z_n,\Phi_n)\right\|_1^2 = \int_0^l \left\{ I_p Z_n^2 + A\Phi_n^2 \right\} dz$ is the associated square norm

Second orthogonality

$$\left\|(Z_n,\Phi_n),(Z_m,\Phi_m)\right\|_2 = \int_0^l \left\{ Ac_{33}^D Z_n' Z_m' + 2A(c_{11}^D + c_{12}^D)\Phi_n\Phi_m + c_{44}^D I_p \Phi_n'\Phi_m' + 2c_{13}^D A(Z_n'\Phi_m + Z_m'\Phi_n) \right\} d \;\; = \left\|(Zz_n,\Phi_n)\right\|_2^2 \delta_{nm}$$

Where $\left\|(Z_n,\Phi_n)\right\|_2^2 = \int_0^l \left\{ Ac_{33}^D Z_n'^2 + 2A(c_{11}^D + c_{12}^D)\Phi_n^2 + c_{44}^D I_p \Phi_n'^2 + 4c_{13}^D A Z_n'\Phi_n \right\} dz$ is the associated square norm.

4.2. Solution of the problem

In this section, we give the solution of problem (17)-(20) on the basis of the method of eigenfunction orthogonalities for vibration problem developed by Djouosseu (2008) and Fedotov *et al.* (2009).

$$w(z,t) = A\int_0^l \left\{ g(\xi)\frac{\partial G_1(z,\xi,t)}{\partial t} + h(\xi)G_1(z,\xi,t) \right\} d\xi + I_p \int_0^l \left\{ \phi(\xi)\frac{\partial G_2(z,\xi,t)}{\partial t} + q(\xi)G_2(z,\xi,t) \right\} d\xi + +\frac{A}{\rho}\int_0^l\int_0^l F(\tau)G_1(z,\xi,t-\tau)\,d\xi\, d\tau$$

$$\varphi(z,t) = A\int_0^l \left\{ g(\xi)\frac{\partial G_3(z,\xi,t)}{\partial t} + h(\xi)G_3(z,\xi,t) \right\} d\xi + I_p \int_0^l \left\{ \phi(\xi)\frac{\partial G_4(z,\xi,t)}{\partial t} + q(\xi)G_4(z,\xi,t) \right\} d\xi + +\frac{A}{\rho}\int_0^l\int_0^l F(\tau)G_3(z,\xi,t-\tau)\,d\xi\, d\tau$$

where $G_1(z,\xi,t) = \sum_{n=1}^{\infty}\left(\frac{Z_n(z)Z_n(\xi)\sin\Omega_n t}{\Omega_n \left\|(Z_n,\Phi_n)\right\|_1^2} \right)$, $G_2(z,\xi,t) = \sum_{n=1}^{\infty}\left(\frac{Z_n(z)\Phi_n(\xi)\sin\Omega_n t}{\Omega_n \left\|(Z_n,\Phi_n)\right\|_1^2} \right)$,

$G_3(z,\xi,t)=\sum_{n=1}^{\infty}\left(\frac{\Phi_n(z)Z_n(\xi)\sin\Omega_n t}{\Omega_n\left\|(Z_n,\Phi_n)\right\|_1^2}\right)$, and $G_4(z,\xi,t)=\sum_{n=1}^{\infty}\left(\frac{\Phi_n(z)\Phi_n(\xi)\sin\Omega_n t}{\Omega_n\left\|(Z_n,\Phi_n)\right\|_1^2}\right)$ are Green's function in which $\Omega_n=\frac{1}{\rho}\frac{\left\|(Z_n,\Phi_n)\right\|_2}{\left\|(Z_n,\Phi_n)\right\|_1}$ is the natural frequency and $F(t)=\frac{2}{l}Ah_{31}D_3=\frac{2}{l}Ah_{31}D_{30}e^{\omega t}$.

4.3. Electrical response of the Rod

The electrical response of the rod to the excitation voltage $V(t)$ applied at the edge of its thickness is characterised by:

- the current through this thickness
  $I(\omega)=A\dot{D}_3=i\omega A\left|D_3\right|$ and
- the associated electric impedance
  $Z_{imp}(\omega)=\frac{V(t)}{I(\omega)}$

5. CONCLUSIONS

1. The Mindlin-Herrmann approach was used to build a model describing a thick and short piezocomposite rod longitudinally polarised.
2. The Hamilton variational principle was used to derive the system of equation of motion in the process of which the electromechanical boundary conditions were obtained. The method of eigenfunction orthogonalies based on the variational principle was used to obtain the exact solution of the problem in terms of the Green function.
3. The electric impedance through the thickness of the piezocomposite rod is formulated in terms of the excitation frequency.

6. REFERENCES

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