The role of energy conservation and vacuum energy in the evolution of the universe

Jan M. Greben

CSIR, PO Box 395, Pretoria 0001, South Africa

E-mail: `jgreben@csir.co.za`

Abstract. We discuss a new theory of the universe in which the vacuum energy is of classical origin and dominates the energy content of the universe. As usual, the Einstein equations determine the metric of the universe. However, the scale factor is controlled by total energy conservation in contrast to the practice in the Robertson-Walker formulation. This theory naturally leads to an explanation for the Big Bang and is not plagued by the horizon and cosmological constant problem. It naturally accommodates the notion of dark energy and proposes a possible explanation for dark matter. It leads to a dual description of the universe, which is reminiscent of the dual theory proposed by Milne in 1937. On the one hand one can describe the universe in terms of the original Einstein coordinates in which the universe is expanding, on the other hand one can describe it in terms of co-moving coordinates which feature in measurements. In the latter representation the universe looks stationary and the age of the universe appears constant.

The paper describes the evolution of this universe. It starts out in a classical state with perfect symmetry and zero entropy. Due to the vacuum metric the effective energy density is infinite at the beginning, but diminishes rapidly. Once it reaches the Planck energy density of elementary particles, the formation of particles can commence. Because of the quantum nature of creation and annihilation processes spatial and temporal inhomogeneities appear in the matter distributions, resulting in residual proton (neutron) and electron densities. Hence, quantum uncertainty plays an essential role in the creation of a diversified complex universe with increasing entropy. It thus seems that quantum fluctuations play a role in cosmology similar to that of random mutations in biology. Other analogies to biological principles, such as recapitulation, are also discussed.

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1. Introduction

In standard quantum mechanics conservation of energy is related to the invariance of the Lagrangian under space-time translations and is expressed as a divergence equation for the energy-momentum tensor. In General Relativity (GR) this divergence equation is replaced by a covariant equation and is equivalent to the Bianchi identities satisfied by GR [1]. However, in an expanding universe, this symmetry is no longer equivalent to energy conservation. For example, the popular de Sitter universe violates energy conservation ([2], p.120).

In view of the importance of the principle of total energy conservation we propose to impose this principle as a separate condition in GR. For a non-expanding universe this condition reduces to the usual divergence equation. However, the general consensus is that the universe is expanding, in which case this principle becomes a separate condition. It can be imposed by demanding that the spatial integral over the energy component of the energy-momentum tensor is constant over time. However, in the standard Robertson Walker (RW) metric this procedure leads to a problem. Given an energy-momentum tensor, the metric of the universe is fixed by the GR equations. In the usual RW metric, the expansion is incorporated in the metric via the scale factor, so that the expansion is fixed by the GR equations. Hence, there is no room for another condition for total energy conservation as the whole dynamics is already fixed (apart from boundary conditions). It is thus not surprising that the solution of the GR equations for a universe with a constant cosmological constant violates energy conservation ([2], p.120). Our solution to this conundrum is to remove the scale factor from the metric. This means that the expansion now has to be derived in a different way, and this is done via a scale factor $a(t)$ fixed by energy conservation. If the universe does not expand, this scale factor reduces to unity, and the extra condition merely represents a consistency condition for two equivalent definitions of energy conservation. This formulation ensures that the scale factor is a truly global function as it is fixed by the total energy, which is a property of the whole universe. It also means that the metric tensor exclusively serves its natural function of reflecting the (local) distribution of energy.

For a flat universe with constant vacuum energy density, this new formulation leads to a linear expansion. This is clearly the simplest possible mode of expansion of the universe and provides a natural representation of the observed Hubble expansion. It should be noted that such a simple solution is impossible in the RW metric, as the linear case represents a singular limit in that framework (only by setting the curvature $k = -1$ can one find a linear solution, the so-called Milne universe [2]). The vacuum energy, which dominates the energy content of the universe in our picture, is easily identified with the so-called dark energy, both having a pressure-to-density ratio of -1. Hence, this model automatically incorporates the present consensus that dark energy dominates the energy content of the universe. In addition to explaining dark energy, the constant vacuum energy density has many other important consequences and plays a central role in the dynamics of the universe, as we will demonstrate amply in this paper.

The original theoretical motivation for constructing a universe with a constant vacuum energy is that our analyses in quantum field theory (QFT) (see Section 2) suggest that the vacuum energy has no quantum contributions, in contrast to generally held beliefs. It is then natural to identify dark energy with a classical vacuum energy, however this forces a new approach to cosmology, as the conventional solution in this case - the de Sitter universe - does not feature a big bang singularity.

Current theoretical scenarios often contain an inflationary period at $t = 10^{-35}$ s. In our theory an inflationary period is not mandatory, as the vacuum metric leads to an infinite horizon, so that one of the main motivations for inflation falls away. This also obviates the need for unknown forces to explain the inflationary epoch, in particular those forces which derive from QFT vacuum energy and which would be excluded by our QFT findings. Most scenarios agree that this early period is followed by a period of linear expansion, moderated by a slight deceleration initially and a slight acceleration in the current epoch. Hence,the dominant form of the expansion (linear) is already accounted for in our approach. Phases of deceleration and acceleration can occur in our model because of known quantum field theoretic processes, such as the creation and annihilation of particles. The presence of matter and radiation does not change the essential linear evolution of the universe in our description, in contrast to universes described by Friedmann-Robertson-Walker(FRW) models, which are vastly different in radiation - and matter - dominated situations. Hence, the simple vacuum metric still dominates the universe in the presence of matter and radiation. Another important difference with the Friedmann models is that in the solution of the Einstein equations, the localized nature of matter distributions is taken into account. Since our scale factor is controlled by energy conservation rather than by the Einstein equations, such a refinement of the cosmological treatment is now feasible. This leads to a unified treatment of cosmological and local astronomical phenomena. Another consequence of this improvement is the emergence of a new tentative explanation of dark matter, which does not require any exotic new particle assumptions.

The evolution of the universe in this theory has rather definite characteristics, with some of the details still to be developed. Contrary to most popular scenarios, the universe starts out as a classical system, with an effective energy density proportional to t^{-3}. The universe with positive and negative time are exact replicas of each other, answering the question what happens "prior" to time zero. The cosmological principle (i.e. the homogeneity and isotropy of the density) is satisfied exactly in this initial classical period, so that the entropy is zero. After about 5×10^{-24} seconds quantum field theory becomes effective and the first physical (rather than virtual) particles are created. The particle physics scale 5×10^{-24} naturally emerges from our formulation and is expressed in terms of the fundamental cosmological constants. This epoch is characterized by deflation, as the universe has to contract to supply the energy for the production of matter. The emergence of physical particles in this epoch also allows the expression of the subsequent evolution in thermodynamic language. The particle-creation epoch is characterized by the Planck energy and by a corresponding temperature of 10^{32} ^{0}K. In the subsequent epoch particles and anti-particles annihilate, leaving a residue of protons, neutrons and electrons. This period is accompanied by an inflationary expansion to counter the loss of energy in the annihilation phase due to the changing metric. After these processes, the normal Big Bang dynamics sets in which is again characterized by a linear scale factor.

The uncertain outcome of quantum events plays an essential role in the creative epoch. Firstly, it is responsible for breaking the symmetry of the matter distribution, leading to sufficient inhomogeneities for localized matter concentrations to form. In later epochs this asymmetry will initiate the creation of massive astronomical objects. In a classical universe this spatial symmetry would be maintained and no concentrations of matter could possibly emerge. Secondly, we expect the quantum fluctuations to be responsible for the imbalance between the different particle and anti-particle populations after the annihilation epoch. Hence, the current universe is a consequence of the physical laws and historical accidents, caused by the outcome of quantum processes in our world. Thus, randomness is as much a factor in the evolution of the universe as it is in biological (mutation) processes. To some extent the principle of the survival of the fittest is carried in quantum physics by the probability functions. Objects or configurations that form in the early universe, but then decay and vanish from the universe, appear to play a role similar to that of unsuccessful species in biology.

2. A universe with constant vacuum energy density

We will assume that the vacuum energy density ϵ is constant and is a basic property of Nature. This assumption is equivalent to the presence of a non-zero cosmological constant and leads to the usual de Sitter solution for the common Robertson-Walker (RW) metric. The assumption that the vacuum energy density is constant and small appears in conflict with standard QFT estimates, which quote vacuum energy densities of between 40 and 120 orders of magnitudes larger than the "observed" value. This problem of standard QFT is known as the cosmological constant problem ([3], [4], [5]).

Our hypothesis therefore implies that the vacuum energy does not derive from such QFT processes. To put it more bluntly: it suggests that the usual QFT derivations of vacuum processes contain serious flaws. Although this may be a natural conclusion to draw because of the phenomenal discrepancy between the standard QFT result and experiment, various practices with vacuum expectation values (vev's) in QFT have been so ingrained that the acceptance of this conclusion will require much debate. It therefore appears opportune to present some consequences of this hypothesis (such as in cosmology), before engaging in a full debate on its theoretical motivation. Our hypothesis is based on a study of the role of creation and annihilation operators of particles and anti-particles in QFT. We found that many vacuum phenomena, such as the definition of the propagator as a time ordered product, survive under our reformulated operator algebra. Also, the Casimir effect [6], which is often seen as a consequence of QFT vacuum energy, can be derived without invoking any vacuum energy

[7]. We contend that other phenomena, such as the vacuum condensates in QFT [8] have been misinterpreted, and can possibly be reformulated with an equivalent quantitative formulation without resorting to the QFT interpretations of the vacuum used presently. Hence, the purpose of the present study is to derive a realistic cosmological theory for a constant (classical) vacuum energy density. The fact that the hypothesis avoids the cosmological constant problem and leads to a very elegant theory which explains many cosmological phenomena in a simple way, is then seen as a strong endorsement of the correctness of this hypothesis. We decided to test this hypothesis first in the cosmological context

Accepting this hypothesis we are now confronted with the standard de Sitter solution of the GR equations for a non-zero cosmological constant. This solution has no singular beginning. It also leads to a violation of total energy conservation, as the expanding vacuum universe will increase its energy content with time [2]. The solution to this problem is to employ a metric distinct from the usual Robertson-Walker metric. The proposed solution is a good candidate for the description of the actual universe, as it features the expected singularity at time $t = 0$. In addition energy conservation and the expansion of the universe will be compatible rather than in conflict with each other.

Let us briefly discuss this solution. The vacuum energy is represented by the following energy-momentum tensor:

$$T_{\mu\nu} = -\epsilon g_{\mu\nu}, \tag{1}$$

where we use the metric - popular in cosmology - with g_{00} negative, so that ϵ is positive for positive vacuum energy. The Einstein equations then read:

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} = -8\pi G\epsilon g_{\mu\nu}. \tag{2}$$

In view of the observed (approximate) spatial flatness of the universe [9] we try to solve this equation (2) with a metric tensor that is conformally flat:

$$g_{\mu\nu} = -g(x)\eta_{\mu\nu}, \tag{3}$$

where $\eta_{\mu\nu}$ is the Minkowski metric. In contrast to the Robertson-Walker (RW) metric, we do not introduce a scale factor in the metric. This raises the question how to account for the expansion of the universe in the current parametrization. As we will see shortly, we will account for the expansion without abandoning the Minkowski metric $\eta_{\mu\nu}$. Since the vacuum is expected to be spatially homogeneous, we restrict the dependence of $g(x)$ to the time coordinate t. We then obtain the following solution of the Einstein equations:

$$g(t) = \frac{3}{8\pi G\epsilon t^2} = \frac{t_s^2}{t^2}, \tag{4}$$

where t_s is a characteristic time, which will play an important role in the following. Hence, the conformally flat metric of this vacuum universe now reads explicitly as follows:

$$ds^2 = \frac{t_s^2}{t^2}\left(-dt^2 + dx^2 + dy^2 + dz^2\right). \tag{5}$$

This can be contrasted with the usual Robertson-Walker metric where:

$$ds^2 = -d\tau^2 + a_{RW}(\tau)^2(dx^2 + dy^2 + dz^2), \tag{6}$$

The two representations are mathematically related by the following transformations:

$$\tau = \pm t_s \; ln(t/t_s) \longrightarrow a_{RW}(\tau) = exp(\mp\tau/t_s), \tag{7}$$

where the latter can be recognized as the de Sitter solution. However, the different choices of the physical variables lead to very different universes. For example, an expanding de Sitter universe ($a_{RW}(\tau) = exp(\tau/t_s)$ with τ positive and increasing towards the future) corresponds to a decreasing t in our formulation, and therefore to a contracting universe. Our definition of the scale factor will accordingly follow a very different route from that in the RW formulation. It will not be based on the metric (which is left in its Minkowski form) and rather being based on the demand of energy conservation. The $1/\epsilon$ dependence of $g(t)$ emphasizes the non-perturbative nature of this vacuum solution, typical of complex systems. Hence, in a cosmological context it is incorrect to neglect the small vacuum energy ϵ, or treat it perturbatively. The $1/t^2$ singularity of the metric at $t = 0$ makes this vacuum solution a good candidate for the description of the Big Bang.

An important property of this vacuum solution is that the geodesics represent either stationary points or test particles that move with the speed of light. Instead, in an ordinary flat universe (without vacuum energy) any (constant) speeds not exceeding the velocity of light are allowed [10]. It could thus be argued that the presence of the (classical) vacuum energy is responsible for both the origin of the velocity of light and for the apparent stationary nature of astronomical objects in the cosmos. Other interesting perspectives on the significance of ϵ, or equivalently (if G is constant) the cosmological constant Λ, are reviewed by Padmanabhan [11]. We quote: "the innocuous looking addition of a constant to the matter Lagrangian (which does not does not affect the dynamics of matter) leads to one of the most fundamental and fascinating problems of theoretical physics". What the present author finds particularly interesting is the suggestion [11] to consider the cosmological constant as a Lagrange multiplier, ensuring the constancy of the 4-volume of the universe when the metric is varied. Since we will see that the constancy of the (invariant) volume in the current approach is closely related to energy conservation, we suspect that there are deeper connections to be resolved.

It is interesting to note that Einstein originally introduced the cosmological constant to ensure the stationary nature of the universe, while we use it to generate an expanding universe. However, our analysis will show that from the perspective of a co-moving observer the universe does look stationary. This result also suggests possible links between our description and Hoyle's steady state universe, as the latter also makes use of a constant cosmological constant (see a paper by McCrea [12]).

In the next section we will demonstrate the origin of the linear expansion of the universe and the prescription for determining the scale factor.

3. Energy Conservation in General Relativity

The energy-momentum conservation condition from ordinary quantum mechanics is generalized in GR by replacing the derivative of the energy-momentum tensor by its covariant counterpart:

$$\nabla_\mu T^{\mu\nu} = 0. \tag{8}$$

This condition is automatically satisfied for a metric satisfying the Einstein equations, and can be shown to follow from the Bianchi identities [1]. Equation (8) is trivially satisfied by the vacuum energy density (Eq. (1)). However, in an expanding universe condition Eq. (8) is not sufficient to guarantee energy conservation. The total energy content of the vacuum universe is obtained by integrating $-T_0{}^0 = \epsilon$ over the invariant volume:

$$E = \int\limits_V d^3x \sqrt{^3g}\ \epsilon = \int\limits_V d^3x \frac{t_s^3}{t^3}\ \epsilon. \tag{9}$$

Here ^3g is the induced spatial metric, i.e. it is the spatial component of the determinant of the metric tensor (Ref. [2], p.120), which in our diagonal case equals $^3g = g_{11}g_{22}g_{33}$. In order to ensure energy conservation, the spatial volume V in (9) must expand like t^3:

$$V(t) = \frac{t^3}{t_s^3} V_s. \tag{10}$$

The proportionality constant V_s can be interpreted as the invariant volume since:

$$\int\limits_V d^3x \sqrt{^3g} = V_s, \tag{11}$$

is constant. This invariant volume also equals the physical volume of the universe at the characteristic time t_s . This volume also features in the expression for the total energy: ϵV_s, which therefore is invariant, as it should be. The expansion of the volume of the universe is best interpreted in terms of a scale factor that rescales distances, especially since it remains finite if V_s is infinite. Hence, we write:

$$V(t) = a(t)^3 V_s, \tag{12}$$

where the scale factor for the vacuum equals:

$$a(t) = \frac{t}{t_s}. \tag{13}$$

Even after the introduction of matter and radiation, $V(t)$ will still display this cubic time dependence. In addition, however, V_s will change every time a creation or annihilation process takes place. Hence, in the real universe $a(t)$ will also have to reflect these QFT processes. We will come back to this aspect in Section 6.

We note finally that the linear scale factor in (13) is unique to our approach as is normally forbidden in the RW metric [2]. The only other situation in which a linear scale factor occurs is in the Milne universe [2]. However, as this universe has zero vacuum energy and non-zero curvature, it is not related to our universe and is not an acceptable model of the universe.

4. The modified metric in the presence of matter

The vacuum universe can be described as an ideal fluid with a pressure-to-density ratio of -1. This value is in excellent agreement with the Supernova Legacy Survey ($w = -1.023 \pm 0.090(stat) \pm 0.054(sys)$ [13] for the dark energy equation of state). Hence, this strongly suggests that dark energy and vacuum energy are one and the same thing. Since dark energy appears to dominate the energy content of the universe, by implication vacuum energy dominates the global dynamics of the real universe. However, as the presence of matter and radiation is a consequence of QFT processes and make the universe interesting, our next task is to include these aspects as well. In view of the dominance of the vacuum energy it seems reasonable to treat the matter and radiation terms to first order, i.e. to linearize the Einstein field equations within the non-perturbative vacuum background. This approach has additional advantages, as it allows us to solve the Einstein equations exactly for the proposed representations of matter and radiation, and allows us to sidestep certain problems arising from the quantum nature of these terms.

The usual way to characterize the universe in the presence of matter and radiation is as an ideal fluid. However, instead of the constant pressure and density appropriate for a vacuum universe, one must now consider the pressure and density as being *time dependent* [14]. In our opinion, even this generalization is not sufficient for the matter in the universe: an important characteristic of matter is that it is localized, whereas the perfect fluid description does not take into account any spatial dependence. This localized nature of matter is true, irrespective of whether matter is in the form of fermions, planets, stars or galaxies. Astronomical objects are separated by vast empty areas and the matter distribution is thus far from being locally homogeneous. Neglect of this spatial dependence of matter is unlikely to provide the correct solution of the differential Einstein equations, where the spatial derivatives are expected to play a prominent role. Hence, we propose a matter density representation which emphasizes this local inhomogeneity:

$$T^{matter}_{\mu\nu}(x) = -\rho^{matter}(x)\hat{g}_{00}\delta_{\mu 0}\delta_{\nu 0}, \tag{14}$$

where the matter density is represented by

$$\rho^{matter}(t, \vec{x}) = \sum_i \frac{M_i}{\sqrt{{}^3\hat{g}(t, \vec{x})}}\delta^{(3)}(\vec{x} - \vec{x}_i). \tag{15}$$

A similar form for a source term expressed in terms of delta functions accompanied by a suitable function of the metric, was already suggested by Weinberg ([14], (5.2.13)). The appearance of the metric in the energy expression is not unexpected: the covariant condition, Eq.(8), clearly shows that a consistent definition of the energy-momentum tensor requires a particular dependence on the metric. In fact, the form (14) satisfies the covariant energy conservation condition (8) to the required order. We also introduced the new metric $\hat{g}_{\mu\nu}$ which accounts for the presence of matter. Since we treat the corrections to the vacuum metric to first order, we can approximate the exact metric in Eqs.(14) and (15) in most analyses by the vacuum metric $g_{\mu\nu}$. If we integrate this density (i.e. $-T_0{}^0$) over the whole universe we get the sum of all masses, as desired.

In order to calculate the first order effect of the matter term on the metric we write the metric tensor as follows:

$$\hat{g}_{\mu\nu}(t,\vec{x}) = g(t)\left\{\eta_{\mu\nu} + h^{matter}_{\mu\nu}(x)\right\} \ , \tag{16}$$

where $g(t)$ is given by (4). The inverse metric to first order then equals:

$$\hat{g}^{\mu\nu}(t,\vec{x}) = g(t)^{-1}\left\{\eta^{\mu\nu} - \eta^{\mu\alpha}h^{matter}_{\alpha\beta}(x)\eta^{\beta\nu}\right\}. \tag{17}$$

The linearized Einstein field equation reads:

$$\begin{aligned}
&-\eta^{\lambda\lambda}\partial_\lambda\partial_\lambda h_{\nu\mu} + \eta^{\lambda\lambda}\partial_\lambda\partial_\nu h_{\mu\lambda} + \eta^{\lambda\lambda}\partial_\lambda\partial_\mu h_{\lambda\nu} - \eta^{\lambda\lambda}\partial_\nu\partial_\mu \ h_{\lambda\lambda} \\
&+\frac{2}{t}\left(\partial_\nu h_{\mu 0} + \partial_\mu h_{0\nu} - \partial_0 h_{\nu\mu} + \eta_{\nu\mu}\eta_{\lambda\lambda}\partial_\lambda h_{\lambda 0} - \frac{1}{2}\eta_{\nu\mu}\eta^{\lambda\lambda}\partial_0 h_{\lambda\lambda}\right) + \frac{6}{t^2}h_{00}\eta_{\nu\mu} \\
&= 16\pi G\left(T^{matter}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T^{matter}\right) = 16\pi G g\rho^{matter}\left(\delta_{\mu 0}\delta_{0\nu} + \frac{1}{2}\eta_{\mu\nu}\right),
\end{aligned} \tag{18}$$

where some vacuum terms cancelled out. The solution can be expressed in terms of a single function $h(x)$:

$$h^{matter}_{\mu\nu}(x) = \delta_{\mu\nu}h(x), \tag{19}$$

with:

$$h(x) = 2Gg\int\limits_{\hat{V}} d^3x' \frac{\rho^{matter}(t,\vec{x}')}{|\vec{x}-\vec{x}'|} = 2G\frac{t}{t_s}\sum_i M_i\frac{1}{|\vec{x}-\vec{x}_i|} \ . \tag{20}$$

Here the original volume V is replaced by the volume $\hat{V}$, associated with the new state vector of the universe. This new volume (and hence the corresponding scale factor) is determined by the demand of global energy conservation, and is not fixed by the Einstein equations (see Section 6).

The only difference between (20) and the standard result in a flat background metric is the factor t/t_s. This factor counters the expansion of the universe and ensures that astronomical objects are in stable orbits despite the expansion of the universe. If we replace the coordinates $\vec{x}$ by co-moving coordinates $\vec{\tilde{x}}$, we get the standard result [14]:

$$h(x) = 2G\sum_i M_i\frac{1}{\left|\vec{\tilde{x}}-\vec{\tilde{x}}_i\right|}, \tag{21}$$

where:

$$\vec{\tilde{x}} = \frac{t_s}{t}\vec{x}. \tag{22}$$

Hence, two related representations are possible of the space-time characteristics of the universe. The first one is the co-moving representation, which is closest to our observations, as we cannot directly observe the scale factor t/t_s, whereas our astronomical observations are in agreement with (21). However, we have to use the original variables and the explicit form (20) in the Einstein equations, since $\vec{x}$ and not $\vec{\tilde{x}}$ is the independent variable in those equations. A similar duality occurs with respect to the time coordinate and the fourmomentum of particles, as we will discuss in more detail in Section 9.

Finally, we note that the sum (20) would be infinite for an infinite universe. The problem is that we have used an instantaneous solution. By imposing causality we can limit the contributions to

$$|\vec{x} - \vec{x}_i| < ct \ , \tag{23}$$

when applying (20). In terms of co-moving coordinates condition (23) implies:

$$\left|\vec{\tilde{x}} - \vec{\tilde{x}}_i\right| < ct_s \ . \tag{24}$$

The average correction to the flat metric is then:

$$< h(x) >= \frac{3}{2}\frac{\rho^m}{\epsilon} = \frac{3}{2}\frac{\sum_i M_i}{\epsilon \hat{V}_s} \ , \tag{25}$$

where ρ^m is the average matter density in the universe. For the definition of the adjusted invariant volume $\hat{V}_s$ we refer to Section 6.

5. The modified metric in the presence of radiation

As we only consider $h_{\mu\nu}$ to first order, we can solve the equations of general relativity separately for the matter and electro-magnetic contributions in the vacuum background. The radiation density can be written in the perfect fluid form, as the QFT expression for the energy density is not localized (photons are represented by plane waves). Taking account of the metric factors so that the resulting expression satisfies the covariance condition (8), we arrive at:

$$T^{rad}_{\mu\nu}(x) = \frac{1}{g}\frac{\sum_j p^{(j)}}{\hat{V}} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & \frac{1}{3} \end{pmatrix} = g \begin{pmatrix} \rho^{rad} & 0 & 0 & 0 \\ 0 & p^{rad} & 0 & 0 \\ 0 & 0 & p^{rad} & 0 \\ 0 & 0 & 0 & p^{rad} \end{pmatrix} . \tag{26}$$

In (26) we included all the photons in the universe at time t. Both $p^{(j)}$ and $\hat{V}$ have an effective time dependence owing to the expansion of the universe: $p^{(j)}$ is complementary to the spatial Einstein coordinate and thus decreases like $1/t$ (see Section 9), whereas $\hat{V}$ increases like t^3 (see Section 6). Hence, although the explicit time dependence of $T^{rad}_{\mu\nu}(x)$ is like t^2, its effective time dependence after accounting for the expansion of the universe is like t^{-2}. Similarly, ρ^{rad} and p^{rad} have the explicit time dependence t^4, but after the expansion of the universe is taken into account, its effective time dependence is constant. In analogy to the co-moving coordinates $\vec{\tilde{x}}$ introduced in the matter case (see (22)), we can introduce momenta as observed by a co-moving observer:

$$\vec{\tilde{p}} = \frac{t}{t_s}\vec{p}. \tag{27}$$

This behaviour will be further discussed in Section 9, in which we discuss the two dual representations in detail. If we integrate ρ^{rad} (or $-{T_0}^{0,rad}$) over all of space, we get the sum over all momenta $p^{(j)}$ at time t, multiplied by t/t_s; i.e the sum over all co-moving momenta. As we will see in Section 6 this implies that the radiative contribution to the total energy is constant over time, unless creation or annihilation events change the number and/or nature of the participating photons. Hence, this situation is similar to the matter case where the energy integral is also constant, as long as the state vector remains the same.

We can obtain $h^{rad}_{\mu\nu}$ by solving the first order equation (18) with the electro-magnetic source term. We find:

$$h^{rad}_{\mu\nu}(x) = -8\pi G t^2 T^{rad}_{\mu\nu}(x) \ . \tag{28}$$

Because of the effective t^{-2} time dependence of $T^{rad}_{\mu\nu}(x)$, $h^{rad}_{\mu\nu}(x)$ behaves effectively like a constant. Naturally, when solving for $h^{rad}_{\mu\nu}(x)$ in the Einstein equations, we must use the explicit t^4 dependence of this function. In other words, the decrease of the radiation density with time caused by the expansion of the universe is countered by the t^4 dependence, arising from the background vacuum metric. The combination of these two factors is the cause of the constancy of the effective contribution to the total energy. The effective constancy of the radiation and matter terms also ensures the continued basic linear expansion of the universe in the presence of matter and radiation, as we will demonstrate in Section 6. It should be noted that $h^{rad}_{\mu\nu}$ has a sign opposite to that of the matter contribution, because of the minus sign in (28). Thus radiation has a gravitational effect opposite to that of matter. Because light cannot solidify into a massive astronomical body these effects are hard to measure, but this opposite sign of the radiation contribution has a distinct effect on the average metric in the universe.

Just as in the case of matter we can evaluate the average value of $h_{\mu\nu}$ in the radiative case. We find:

$$< h^{rad}_{00}(x) > = -3\rho^{rad}/\epsilon \ . \tag{29}$$

The two results, Eq.(25) and Eq.(29), have nearly the same form and can be used to assess the flatness of the universe. As noted above, matter and radiation have opposite effects on the metric.

6. Energy Balance and the Expansion of the Universe

After the introduction of matter and radiation the total energy of the universe is given by:

$$E = \int\limits_{\hat{V}} d^3x \sqrt{{}^3\hat{g}(t,\vec{x})}\ \epsilon + \int\limits_{\hat{V}} d^3x \sqrt{{}^3\hat{g}(t,\vec{x})}\ \rho^{matter}(x) + \int\limits_{\hat{V}} d^3x \sqrt{{}^3\hat{g}(t,\vec{x})}\ \rho^{rad}. \tag{30}$$

Expanding (30) to first order in $h_{\mu\nu}$, we have:

$$\begin{aligned} E = \epsilon V_s &= \epsilon \frac{t_s^3}{t^3}\hat{V}(t) + \epsilon \frac{t_s^3}{t^3}\frac{3}{2}\int\limits_{\hat{V}} d^3x \left\{ h(x) + \frac{1}{3}\sum_i h^{rad}_{ii} \right\} + \sum_i M_i + \frac{t}{t_s}\sum_i p^{(i)} \\ &= \epsilon \frac{t_s^3}{t^3}\hat{V}(t) + \epsilon \frac{t_s^3}{t^3}\frac{3}{2}\int\limits_{\hat{V}} d^3x\ h(x) + \sum_i M_i + \frac{t}{t_s}\sum_i p^{(i)}(1 - \frac{3}{2}) \ . \end{aligned} \tag{31}$$

We now extend (10) to the volume in the presence of radiation and matter:

$$\hat{V}(t) = \frac{t^3}{t_s^3}\hat{V}_s \ , \tag{32}$$

i.e. we assume that the introduction of matter and radiation only requires a change of the original invariant volume V_s into $\hat{V}_s$. Using (32) and (27) we can then show that all terms in (31) are constant for a given state vector, so that Ansatz (32) is consistent. Therefore, the volume $\hat{V}_s$ is indeed invariant under the basic linear expansion of the universe, although it adjusts itself whenever the state vector of the universe changes if particles are created or destroyed. Using the definition (32) and expression (25) for $< h >$, we get:

$$E = \epsilon V_s = \left(\epsilon + \rho^m + \rho^{rad} + \frac{9}{4}\rho^m - \frac{3}{2}\rho^{rad}\right)\hat{V}_s \ . \tag{33}$$

The last two terms originate from the modifications to the metric after the introduction of matter and radiation. Conveniently, they have the same form as the original matter and radiation terms, with only the coefficients being different. If, as is usually assumed, the matter term dominates over the radiation term, then the new invariant volume $\hat{V}_s$ is smaller than the original invariant volume V_s.

We can now generalize the scale factor in the presence of matter and radiation:

$$a(t) = \frac{t}{t_s}\left(\frac{\hat{V}_s}{V_s}\right)^{1/3} = \left(\frac{\hat{V}(t)}{V_s}\right)^{1/3} \ . \tag{34}$$

Hence, a change in $\hat{V}_s$ also implies a change in the scale factor. So, in addition to the linear time dependence, there is a further implicit time dependence, which depends on the evolving state vector of the universe. Whenever a QFT transition takes place, and the state vector is changed, the volume $\hat{V}_s$ is also adjusted, and, consequently, the scale factor. Together this leads to an effective time dependence of the scale factor. Thus, the creation and/or decay of matter and the creation or absorbtion of radiation in the universe has rather specific consequences for the acceleration or contraction of the universe over and above the linear expansion. Since the volume $\hat{V}_s$ is determined by the energy equation (33), the scale factor is no longer determined by the Einstein equations as in the FRW case, but rather by total energy conservation. This is an important difference, which allows us to consider local gravitational effects and effects of the global expansion together, as the scale factor no longer appears in the metric. The scale factor is now also a truly cosmological property, as it is the same everywhere in the universe, being defined in terms of integrals over the whole universe. It may be hard to accept how a transition at a distinct location can influence the scale factor in our neighbourhood, especially for an infinite universe. However, if we replace individual transitions by rates and assume that the universe looks the same everywhere on a large scale (the cosmological principle), then we can view our visible universe as a finite representation of the whole universe. The additional time dependence $\left(\hat{V}_s/V_s\right)^{1/3}$ then becomes continuous and represents the acceleration or deceleration of the universe as a deviation from the basic linear expansion of the universe. Obviously, this continuous time dependence does not feature explicitly in the Einstein equations, although the solution of these equations at a particular time must use the state vector pertaining to that particular moment.

One could call (33) the co-moving form of the energy balance equation. If we go back to the original volume using (32), we obtain:

$$E = \epsilon V_s = \left(\frac{t_s^3}{t^3}\epsilon + \frac{t_s^3}{t^3}\rho^m + \frac{t_s^3}{t^3}\rho^{rad} + \frac{9}{4}\frac{t_s^3}{t^3}\rho^m - \frac{3}{2}\frac{t_s^3}{t^3}\rho^{rad} \right) \hat{V}(t) \ . \tag{35}$$

This form clearly illustrates the high densities in the initial universe and the decreasing densities with time. It is these time-dependent densities which have to be considered in descriptions of the evolution of the universe and of the hot Big Bang, because they have to be compared to QFT processes that do not depend on the expansion of the universe and therefore play different roles in different epochs.

One question is now whether we can determine the contributions of the matter and radiation components to the energy balance (33). As stated previously, the unperturbed vacuum energy (i.e. the first term in (33)) has the same density to pressure ratio ($w = -1$, [13]) as dark energy, suggesting that this dark energy can be identified with the unperturbed vacuum energy. Since the ratio w will change if $\hat{g}_{\mu\nu}$ differs considerably from the vacuum metric $g_{\mu\nu}$, the dominance of dark energy and the observed flatness of the universe suggest that the average matter and radiation densities ρ^{matter} and ρ^{rad} are small compared to the vacuum energy density ϵ. The current estimates of the matter content of the universe (about 24% including dark matter, [15]) rely heavily on the energy balance as formulated in the RW framework ([4], [1]). Hence, these estimates should be re-examined in the context of the current framework and might actually be much less certain than usually is assumed. Furthermore, the usual estimate of the radiation content is based on the decrease of this density owing to the expansion of the universe and the red shift, and does not take account of the metric factor t^4, which completely compensates for this decrease, leading to a constant ρ^{rad}. Hence, the contribution of radiation to the total energy could well be comparable to the matter density, rather than merely having the tiny value of 5×10^{-5} quoted in the literature [16]. It should also be noted that the observed decrease of photon momenta with time, popularly called the red-shift effect, has a rather different interpretation in our formulation, as we will see in Section 8. We attribute this to the dual nature of the vacuum universe and to our role as a co-moving observer therein. So one can expect that this "red-shift" effect is compensated for in energy expressions and that it will not lead to a reduction in the contribution of radiation with the passing of time.

In the standard picture the initial universe experienced a radiation-dominated phase, followed by the current matter-dominated phase. However, in our picture the contribution of these phases to the energy balance are more or less constant over time, owing to the influence of the background metric. Hence, both the dominance of radiation over matter in the early stages, and the dominance of matter over radiation at the present time must be re-examined in our approach.

If these densities are indeed of comparable magnitude then many interesting scenarios for the evolution of the universe become possible. For example, if:

$$< h_{00}^{rad} >= -\frac{3}{2} < h > \quad , \tag{36}$$

or in terms of densities

$$\rho^{rad} = \frac{3}{4}\rho^{m} \quad , \tag{37}$$

then the average metric becomes proportional to the flat Minkowski metric, even in the presence of large matter and radiation densities. Since $< h_{00}^{rad} >$ increases and $< h >$ decreases in size, whenever matter is converted into radiation, condition (36) cannot be satisfied at all times, unless the creation and annihilation processes are in equilibrium. However, the universe may have been close to this point for most of its existence, in which case (36) would explain why the universe appears so flat, despite containing a considerable amount of matter. Clearly, further analyses are required to examine these possibilities.

Using Eq.(24) it is also possible to calculate the approximate energy content of the visible universe. We find

$$E^{visible} = \epsilon \times \frac{4\pi}{3} t_s^3 = \frac{1}{2}\left(\frac{3}{8\pi}\right)^{1/2} \frac{1}{\epsilon^{1/2} G^{3/2}} \approx 5 \times 10^{79} \text{ GeV}. \tag{38}$$

where we used the value $\epsilon = 4.06 \times 10^{-47}$ GeV^4 derived in Section 9. Again, this emphasizes the important role of the two fundamental dimensionfull constants of Nature, ϵ and G. This result also allows one to make a rough estimate of the number of massive particles in the visible universe: about 10^{79} protons and the same number of electrons. Other independent estimates of this number will put further constraints on the value of ϵ or t_s.

7. A possible explanation for dark matter

An important cosmological problem is the nature of dark matter. It may be tempting to consider the mixed vacuum-matter term in (31) as a dark matter term. Firstly, it is closely related to the matter distribution and is localized near matter concentrations, on account of the form of $h(x)$. Secondly, its contribution to the total energy is much larger than that of the original mass term, as is the case for dark matter by comparison with ordinary matter. However, since the mixed term does not influence the metric in lowest order (it being rather a consequence of the metric) it could only have gravitational effects in higher order. Furthermore, the localization of the equivalent "mass" it represents is so weak that it cannot explain the distribution of dark matter near galaxies. Finally, the enhancement factor 9/4 differs considerably from the usual ratio of dark matter to ordinary baryonic matter (a factor of about 4.8, see [17]).

We will discuss another interesting possible explanation for dark matter. This explanation is based on certain second-order effects, which are unique to our approach. Since we have neglected second order effects up to now, this analysis is somewhat tenuous. However, it shows encouraging agreement with some observations. As we see from Eq.(15), and the integral in Eq.(20), the gravitational potential is inversely proportional to $\sqrt{^3\hat{g}(t,\vec{x})}$ owing to the form of the matter energy-momentum tensor. In first order we replaced $\sqrt{^3\hat{g}}$ by $\sqrt{^3g}$. However, in higher order we would need to consider the corrected metric factor. For a black hole at the center of a galaxy this would effectively mean that instead of experiencing the gravitational pull of the real mass M, one would experience the reduced effect of the apparent mass M^{app} at a distance r:

$$M^{app}(r) = \frac{M}{\prod_{i=1}^{3}\{1+h_{ii}^{matter}(x)\}^{1/2}} = \frac{M}{(1+2GM/r)^{3/2}} \ , \qquad r > R_{BH} \ , \quad (39)$$

where R_{BH} is the radius of the black hole. At small distances M^{app} could be much smaller than M, whereas at large distances the black hole mass would have its normal effect as the screening becomes negligible. Since this mass does not correspond to any visible material, it could represent dark matter. Assuming this to be the case, we can define the dark matter density by subtracting the observed mass near the black hole from the effective mass distribution:

$$4\pi\int_0^r dr' \ r'^2 \ \rho^{dm}(r') = \frac{M}{(1+2GM/r)^{3/2}} - M^{app}(R_{BH}) \ . \quad (40)$$

Differentiating with respect to r we obtain for small r and large $2GM/r$:

$$\rho^{dm}(r) \sim r^{-3/2} \qquad r > R_{BH} \ . \quad (41)$$

Possible support for this picture comes from analyses of dark matter ([18], [19]), which also indicate a singular behavior of dark matter density in the center of galaxies. For example, Krauss and Starkmann [18] find that the dark matter density near the centre behaves like $r^{-3/2}$, which is exactly in agreement with our explanation. In addition, thermal models of galaxy densities [19] give a constant core density for normal matter, so that our effective mass distribution cannot be interpreted as normal matter. At large distances we obtain:

$$\rho^{dm}(r) \approx \frac{3}{4\pi}\frac{GM^2}{r^4} \qquad r \gg R_{BH} \ , \quad (42)$$

which gives the required localization near existing galaxies.

A possible objection to this explanation is that a current survey [16] only gives a black hole contribution of 7×10^{-5} to the energy content of the universe, although this number may be surrounded by uncertainties similar to those around other estimates in the RW framework. If this number is based on the apparent mass of black holes as measured in the vicinity of these objects, then a huge screening effect is required to explain the large dark matter component in terms of black holes. However, as (39) allows such effects, the true mass of the black holes at the center of galaxies may well be order of magnitudes greater than is currently assumed. This possibility may also have an important influence on considerations of the evolution of the early universe, as enormous black holes are usually considered fatal to the development of galaxies. This need no longer be the case owing to the screening effects suggested in the current framework. Clearly, non-perturbative calculations are required to test this dark matter theory, as large second order corrections would in turn induce important third or even high-order terms, which could either moderate or enhance the effect observed in second order.

8. Description of red shift data and other observables

Let us now discuss a number of astronomical observables. Firstly we discuss the Hubble constant. This quantity is defined as the relative increase of the scale factor with time [14], which in our formulation reads:

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{1}{t}, \tag{43}$$

where the last transition follows from Eq.(34) if we ignore the time dependence of $\hat{V}_s$. As we will see in the following discussion, one actually measures the inverse of the Hubble constant, because one detemriens the luminosity distance d_L. As a co-moving observer would measure the co-moving distance $(t_s/t)d_L$, the Hubble constant measured would also be rescaled and would equal $(t/t_s)1/t = 1/t_s$. This leads to the pleasing result that the measured Hubble constant is indeed constant, since t_s is constant! So this to some extent justifies the name Hubble *constant*. Since Eq.(43) shows that the inverse Hubble constant represents the age of the universe, we find that for a co-moving observer the age of the universe is constant and equals t_s. Unfortunately, this also implies that the Hubble constant does not provide us with the actual age of the universe in terms of GR coordinates $t = t_0$. The value t_0 is of importance, since it tells us in which epoch we are living, as it is expressed in the same representation as the elementary particle properties (for example the particle physics scale t_c derived in the Section 10). In Section 9 we will discuss how one can get information about the value of t_0. It should be noted that the measurement of the Hubble constant gives information on the (constant) vacuum energy density ϵ, because of the relationship between ϵ and t_s.

The Hubble constant is determined from supernovae measurements. As shown below the fit to the Cepheid data suggests a value of $t_s = 13.8 \times 10^9$ years, corresponding to $H_0 = 71\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. Current best estimates by Freedman *et al* [20] provide the value $H_0 = 72 \pm 8\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. The identity of H_0^{-1} and the age of the universe in our theory is in good agreement with observations, as most analyses favour values which are close together for these two quantities. Although decay processes contribute towards an acceleration of the expansion, we do not expect such changes to upset this agreement. In matter dominated FRW universes the age of the universe equals $\frac{2}{3}\,H_0^{-1}$, whereas in radiation dominated FRW universes it equals $\frac{1}{2}\,H_0^{-1}$, both possibilities differing from the accepted values of H_0 and the age of the universe.

The increase in wavelength of photons originating from distant galaxies or supernovae, as first observed by Hubble, is known as red shift. This name suggests that the phenomenon is due to the Doppler effect of receding galaxies. However, as is well-known [14], the correct explanation should be based on the framework of GR.

Weinberg [14] gives the standard explanation in terms of the RW metric, leading to the relationship:

$$z = \frac{\lambda_{obs} - \lambda_1}{\lambda_1} = \frac{a_{RW}(t_0)}{a_{RW}(t_1)} - 1, \tag{44}$$

where the source is characterized by t_1 and λ_1, the observer being characterized by t_0 and the observed wavelength λ_{obs}. Although we do not use the RW representation, we still get the same final result. Our explanation is based on the dual representation of space-time, with the observer measuring the wave length in terms of co-moving variables $\vec{\tilde{x}}$ and $\vec{\tilde{p}}$.

Firstly, the atomic transition giving rise to the emission of the light is defined by a characteristic wavelength λ or by a characteristic time interval Δt, which remain constant over time. The wave length measured by a co-moving observer at the source is then:

$$\lambda_{source} = \frac{t_s}{t_1} \lambda \ , \tag{45}$$

or alternatively by a time interval $\frac{t_s}{t_1}\Delta t$. Because of the invariance of the interval $ds = \frac{t_s}{t_1}\Delta t$, we will measure the same time interval $\frac{t_s}{t_1}\Delta t$ when the photon finally reaches our equipment. Hence we will also observe the wave length:

$$\lambda_{observed} = \frac{t_s}{t_1} \lambda \ , \tag{46}$$

at our current location at time t_0. However, if we measure the same transition at our terrestrial location, we observe the wave length:

$$\lambda_{terrestrial} = \frac{t_s}{t_0} \lambda \ . \tag{47}$$

Now, in the standard interpretation ([14], p. 417) the wavelength at the source, λ_1, is supposed to be equal to the wavelength currently measured in a terrestrial laboratory, which we indicate by $\lambda_{terrestrial}$. So the unknown λ_1 in Eq.(44) is replaced by $\lambda_{terrestrial}$. Hence, what one really tests in the red shift analysis is an expression involving $\lambda_{terrestrial}$, not the unmeasured λ_1. Therefore, we introduce the terrestrial wave length directly into our expression for z. We are then led to the relationship:

$$z = \frac{\lambda_{observed} - \lambda_{terrestrial}}{\lambda_{terrestrial}} = \frac{\frac{t_s}{t_1}\lambda - \frac{t_s}{t_0}\lambda}{\frac{t_s}{t_0}\lambda} = \frac{t_0}{t_1} - 1 = \frac{a(t_0)}{a(t_1)} - 1, \tag{48}$$

in which the last identity is valid if we ignore the time dependence resulting from the change in $\hat{V}_s$. As we see, the final relationship is identical to Eq. (44) derived in [14]. Hence, totally different philosophies can still lead to the same result and consequently to the same agreement with experiment. The relationships Eq.(45) and Eq.(47), which involve $1/t$, seem to suggest that the wavelength decreases rather than increases with time. However, this conclusion is wrong: because of the expansion of the universe all lengths such as x and λ are increasing with time (although this increase is not explicit in the GR equations), and the indicated time dependence in these equations merely compensates for this increase to make the effective wave length constant over time.

In deriving the simple expression $t_0/t_1 - 1$ in Eq.(48) we have ignored the time dependence of h_{00} in the metric. This is justified by the fact that the contributions of both matter and radiation to h_{00} are effectively constant. We would only get deviations from this identity if creation or decay processes substantially affect the time dependence of h_{00}.

In order to compare our theory with the Cepheid observations we have to express the luminosity distance in terms of z. We have [14]:

$$d_L = \frac{a^2(t_0)}{a(t_1)} d(t_0) \; , \tag{49}$$

where the distance $d(t_0)$ can be expressed in terms of the time of emission and the time of observation:

$$d(t_0) = c \int_{t_1}^{t_0} \frac{dt}{a(t)} \; . \tag{50}$$

Using the vacuum metric and eliminating t_1 in favour of z, we have:

$$d_L = cH_0^{-1}(z+1)ln(z+1) = cH_0^{-1} z \left(1 + \frac{1}{2}z - \frac{1}{6}z^2 + \cdots \right) \; . \tag{51}$$

This corresponds to a deceleration parameter q_0 and to a jerk parameter j_0 both of which are zero (see Visser [21] for a definition of these parameters and the corresponding red shift formula). Since a co-moving observer will measure $t_s/t_0 \; d_L$ rather than d_L itself, we still have to multiply this expression by the factor t_s/t_0. The current notation can be retained if we understand that $H_0^{-1} \rightarrow t_s/t_0 \; H_0^{-1} = t_s$.

We considered recent Cepheid data for distance moduli [22], which have to be corrected by -0.27 according to a recent analysis by the same authors [23]. The best fit for the vacuum solution is obtained for the value of $H_0 = 71$ km s^{-1} Mpc^{-1}, which corresponds to a measured lifetime of $t_s = 13.8$ billion years. This agrees well with a recent WMAP analysis by Hinshaw et al. [24], who state that: $H_0 = 70.5 \pm 1.3$ km s^{-1} Mpc^{-1}. In Fig. 1 we show a comparison between the observations and our vacuum solution result, together with some additional fits. The ratio $H_0 \, d_L/z$ is displayed, rather than d_L itself, so as not to obscure the deviations between experiment and theory at small values of z. The vacuum result fits the data very well. In order to put this result in perspective, we have also fitted various power expansions of the expression in brackets in the right-hand-side of Eq. (51). These will allow us to determine the values for q_0 and j_0 preferred by the data and give an indication of the uncertainty in these parameters. The linear fit gives $q_0 = .038$, which is close to the value of zero obtained in the vacuum solution. The quadratic fit yields $q_0 = -.63$ and $j_0 = 1.26$. The Hubble parameters for the linear and quadratic fit are 73.0 and 75.9 km s^{-1} Mpc^{-1}, respectively, both falling outside the range given by Hinshaw et al. [24]. The corresponding ages are 13.4 and 12.9 billion years. We see that the parameters obtained depend quite strongly on the nature of the fit, casting some doubt on the strength of evidence for a pronounced acceleration ($q_0 < 0$) in the current universe. As stated above, our model with a linear expansion already fits the data very well.

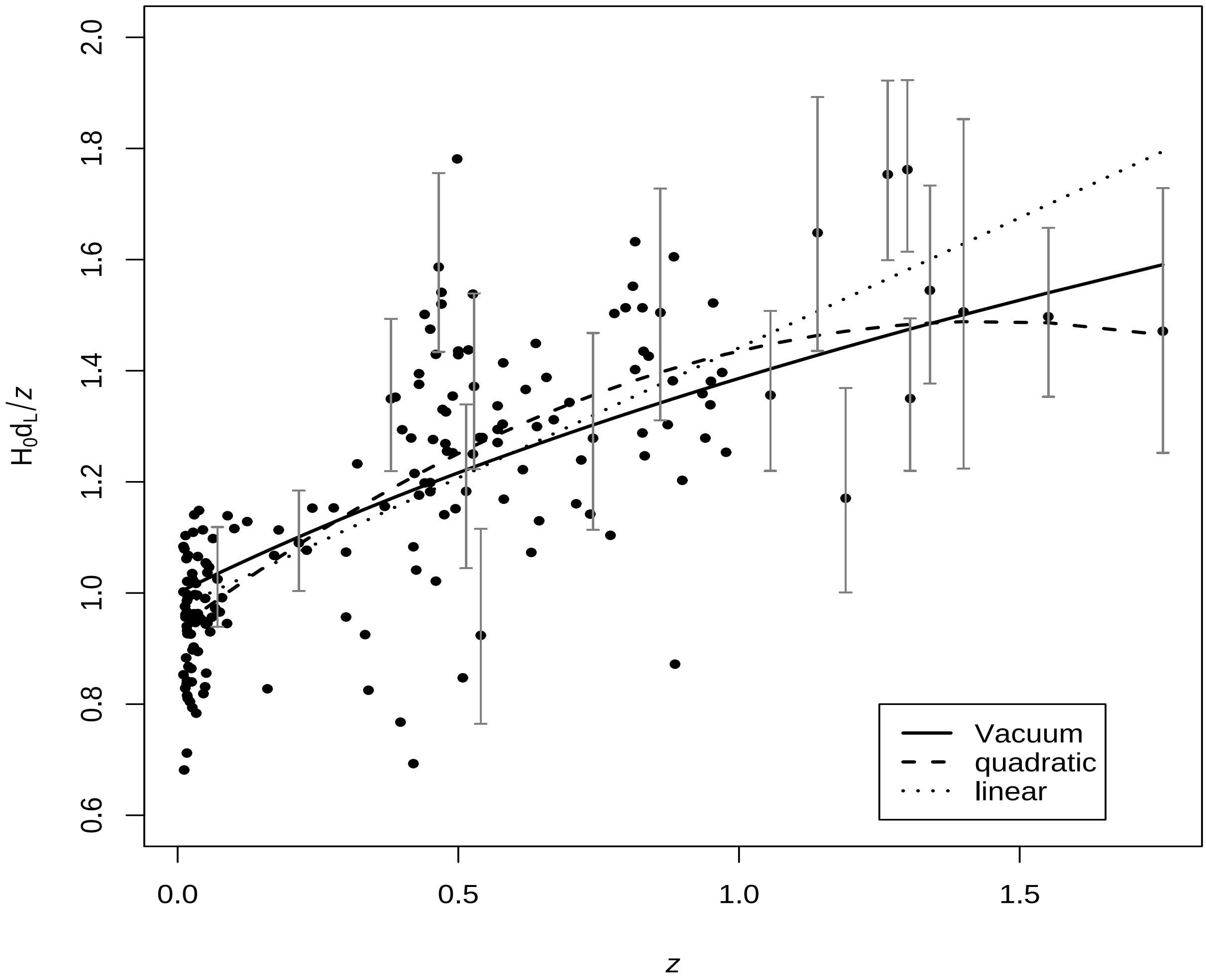

Figure 1. Comparison between Supernovae data ([22], [23]) and various theoretical descriptions. Plotted are the distance moduli multiplied by H_0 (as determined from our model) and divided by z as a function of the red shift z. Some of the data errors are shown to illustrate the quality of the data and the fit.

It can be expected that supernovae data at higher values of z will put stronger constraints on these parameters. The negative q_0 in the quadratic fit suggests that the universe is currently accelerating, whereas the positive jerk parameter suggests that there might have been a deceleration in the past. This agrees with the detailed statistical analysis carried out in Refs. ([22], [23]). Clearly, the present theory can explain the average expansion (linear). Within this theory it is natural to explain a possible current acceleration by means of the presence of decay processes. The standard decay process is radiation, as this process transforms matter into radiation. Other - more speculative - decay processes are possible as well. The decay of WIMP like particles in dark matter would contribute towards acceleration. Also, the annihilation of particles by black holes would be a possible source of acceleration. However, deviations from the linear expansion may also be attributed to the change in h_{00} over time, as noted previously. Hence, a more detailed theoretical analysis is required, in which the role of decay processes in the current universe is elucidated. Higher order consequences of the abandonment of the RW formulation on the red-shift formula also have to be examined. In any case, it is clear that accurate supernovae data will yield strong constraints on the theoretical description.

Another important issue in cosmology is the horizon problem. The horizon is defined as the distance a photon traveled since the Big Bang to a particular point in time [2]. Obviously, this is infinite for the expression (50), as $t_1 = 0$. Since we only expect virtual photons to exist a finite time after the Big Bang (see Section 10), the physical horizon is not infinite. However, it is still true that in our description the horizon is much larger than in the typical FRW models, where the lower limit in the integral (50) vanishes. Hence there does not appear to be a horizon problem in our approach. This eliminates the main reason for the introduction of the inflationary hypothesis, although we predict an inflationary phase naturally in our approach, when particles and antiparticles were annihilated soon after they were created (see again Section 10).

9. The role of co-moving coordinates and the dual representation of space-time

As we have seen in previous sections, co-moving observers measure physical quantities in terms of the coordinates $\vec{\tilde{x}}$ and $\vec{\tilde{p}}$. These coordinates are invariant under the expansion of the universe (naturally they still function as variables in regard to local physical processes), and make it hard to measure this universal expansion directly. For example, the gravitational potential given in Eq.(20) remains constant in spite of the expansion of the universe, if it is expressed in terms of co-moving coordinates $\vec{\tilde{x}}$. We only measure the expansion indirectly via the red shift observations discussed in the previous section. These features appear to be especially simple for - and perhaps unique to - the vacuum metric, on account of the linear nature of the transformations.

The local co-moving representation is concisely given by:

$$\vec{\tilde{x}} = \frac{t_s}{t_0}\vec{x}, \tag{52}$$

$$\tilde{t} = \frac{t_s}{t_0}t\,, \tag{53}$$

while the conjugate relationships for the momentum reads:

$$\vec{\tilde{p}} = \frac{t_0}{t_s}\vec{p}, \tag{54}$$

$$\tilde{E} = \frac{t_0}{t_s}E\,. \tag{55}$$

So far we have avoided the use of the symbol $\tilde{E}$ for the co-moving energy. The energy E used in previous equations (e.g. in Section 6) can be identified with the co-moving energy if we use the co-variant metric $\sqrt{-g}$, rather than ${}^3g$, in the relevant energy integrals. In that case we have to multiply the resulting expression by the factor t_0/t_s (or by the global factor t/t_s) in order to obtain the standard result, called E in Section 6. We will not discuss the implications of this modification further. Eq.(53) is a local representation of the global relationship $\tau = t_s\ ln(t/t_s)$, which was already mentioned in Eq.(7). We can also replace t_0 by t in Eqs.(52, 54 and 55) to make the equations more global. However, for a local co-moving observer, the given equations are the relevant ones, since they simply amount to a rescaling of the original coordinates. This local representation is also natural in the context of the Einstein equations, since the (temporary) replacement of t by t_0 ensures that the time-dependence of $\vec{x}$, t and p is not explicit in the Einstein equations.

Under the transformation Eqs.(52 and 53) the metric expression Eq.(5) near t_0 reads:

$$ds^2 = -d\tilde{t}^2 + d\tilde{x}^2 + d\tilde{y}^2 + d\tilde{z}^2. \tag{56}$$

It is natural that an observer would use a locally flat metric to carry out his observations, which explains the important role of this co-moving representation in measurements. The importance of such transformed variables in cosmology had already been recognized very early on in the development of cosmology. Milne [25] introduced dual variables in 1937, although he did not base himself on a universe with a finite vacuum energy density, so that he did not have a natural time scale t_s. It would be of interest to study the analogies further, although Milne did not use a relativistic formulation in his analysis.

As stated in the Section 8, one of the results of our particular measuring process is that the measured Hubble constant equals $(t/t_s)1/t = 1/t_s$. Given the value of the Hubble constant derived in the previous section from supernovae data, we find that the vacuum energy density acquires the value $\epsilon = 3/8\pi G t_s^2 = 4.06 \times 10^{-47}$ GeV^4. This is the value used in previous sections of this paper, e.g. in Eq.(38). This value is very close to the one given by Weinberg ([14], p. 476): 10^{-29}g/cm^3, which corresponds to 4.31×10^{-47} GeV^4. Actually, the value quoted by Weinberg represents the *critical* energy density of the universe, which must be close to the actual energy density for a universe that is flat (observations have shown that the geometry of the universe is very close to being spatially flat [1], [9]). Hence, this critical density should coincide with the vacuum energy density in a universe dominated by vacuum energy. In our theory there is nothing critical or accidental about the value of ϵ, as slightly larger or smaller values would describe the universe equally well. Hence, this is another puzzle (why the critical and actual energy density are so close at this very moment) that is solved by the current theory. Carroll gives the rough estimate 10^{-8}erg/cm^3 ([2], Eq. (8.162)), corresponding to 5×10^{-47} GeV^4, which is also consistent with the current estimate.

The constancy of the age of the universe t_s also indicates that an observer could never reach the beginning of time by moving backward in time (apart from the practical aspect that the Big Bang defines a direction in time which allows us only to "move" forward). This property also is evident if we use the global transformation variable τ

defined in Eq.(7): $\tau = t_s\ ln(t/t_s)$. This suggests that $t = 0$ refers to the infinite past.

As the co-moving variables Eqs.(52, 53, 54 and 55) are essential for our measurements, the question can now be asked whether the magnitude of the original variables, as represented by our current time t_0, plays any (absolute) role. For example, in the red shift discussion, only the relative quantity t_1/t_0 played a role in the definition of z. However, in Section 10 we will see that different epochs in the evolution of the universe can be distinguished despite the "relativity" of the concept of time. Since properties of particles are expressed in the original time units, as they are independent of the expansion of the universe, the particle scale represents an independent way of measuring time. As a consequence, there are ways of inferring t_0, provided our particle models are sufficiently accurate. Since the currently accepted elementary particle models do not predict the masses of quarks and leptons, and in particular do not relate them to G and ϵ, we are a long way away from the situation that t_0 can be determined accurately. Nonetheless we will argue in Section 10 that t_0 is of the same order of magnitude as t_s. In other words: our current epoch is characterized approximately by the time scale t_s of the vacuum universe.

10. Evolution and Development of a Vacuum Dominated Universe

It is common to consider the first moments after the Big Bang as a period of extreme complexity, during which particles are compressed into an extremely small space and carry enormous kinetic energy. This scenario is sketched in many articles and popular books, e.g. in a recent book by Martin Rees [27]. It also leads to the idea, often heard these days, that the LHC experiment at CERN will reproduce the early moments of the Big Bang [28]. Such a densely populated state of the early universe requires a reliable unified theory of QFT and GR. Since that does not (yet) exist, a reliable picture of this initial epoch is lacking. Our solution to the GR equations suggests another scenario. The singularity in the classical vacuum metric implies that the universe started out in the simple "classical" vacuum state. The initial density of the universe was so high and the distance scale so small that physical fermions, which we expect to have a finite - although extremely small - size, could not form. The creation of real photons, which is linked to the fermionic processes by the standard model Lagrangian, was likewise suppressed. Under these circumstances, the quantum fluctuations in the early universe do not lead to the creation of any physical particles and thus leave the vacuum state vector of the universe unaffected, as this state will only change once physical particles have been formed. As a consequence, in this early epoch the state vector of the universe equals the vacuum state and displays perfect homogeneity and zero entropy. The energy of this state is given by $\epsilon\ V_s$, which at the same time represents the total energy of the universe at all times and the permanent entry in the right-hand side of Eq. (33).

After the Big Bang the density decreases according to the formula $\epsilon t_s^3/t^3$ until the first particle epoch arrives when the energy density has diminished to such an extent that it matches the energy density of physical (as opposed to bare or virtual) quarks and leptons. The size of strings in the string model is of the order of the Planck length $G^{1/2}$. Similarly, we expect the volume of finite elementary particles (which in our theory are spherical finite objects) to be $O(G^{3/2})$, with a corresponding energy density of $O(G^{-2})$. Our theory of isolated elementary particles is based on non-linear self-consistent solutions of the field equations of QFT. We intend to publish this work in the near future. However, for now we just use the hypothesis that the physical particles are characterized by the Planck length. The creation of particles out of the vacuum is likely to require a matching of the energy density of the vacuum universe and the particle energy density. This occurs at a time t_c given by:

$$\epsilon\sqrt{{}^3g(t_c)} = \epsilon\frac{t_s^3}{t_c^3} = G^{-2}, \tag{57}$$

leading to:

$$t_c = \left(\frac{G}{\epsilon}\right)^{1/6}\left(\frac{3}{8\pi}\right)^{1/2} \approx 5\times 10^{-24}\text{sec} = (125\text{MeV})^{-1}\ . \tag{58}$$

In Eq. (58) we ignored the modification of the metric due to the presence of the created matter. From Eq.(25) we see that $h(x)$ will rapidly increase with the creation of particles, in particular as the volume $\hat{V}_s$ will decrease to compensate for the energy increase resulting from particle formation and from the fact that physical particles now consume space. Hence, this will extend the creation epoch beyond t_c, as we will still satisfy Eq.(57) for some time after t_c, if we replace ${}^3g$ by ${}^3\hat{g}$ and t_c by $\hat{t}_c$, where $\hat{t}_c > t_c$. Since the large size of $h(x)$ invalidates perturbative calculations, a more extensive theoretical investigation is required to describe the later stages of this epoch in detail. Dimensional considerations indicate that the initial constraints on particle creation have a spatial nature, so that during this epoch available space must be divided homogeneously, thus, providing a possible explanation for the homogeneous nature of our universe. This initial epoch has both developmental and evolutionary aspects to it. The evolutionary label is best reserved for processes with a degree of randomness (where and when the particles are created). The quantum fluctuations responsible for these aspects are discussed later in this section. The developmental aspects cover the fact that the universe expands (develops) linearly with time and that the states into which it can develop are fixed by QFT and GR together.

Eq.(58) illustrates how the particle physics scale can arise from the two fundamental dimensionfull constants of Nature, ϵ and G, and gives credence to our expectation that a truly fundamental understanding of elementary particles requires consideration of GR. Since the equations of motion in QFT do not contain any fundamental dimensional constants, it is not unexpected that the particle physics scale in QFT only emerges when QFT is unified with GR. As part of our model of particle creation, we also suggest that a creation process in QFT mimicks the creation process of particles at the t_c epoch after the Big Bang. Such a mechanism is required in our theory of isolated elementary particles in order to stabilize the solution. The distortion of space resulting from the formation of a physical particle of Planck size must counter the collapse of the dressed particle to a singular point. This is roughly opposite to the situation of a black hole, where the metric induces rather than prevents the collapse. The creation terminates at the time t_c and thereby explains the typical creation time of elementary particles. The annihilation of a particle is the reverse process, again characterized by the same time t_c. Although many aspects of our particle theory are still under development, the possibility of being able to explain the nature of particles and to give an explanation of their masses and of their creation and annihilation properties is a very exciting prospect, indeed.

If these notions are confirmed after further development, we see a phenomenon in elementary particle physics that reminds us of a mechanism known in biology, namely that the development of a current entity recapitulates a (series of) historical process(es). In biology these processes are called ontogeny (the development of an organism) and phylogeny (ancestor-dependent relationships in a group) and the biogenetic law to which we refer states that ontogeny recapitulates phylogeny (Haeckel [29]). In the physics analogy the historical process would be the creation of particles from the vacuum at the appropriate epoch t_c, whereas the current process of creation repeats this process as part of the full process of particle creation. The epoch at t_c is associated with an increase in entropy and is irreversible, whereas the current physical process of particle creation does not increase entropy and is reversible (annihilation is the reverse process). Of course, the analogy with biology is to a large extent symbolic, still apart from the fact that Haeckel's law itself has a very limited range of validity in biology and has been severely criticized [30]. Nonetheless it is gratifying that Nature finds ways of expressing similar mechanisms under very different circumstances. To emphasize the limitations of this principle, we note that the annihilation of particles does not have a corresponding epoch in the development of the universe, unless the universe were to die in a big crunch in which particles are converted back into vacuum energy. This would require the decrease of entropy in order to return to a state of zero entropy and would violate thermodynamic laws.

It should be noted that the derivation of the particle physics scale only is only valid for our particular vacuum metric, confirming again the unique role of the current vacuum solution. The result is also contingent on physical elementary particles being three dimensional (spherical) objects, and therefore cannot be derived in the common form of string theory. Within our picture the creation epoch starts much later than the Planck time, which is often considered to be the critical time period for events near the Big Bang. In this way we have avoided the difficult question of the unification of GR and QFT at the Planck scale, although this question returns in a more controlled form in the treatment of elementary particles of size $G^{1/2}$ and energy density G^{-2}. It should also be noted that the particle physics scale $(t_s/t_0) \times t_c$, rather than t_c, is measured at present. An accurate model of particle properties will therefore give information on the ratio t_s/t_0, and thus on t_0. Since t_c is of the order of the (currently) measured particle scale, we conclude that t_0 is currently of the same order of magnitude as t_s. Hence, we are living in a time and age t_0 characterized by the typical cosmological time unit t_s.

Naturally this is a rather qualitative statement as there is a large difference between the hadronic and the leptonic scale, making the definition of the particle physics scale rather uncertain. If we go back to the time $t = t_c$ when the first particles were created, then the measured age of the universe would still be t_s. However, the measured particle properties would be characterized by an interaction time $(t_s/t_c) \times t_c = t_s$, which is comparable to the age of the universe. Hence, the changing factor t_s/t_0 ensures that the universe proceeds through different physical epochs.

Since the first creation process of physical particles takes place in a homogenous vacuum universe, we would expect the created particles to be distributed homogenously, disturbed only slightly by the quantum fluctuations and randomness of the quantum processes that created them. It is only through these random processes that we can break the initial perfect symmetry and form increasingly complex and diverse structures, allowing an increase in the value of the entropy. Such a creation of entropy is discussed elsewhere in the literature [26]. Initially the linear expansion of space will be halted - or even reversed - when the creation of particles increases the mass terms in the energy balance, an energy increase which has to be matched by a corresponding decrease in the total vacuum energy, i.e. a decrease in $\hat{V}_s$. However, after the initial creation of particles and anti-particles, we would expect an inflationary period, when most of the particle-anti-particle pairs annihilate. These processes destroy most of the initial mass energy and the induced matter-vacuum energy, leaving only a small residue of "particles" and converting some of the energy into radiation and its associated negative mixed radiation-vacuum energy. To compensate for this energy loss the universe would have to expand very rapidly in a short time (an inflationary phase). Clearly, this inflationary period has an origin and nature quite different from that considered in currently popular inflationary scenarios. A phase of extremely quick inflation does not seem to be required in the current theory to explain the uniformity of the temperature distribution in the universe, as the infinite horizon in our description allows particles to interact over much larger distances than in the standard picture.

Since the current universe contains only a relatively small percentage of matter, we would expect that after these violent processes have been essentially completed, the universe would return to a state in which the vacuum energy dominates and the expansion is dominated by the linear trend. However, the effective density will still be huge initially, because of the factor t_s^3/t^3, as we saw in (35). Hence, we expect that the usual hot Big Bang phase, which is responsible for primordial nucleosynthesis, can be derived in the usual way, although further study is required to confirm this in detail. There are other characteristic epochs in the evolution of the universe which can be characterized in terms of G and ϵ. For example the epoch that the vacuum energy equals the particle physics scale is characterized by:

$$\epsilon\sqrt{^3g(t_N)} = \epsilon\frac{t_s^3}{t_N^3} = t_c^{-4}, \tag{59}$$

leading to a time $t_N = \epsilon^{-7/18}G^{-5/18} \approx 8$ hrs.

Finally we discuss the observation of early galaxies. In FRW calculations one usually employs a $t^{2/3}$ expansion for the early universe. Using this type of time scale, Bouwens *et al* [31] conclude from certain Hubble observations that the first galaxies were formed about 900 Myr after the Big Bang. Similar conclusions were reached by a Japanese group [32], which found early galaxies dating from 750 Myr after the Big Bang. By demanding that these events take place with the same value of z as they do in the analysis by these authors, we find that in our theory the formation of the early galaxies would take place 1.9 and 1.7 billion years respectively after the Big Bang. Although the creation and annihilation events in the early universe might modify these estimates slightly, the net result is that the early galaxies were formed much later than claimed by the authors above, reducing the mystery of the early formation of galaxies.

11. Summary and Concluding Remarks

We have solved the standard equations of general relativity for the vacuum with a "classical" vacuum energy density. We have shown that this leads to a Big Bang solution with an associated linear expansion of the universe, even after the introduction of matter and radiation. The contributions of matter and radiation to the total energy and the distortions of the metric are effectively constant under this linear expansion. Deviations from this basic behaviour, which is controlled by total energy conservation, can appear through creation and annihilation processes. This model can explain many crucial observations of the universe without the need to introduce new variants of the basic theory of general relativity or extensions beyond the Standard Model. In particular the cosmological constant problem and the horizon problem are absent in this approach. The evolution of this universe proceeds from a classical beginning with perfect spatial symmetry and zero entropy to a diverse and complex future thanks to quantum fluctuations. Although, various details of this picture still have to be worked out, the initial results are very promising.

The abandonment of the RW formalism necessitates a reassessment of various properties of the universe, such as its matter and radiation content. Improved supernovae data will impose strong constraints on the current model and on the nature and intensity of the decay processes in the universe (mainly radiative processes), as in our theory, the latter are seen as the cause of the current acceleration of the expansion of the universe. The explanation we suggest for the observation of "dark matter" should also be studied further, since it will be affected by higher order effects that are not considered in this paper.

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