The Effect Of Temporal Dependence And Seasonality On Return Level Estimates Of Excessive Rainfall

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Outline

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Study Area

Figure: Satellite image of Cape Town

Problem Statement

Figure: Informal settlements are vulnerable to flooding

The need for Extreme Value Theory

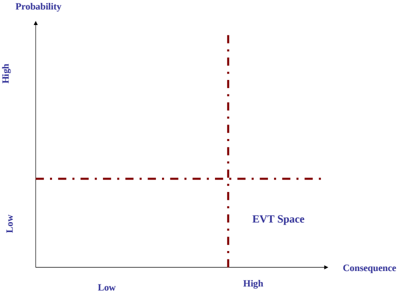

Figure: The domain of application Extreme Value Theory

Outline

The Classical Approach

Assume X_i is a sequence of independent random variables with common distribution F. Without any knowledge of F, a model exists that describes the behaviour of the largest (or smallest) member of the sample

$$M_n = \max(X_1, X_2, \ldots, X_n).$$

Conditional on the existence of $\{a_n\}$ and $\{b_n\} > 0$, the *Fisher-Tippett theorem* states that the re-scaled sample maxima (or minima) converges in distribution to the Generalized Extreme Value (GEV) family of distributions

$$P\left(\frac{M_n - a_n}{b_n} \leq x\right) \longrightarrow \begin{cases} \exp\left(-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right) & 1+\xi\frac{x-\mu}{\sigma} > 0,\ \xi \neq 0 \\ \exp\left(-\exp\left(-\frac{x-\mu}{\sigma}\right)\right) & x \in \mathbb{R},\ \xi \to 0 \end{cases} \tag{1}$$

where $-\infty < \mu < \infty$ and $\sigma > 0$.

From the Classical to the Threshold Exceedance Approach

An important consideration in classical EVT is the choice of block size n.

- Affects the trade-off between bias and variance, i.e. choice between accuracy or precision.

Criticism about the classical approach is that it is wasteful of data.

- Using only one observation per block, discarding the rest.

Alternative approach is the *threshold exceedance* approach.

- Essentially finding an approximate distribution for the series of excesses of a particular level (the threshold).

Approximate Distribution for Threshold Exceedances

Denote X_i by X. Suppose for large n, the Fisher-Tippett theorem holds. Then, for suitable threshold u,

$$P(X - u \leq y | X > u) \sim G(y; \sigma_u, \xi) = 1 - \left(1 + \xi \frac{y}{\sigma_u}\right)^{-\frac{1}{\xi}} \quad (2)$$

defined on $\{y : y > 0 \text{ and } \left(1 + \xi \frac{y}{\sigma_u}\right) > 0\}$, with

$$\sigma_u = \sigma + \xi(u - \mu) \quad (3)$$

- $G(\cdot)$ defines the Generalized Pareto distribution (GPD).

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The Return Level Parameter

The return level is that level of the process, which we expect to be exceeded on average once every N years.

$$q_N = \begin{cases} u + \frac{\sigma_u}{\xi}\left[(\lambda N)^{\xi} - 1\right] & \xi \neq 0 \\ u + \sigma_u \log(\lambda N) & \xi = 0 \end{cases} \quad (4)$$

Exceedance process is assumed to be Poisson with rate λ (per year), estimated by $\hat{\lambda} = m/n$.

Issues to Consider

- The data are often incomplete due to measuring instrument failure, relocation of measuring sites, etc.
- Careful consideration has to be taken in selecting the threshold.
- The length of the data that is available is often shorter than the prediction horizon.

Outline

Data

Rainfall data from Cape Town International Airport ($-33°97'$S, $18°60'$E, 44 m altitude).

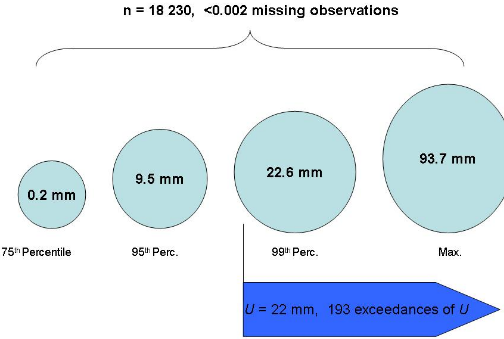


Figure: Description of the rainfall data

Data

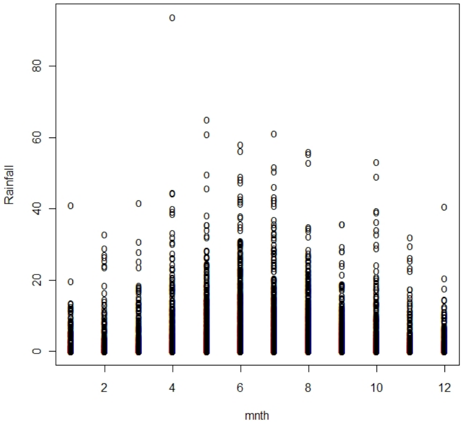


Figure: Rainfall series by month (1958-2007 Cape Town Int)

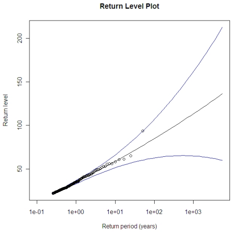
Results

The estimated extremal index $\hat{\theta} = 0.78(0.69, 0.89)$

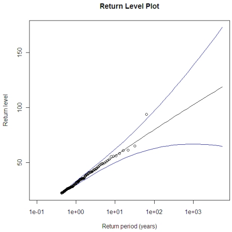
Model (m)	$\hat{\sigma}$(s.e.)	$\hat{\xi}$(c.i.)	$\hat{q}_{25}$	$\hat{q}_{50}$
GP0 (193)	9.37 (0.98)	0.04 (-0.08,0.22)	68.91 (60.36,83.41)	76.79 (65.50,93.72)
GP1 (149)	11.17 (1.25)	-0.02 (-0.14,0.17)	68.69 (58.65,80.09)	72.81 (63.83,89.55)
GP2 (144)	9.74 (1.19)	0.05 (-0.09,0.26)	67.06 (58.38,83.23)	75.47 (63.86,94.65)
GP3 (31)	11.72 (2.95)	-0.26 (-0.61,0.22)	45.07 (40.07,56.81)	48.74 (42.95,67.99)

Table: Parameter estimates and the accompanying measures of uncertainty

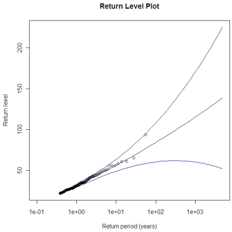
Return Level Plots



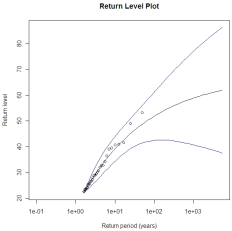
(a) Annual Rainfall (assuming independence)



(b) Annual Rainfall (assuming dependence)



(c) Annual Winter Rainfall



(d) Annual Summer Rainfall

Conclusion

- Clustering extremes should be appropriately treated to ensure better accuracy - especially for long-range return level estimation.
- Where the series shows strong seasonal signal, more insight can be gained by analyzing the behaviour of the extremes of the process for each season.
- To understand the extreme rainfall patterns in the region, extension of the analysis to nearby sites is necessary.

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Acknowledgements

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