Performance Analysis of Super-Orthogonal Space- Frequency Trellis Coded –OFDM System.

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Abstract

This paper presents the performance analysis of Super-Orthogonal Space-Frequency trellis coded OFDM (SOSFTC-OFDM) system using the Gauss-Chebyshev Quadrature technique. SOSFTC-OFDM is a form of the super-orthogonal space-time trellis code that is used with OFDM. SOSFTC-OFDM utilizes the diversities in frequency and space domain by assuming that coding is done along adjacent subcarrier in an OFDM environment. This paper evaluates the exact pairwise error probability (PEP) of the SOSFTC-OFDM system based on the Gauss-Chebyshev Quadrature formula and used the PEP to calculate the approximated average bit error probability. Comparing the calculated average bit error probability to the simulated our results shows 0.2dB SNR deviation for 10^{-2} average bit error probability at error event of length 3..

1. Introduction

Space-time coding techniques are bandwidth and power efficient methods of communication over fading channels. It combines the design of channel coding, modulation, transmit diversity and, receive diversity. Some of the basic examples of space-time codes includes space-time trellis code (STTC) [1], space-time block code (STBC) [2] and super-orthogonal space-time trellis code (SOSTTC) [3]. SOSTTC is a new class of space-time code that combines the set partitioning and a super set of orthogonal space-time block codes in a systematic way to provide full diversity and improved coding gain when compared with earlier space-time trellis construction [1-2]. The orthogonal transmission matrix used in the design of SOSTTC is given as:

$$\mathrm{A}\ (s_1, s_2, \theta) = \begin{pmatrix} s_1 e^{j\theta} & s_2 \\ -s_2^* e^{j\theta} & s_1^* \end{pmatrix}. \qquad (1)$$

For an M-Phase Shift Keying (PSK) modulation with constellation signal represented by $s_i \in \exp(j2\pi a/M)$, i =1,2. , a= 0, 1, …, M-1; one can pick θ = 2πá/M, where á= 0,1, …, M-1.

Although space-time coding schemes were designed for non-frequency selective fading channels, their performance in frequency selective channels have been an ongoing research area. Space-time coding schemes in a frequency selective channel results in severe performance degradation as a result of intersymbol interference (ISI) [4].

Orthogonal frequency division multiplexing (OFDM) technique transforms a frequency selective channel into parallel frequency non-selective subchannels and eliminates the ISI caused by the multipath. The performance of various space-time coding schemes in OFDM systems have been investigated in the literatures [5-6].

In [5], space-time trellis coded OFDM systems with no interleavers over quasi-static frequency selective fading channel was considered. The performance of the code was analyzed under various channel conditions in terms of the coding gain. The work points out that the minimum determinant of the space-time coded OFDM systems increases with the maximum tap delay of the channel, thereby causing an increase in the coding gain of the coded OFDM.

In [6] a full-rate robust Super-orthogonal space-time trellis coded OFDM system was introduced. The scheme provides multipath diversity in wideband-frequency selective fading channels in addition to space and time diversity. The paper assumes that the channel remains constant during the transmission of the orthogonal matrix.

Another possibility of coding as hinted in [7] for a block coding schemes in an OFDM environment is to assume that the channel frequency response is identical across the N_t adjacent subcarrier. We introduce this type of scheme for a super set of orthogonal block codes, as Super-Orthogonal Space-Frequency Trellis Coded (SOSFTC) OFDM system.

In [8], the closed form expression for the average bit error rate (BER) of space-frequency block coded OFDM systems were derived using the instantaneous BER at each channel subchannel and assuming Gray bit-mapping.

In this paper, the approximated expression of the average bit error probability is given using the derived closed form expression for the pairwise error probability (PEP) of a SOSFTC-OFDM system using the Gauss-Chebyshev

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Quadrature technique [9]. The calculated average bit error probability shows good correlated with simulated average bit error probability**.**

The paper is organized as follows. In Section 2, the system model of the SOSFTC-OFDM is discussed. In Section 3, we describe the derivation of the PEP using the Gauss-Chebyshev Quadrature technique and also give numerical examples. In Section 4, we use the PEP obtained in Section 3 to give an approximated expression for the average bit error probability for a quasi-static frequency selective fading channel. In Section 5 a brief discussion on the results obtained from the numerical example of both the PEP and the average bit error probability is given while Section 6 concludes the paper.

2. System Model

An OFDM transmission system with N_t transmit antennas, N_r receive antennas and N subcarriers is considered.

Each transmitted frame consists of $N*N_t$ M-PSK SOSFTC block of codes with each block consisting of N encoded symbols. Let $s_i(n)$ represent the symbol transmitted from subcarrier n ($n \in \{1,2,\ldots,N\}$) at the i^{th} transmit antenna.

After matched filtering, sampling and fast Fourier transform (FFT), the received signal at the j^{th} received antenna is given by:

$$\mathbf{R}_j = \sum_{i=1}^{N_t} \mathbf{G}_{ij}\mathbf{S}_i + \tilde{\mathbf{N}}_j, \qquad (2)$$

where $\mathbf{G}_{ij} = [\,\boldsymbol{G}_{ij}(1), \boldsymbol{G}_{ij}(2), \boldsymbol{G}_{ij}(3),\ldots, \boldsymbol{G}_{ij}(N)]^T$ consist of the set of channel frequency response $G_{ij}(n)$ from the i^{th} transmit antenna to the j^{th} receive antenna for the nth subcarrier, $\mathbf{R}_j = [\,r_j(1), r_j(2), r_j(3),\ldots, r_j(N)]^T$ consist of the received vectors at different subcarriers and $\tilde{\mathbf{N}}_j = [\,\eta_j(1), \eta_j(2), \eta_j(3),\ldots, \eta_j(N)]^T$ consist of the noise component $\eta_j(n)$ at the receive antenna j and subcarrier n. The noise components are independently identical complex Gaussian random variables with zero-mean and variance $N_o/2$ per dimension.

The time domain channel impulse representation between the i^{th} transmit antenna and the j^{th} receive antenna can be modeled as a L tapped-delay line. The channel response at time t with delay τ_s can be expressed as:

$$g_{ij}(\tau_s, t) = \sum_{l=0}^{L-1} \hat{g}_{ij}(l, t)\partial\left(\tau_s - \frac{n_l}{N\Delta f}\right), \qquad (3)$$

where $\partial(\cdot)$ is the Kronecker delta function, L denotes the number of non-zero taps, $\hat{g}_{ij}(l, t)$ is the complex amplitude of the l^{th} non-zero tap with delay of $n_l/N\Delta f$, n_l is an integer and Δf is the tone spacing of the OFDM system. In (3), $\hat{g}_{ij}(l, t)$ is modeled by the wide-sense stationary (WSS) narrowband complex Gaussian processes with power $E[|\hat{g}_{ij}(l, t)|^2] = \sigma_l^2$, which is normalized as $\sum_l^{L-1} \sigma_l^2 = 1$.

For an OFDM system with proper cyclic prefix, prefer sampling, the channel response is expressed as:

$$\boldsymbol{G}_{ij}(n) = \sum_{l=0}^{L-1} \hat{g}_{ij}(l)\ exp(-j2\pi \cdot n \cdot n_l/N)$$
$$= \mathbf{g}_{ij}\, \boldsymbol{w}(n), \qquad (4)$$

where $\mathbf{g}_{ij} = [\,\hat{g}_{ij}(0), \hat{g}_{ij}(1), \hat{g}_{ij}(2), \hat{g}_{ij}(3), \ldots, \hat{g}_{ij}(L-1)]^T$ is the channel vector with narrowband zero-mean complex Gaussian processes and the FFT coefficient is $\boldsymbol{w}(n) = [e^{-j2\pi n n_0/N}, e^{-j2\pi n n_1/N}, \ldots, e^{-j2\pi n n_{L-1}/N}]^T$.

3. Pairwise Error probability of SOSFTC-OFDM Systems

3.1. Mathematical Analysis

For the PEP analysis we consider a SOSFTC-OFDM system with two transmit antenna ($N_t = 2$), using the orthogonal transmission matrix given in (1). The received signal at the j^{th} receiver antenna is given by:

$$\mathbf{R}_j = \mathbf{G}_{1j}\mathbf{S}_1 + \mathbf{G}_{2j}\mathbf{S}_2 + \tilde{\mathbf{N}}_j, \qquad (5)$$

where the super-orthogonal block codes for the two transmit antenna is written in equation (6) below as:

$$\mathbf{S}_1 = [s_1(1), s_1(2), s_1(3),\ldots, s_1(N)\,]^T$$
$$= [s(1)e^{j\theta}, -s^*(2)e^{j\theta}, s(3)e^{j\theta}, -s^*(4)e^{j\theta}, \ldots, s(N-1)e^{j\theta}, -s^*(N)e^{j\theta}\,]^T$$
$$\mathbf{S}_2 = [s_2(1), s_2(2), s_2(3),\ldots, s_2(N)\,]^T$$
$$= [s(2), s^*(1), s(4), s^*(3), \ldots, s(N), s^*(N-1)]^T \qquad (6)$$

To evaluate the PEP of a SOSFTC-OFDM scheme i.e. the probability of choosing the codeword $\hat{\mathbf{S}} = [\hat{s}(1), \hat{s}(2), \hat{s}(3), \hat{s}(4), \hat{s}(5), \ldots, \hat{s}(N)]$, where $\hat{s}(n) = [\hat{s}_1(n), \hat{s}_1(n)]$, when in fact the codeword $\mathbf{S} = [s(1), s(2), s(3), s(4), s(5), \ldots, s(N)]$, where $s(n) = [s_1(n), s_1(n)]$ was transmitted, the maximum likelihood metric corresponding to the correct and the incorrect path will be used. The metric corresponding to the correct path and the incorrect path is given in equation (7) and (8) respectively.

$$m(r, \mathbf{S}) = ||\mathbf{R}_j - (\mathbf{G}_{1j}\mathbf{S}_1 + \mathbf{G}_{2j}\mathbf{S}_2)||^2. \qquad (7)$$

$$m(r, \hat{\mathbf{S}}) = ||\mathbf{R}_j - (\mathbf{G}_{1j}\hat{\mathbf{S}}_1 + \mathbf{G}_{2j}\hat{\mathbf{S}}_2)||^2. \qquad (8)$$

The realization of the PEP over the entire frame length and for a given channel frequency response is given in equation (9).

$$\mathrm{P}(\mathbf{S} \rightarrow \hat{\mathbf{S}} \,|\, \mathbf{G}) = \Pr\{\, m(r, \mathbf{S}) > m(r, \hat{\mathbf{S}})\}$$
$$= \Pr\{\, (m(r, \mathbf{S}) - m(r, \hat{\mathbf{S}})) > 0\} \qquad (9)$$

Simplifying (7) and (8) and substituting it in (9) gives equation (10)

$$\mathrm{P}(\mathbf{S} \rightarrow \hat{\mathbf{S}} \,|\, \mathbf{G}) = \Pr\{\, ||\,\mathbf{G}_{1j}(\mathbf{S}_1 - \hat{\mathbf{S}}_1)\,||^2 + ||\,\mathrm{G}_{2j}(\mathbf{S}_2 - \hat{\mathbf{S}}_2)\,||^2\,\}$$
$$= \Pr\{||\mathbf{G}_j\,\mathbf{\Delta}||^2 > 0\}, \qquad (10)$$

where $\mathbf{G}_j = [\mathbf{G}_{1j}\ \mathbf{G}_{2j}]$, $\mathbf{\Delta}$ is the block codeword matrix that characterize the SOSFTC-OFDM system and ||.|| stands for the norm of the matrix element.. The expression of $\boldsymbol{\Delta}$ is given in equation (11).

$$\boldsymbol{\Delta} = \begin{bmatrix} \mathbf{S}_1 - \hat{\mathbf{S}}_1 \\ \mathbf{S}_2 - \hat{\mathbf{S}}_2 \end{bmatrix} \tag{11}$$

The conditional PEP given in (10) can be expressed in term of the complementary error function [9] as:

$$\mathrm{P}\left(\mathbf{S} \to \hat{\mathbf{S}} \middle| \mathbf{G}\right) = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_s}{4N_0}\sum_{j=1}^{N_r}\mathbf{G}_j\boldsymbol{\Delta\Delta}^H\mathbf{G}_j^H}\right) \tag{12}$$

The function $\boldsymbol{\Delta\Delta}^H$ is a diagonal matrix of the form shown in (13) below and $(\mathbf{.})^H$ represents the conjugate transpose of the matrix element and E_s/N_o stands for the symbol signal-to-noise ratio,

$$\boldsymbol{\Delta\Delta}^H = \begin{bmatrix} \Delta(1)(\Delta(1))^H & 0 & \cdots & 0 \\ 0 & \Delta(2)(\Delta(2))^H & & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & \Delta(N)(\Delta(N))^H \end{bmatrix}. \tag{13}$$

The diagonal element of (13) and further expansion of $\mathbf{G}_j$ is given below.

$$\Delta(n) = \begin{bmatrix} s_1(n) - \hat{s}_1(n) \\ s_2(n) - \hat{s}_2(n) \end{bmatrix} \tag{14}$$

$$\begin{aligned} \mathbf{G}_j &= \begin{bmatrix} \mathbf{G}_{1j} & \mathbf{G}_{2j} \end{bmatrix} \\ &= \begin{bmatrix} \hat{g}_{1j}(0) \cdots \hat{g}_{1j}(L-1) & \hat{g}_{2j}(0) \cdots \hat{g}_{2j}(L-1) \end{bmatrix}_{1 \times LN_t} \\ &\quad * \begin{bmatrix} w(n) & 0 & \cdots & 0 \\ 0 & w(n) & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w(n) \end{bmatrix}_{LN_t \times N_t}, \\ &= [\mathbf{g}_{1j} \ \ \mathbf{g}_{2j}]\mathbf{W}(n). \end{aligned} \tag{15}$$

A complementary error function, as defined integrally in [10, 7.4.11] is given by,

$$\operatorname{erfc}(b) = \frac{2}{\pi}\int_0^{\infty} \frac{e^{-b^2(t^2+1)}}{t^2+1}\, dt \tag{16}$$

Expanding (12) using (16), the conditional PEP can be expressed as an integral. Thus, with $E(x)$ denoting the average of x, one gets:

$$\mathrm{P}\left(\mathbf{S} \to \hat{\mathbf{S}}\right) = \frac{1}{\pi}E\left[\int_0^{\infty} \frac{\exp\left[-(t^2+1)\frac{E_s}{4N_0}\sum_{j=1}^{N_r}\mathbf{G}_j\boldsymbol{\Delta\Delta}^H\mathbf{G}_j^H\right]}{t^2+1}dt\right]. \tag{17}$$

The above expression (17) can be simplified further using the results in [11].

For a complex circularly distributed Gaussian random row vector z with mean μ and covariance matrix $\sigma_z^2 = E[zz^*]-\mu\mu^*$, and a Hermitian matrix $\mathbf{M}$, we have:

$$E_z\left[\exp\left(-z\mathbf{M}(z^*)^T\right)\right] = \frac{\exp\left[-\mu\mathbf{M}\left(\mathbf{I}+\sigma_z^2\mathbf{M}\right)^{-1}(\mu^*)^T\right]}{\det\left(\mathbf{I}+\sigma_z^2\mathbf{M}\right)}. \tag{18}$$

where $\mathbf{I}$ is an identity matrix. Applying (18) and also using (15) in solving (17), (19) is obtained.

Knowing that $z = [\mathbf{g}_{1j} \ \mathbf{g}_{2j}]$, $\mathbf{M} = -(t^2+1)E_s / 4N_o\mathbf{W}(n)\Delta(n)(\Delta(n))^H(\mathbf{W}(n))^H$ (it should be noted that since $\mathbf{W}(n)\Delta(n)(\Delta(n))^H(\mathbf{W}(n))^H$ is a diagonal matrix, $\mathbf{M}$ is an Hermitian matrix i.e. $\mathbf{M} = \mathbf{M}^T$), $\mu=0$ ($[\mathbf{g}_{1j} \ \ \mathbf{g}_{2j}]$ has Rayleigh distribution) and $\sigma_z^2 = \sigma^2_{[\mathbf{g}_{1j} \ \mathbf{g}_{2j}]} = \mathbf{I}_{LN_t}$

$$\mathrm{P}\left(\mathbf{S} \to \hat{\mathbf{S}}\right) = \frac{1}{\pi}\int_0^{\infty} \frac{1}{t^2+1}\prod_{j=1}^{N_r} \frac{1}{\det\left[\mathbf{I}_{LN_t} + \frac{E_s}{4N_0}A(t^2+1)\right]_j}\, dt \tag{19}$$

$$A = \sum_{n=1}^{N}\mathbf{W}(n)\Delta(n)(\Delta(n))^H(\mathbf{W}(n))^H$$

To solve (19), an integral equation is given by (20) considered.

$$I = \frac{1}{\pi}\int_0^{\infty} \frac{1}{t^2+1}\mathrm{f}\left(t^2+1\right)dt \tag{20}$$

where,

$$\mathrm{f}\left(t^2+1\right) = \prod_{j=1}^{N_r} \frac{1}{\det\left[\mathbf{I}_{LN_t} + \frac{E_s}{4N_0}A\left(t^2+1\right)\right]_j}. \tag{21}$$

Substituting $y = 1/t^2+1$ into (20), (20) becomes (22).

$$I = \frac{1}{2\pi}\int_0^1 \frac{1}{\sqrt{y(1-y)}}\mathrm{f}\left(1/y\right)dy \tag{22}$$

The above equation (22) is in the orthogonal polynomial form as given in (23) [10, 25.4.38] and Gauss-Chebyshev Quadrature technique of first kind can be used to solve it.

$$\int_{-1}^{1} \frac{\mathrm{f}(u)}{\sqrt{1-u^2}}du = \sum_{i=1}^{m} B_i\,\mathrm{f}(u_i) + \mathrm{V}_m \tag{23}$$

where

$$u_i = \cos\frac{(2i-1)\pi}{2m},$$

$$B_i = \frac{\pi}{m},$$

$$\mathrm{V}_m \leq \max_{-1<u<+1} \frac{\pi}{(2m)!2^{2m-1}} \left| \mathrm{f}^{2m}(u) \right|.$$

The expression in (22) can take the form of (23) if $2y-1=u$, then

$$2y-1 = \cos\frac{(2i-1)\pi}{2m},$$

$$2y = \cos\frac{(2i-1)\pi}{2m}+1,$$

$$1/y = \sec^2\frac{(2i-1)\pi}{4m} \quad . \tag{24}$$

Therefore

$$I = \sum_{i=1}^{m} w_i \mathrm{f}(u_i) + \mathrm{V}_m$$
$$= \frac{1}{2m}\sum_{i=1}^{m} \mathrm{f}\left(\sec^2\frac{(2i-1)\pi}{4m}\right) + \mathrm{V}_m \tag{25}$$

The closed-form expression of the PEP for SOSFTC-OFDM system using the Gauss-Chebyshev Quadrature formula as enumerated above is now given by (26).

$$\mathrm{P}(\mathbf{S}\rightarrow\hat{\mathbf{S}}) = \frac{1}{2m}\sum_{i=1}^{m}\prod_{j=1}^{N_r}\frac{1}{\det\left[\mathbf{I}_{LN_T}+\frac{E_s}{4N_0}A\sec^2\frac{(2i-1)\pi}{4m}\right]_j} + \mathrm{V}_m \tag{26}$$

As m (which is the order of the polynomial i.e. f (u_i) and also function of degree of precision ≡ $2m$-1) increases the remainder term V_m becomes negligible.

3.2 Numerical Example

A 2-state BPSK trellis given in [12] is used as an example. In our example we assume that L=2, N_r =1 and that the OFDM system has a bandwidth of 1MHz and 256 OFDM subcarrier (N=256). The tone spacing of the OFDM system is 3.9 KHz. Based on this assumption $W(n)$ is given in (27).

$$\mathbf{W}(n)=\begin{bmatrix} 1 & 0 \\ w(n) & 0 \\ 0 & 1 \\ 0 & w(n) \end{bmatrix} \tag{27}$$

The different values of A for the PEP expression of the SOFTC-OFDM system is given below for error event of 1 and 2 .

For Error event of 1, A in (26) is given as A_1.

$$A_1 = \begin{bmatrix} 1 & 0 \\ w(1) & 0 \\ 0 & 1 \\ 0 & w(1) \end{bmatrix}\begin{bmatrix} 2 \\ 2 \end{bmatrix}\begin{bmatrix} 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & w(1) & 0 & 0 \\ 0 & 0 & 1 & w(1) \end{bmatrix} \tag{28}$$

For error event of 2, A in (25) is given as A_2.

$$A_2=\mathbf{K}_1+\mathbf{K}_2+\mathbf{K}_3+\mathbf{K}_4 \tag{29}$$

where:

$$\mathbf{K}_1 = \begin{bmatrix} 1 & 0 \\ w(1) & 0 \\ 0 & 1 \\ 0 & w(1) \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix}\begin{bmatrix} 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & w(1) & 0 & 0 \\ 0 & 0 & 1 & w(1) \end{bmatrix} \tag{30}$$

$$\mathbf{K}_2 = \begin{bmatrix} 1 & 0 \\ w(2) & 0 \\ 0 & 1 \\ 0 & w(2) \end{bmatrix}\begin{bmatrix} -2 \\ 0 \end{bmatrix}\begin{bmatrix} -2 & 0 \end{bmatrix}\begin{bmatrix} 1 & w(2) & 0 & 0 \\ 0 & 0 & 1 & w(2) \end{bmatrix} \tag{31}$$

$$\mathbf{K}_3 = \begin{bmatrix} 1 & 0 \\ w(3) & 0 \\ 0 & 1 \\ 0 & w(3) \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix}\begin{bmatrix} 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & w(3) & 0 & 0 \\ 0 & 0 & 1 & w(3) \end{bmatrix} \tag{32}$$

$$\mathbf{K}_4 = \begin{bmatrix} 1 & 0 \\ w(4) & 0 \\ 0 & 1 \\ 0 & w(4) \end{bmatrix}\begin{bmatrix} -2 \\ 2 \end{bmatrix}\begin{bmatrix} -2 & 2 \end{bmatrix}\begin{bmatrix} 1 & w(4) & 0 & 0 \\ 0 & 0 & 1 & w(4) \end{bmatrix} \tag{33}$$

4. Average bit error probability of the SOSFTC-OFDM system

The average bit error probability is of greater interest than the PEP in analyzing the performance of a communication system. The average bit error probability of the SOSFTC-OFDM systems can be obtained by accounting for error event path up to a pre-determined specific value using equation (34).

$$P_b(E) \approx \frac{1}{h}\sum_{\mathbf{S}\neq\hat{\mathbf{S}}} q(\mathbf{S}\rightarrow\hat{\mathbf{S}})\mathbf{P}(\mathbf{S}\rightarrow\hat{\mathbf{S}}) \tag{34}$$

where h is the number of input bits per trellis transition and q($\mathbf{S}$ → $\hat{\mathbf{S}}$) is the number of bit errors associated with each error event. If the maximum length of error events taken into account is chosen as H, it is sufficient to consider the error event up to H; this represents a truncation of the infinite series used in calculating the union bound on the bit error probability for high SNR values. The choice of H is critical in the sense that most of the dominant error events for the range of SNR of interest should be taken into account by a proper choice while preventing excessive computational complexity (the computational complexity grows exponentially with H).

The PEP previously derived will be used to evaluate in closed form an approximation to the average bit error probability by accounting for error event equal to 3. If the all zero sequence was transmitted i.e. {(0,0)} , using the 2-state code in [12], there is a single error event path of length 1 i.e. {(0,1)} which has a PEP of type PEP_I and contribute only one bit in error. When accounting for error events of length 2 i.e. H = 2 and assuming that the all zero sequence was transmitted i.e. {(0,0)} and {(0,0)}, there are 4 error event paths i.e. {(1,0)(1,0), (1,0)(1,1), (1,1)(1,0) and (1,1)(1,1)} which have PEP of type PEP_{II} and they all contribute a total of 12 bits in error.

Also when accounting for error event of length 3 i.e. H = 3 and the all zero sequence was transmitted i.e. {(0,0), (0,0), (0,0)}, there are 8 error event paths i.e.{(1,0)(0,0)(1,0), (1,0)(0,0)(1,1), (1,0)(0,1)(1,0), (1,0)(0,1)(1,1), (1,1)(0,1)(1,0), (1,1)(0,1)(1,1), (1,1)(0,0)(1,0), (1,1)(0,0)(1,1)}. All these error event paths have PEP of type PEP_{III} and they contribute in total 28 bits in error

To approximate the average bit error probability by considering only the error event path of 1, 2 and 3 we use P_{b1}, P_{b2} and P_{b3} respectively.

$$P_{b1} \approx ½ (PEP_I) \quad (35)$$

$$P_{b2} \approx ½ (PEP_I + 12{*}PEP_{II}) \quad (36)$$

$$P_{b2} \approx ½ (PEP_{I} + 12{*}PEP_{II} + 28{*}PEP_{III}) \quad (37)$$

5. Performance results

The performance of the SOSFTC-OFDM system in a quasi-static frequency selective channel is evaluated by simulation and analysis using the same parameters stated in section 3.2. Figure 1 shows the PEP for of the scheme for various error event lengths using the derived closed form expression of the pairwise error probability of the 2-state SOSFTC-OFDM system. The graph shows that an increase in the error event (i.e. error event of length 3) gives a better PEP evaluation which corresponds to more accurate average bit error probability as show in Figure 2.

6. Conclusions

The paper derived a closed form expression of the pairwise error probability of a 2-state SOSFTC-OFDM system using Gauss Chebyshev Quadrature technique. The pairwise derived is use to approximate the BER of the SOSFTC-OFDM system. The approach used in this paper is different from the one use in [8] for space-frequency block code as our approach took into account the codeword matrix of the transmitted block. The method proposed in this paper can be used for other form of space-frequency block coded-OFDM schemes.

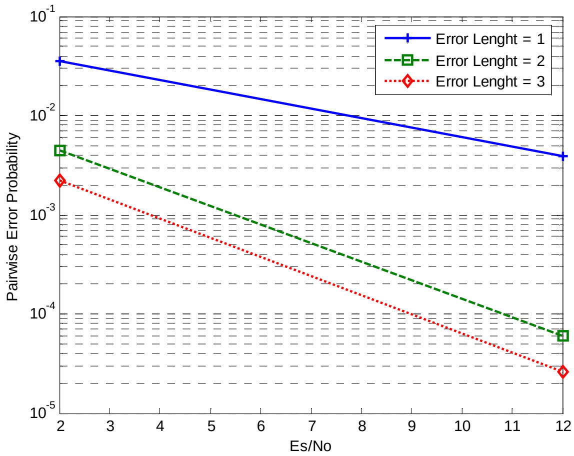

Figure 1: PEP performance of a 2-State BPSK SOSFTC-OFDM system.

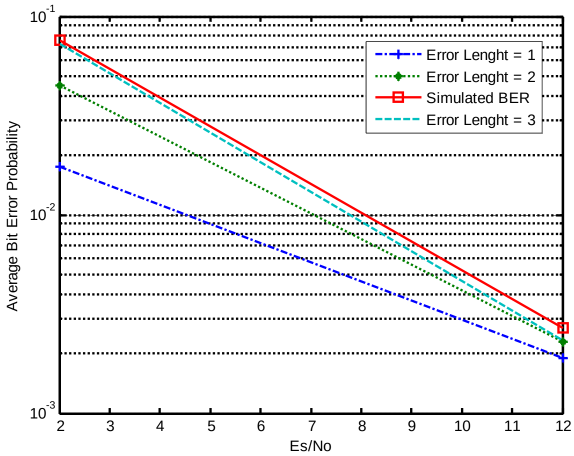

Figure 2: Average Bit error Probability 2-State BPSK SOSFTC-OFDM system.

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