

Modelling of Coil-loaded Wire Antenna Using Composite Multiple Domain Basis Functions

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Abstract— The paper discusses aspects of a novel impedance matrix compressing technique and applies the technique to a coil-loaded monopole. The technique can reduce the number of variables required for modeling of structures with curvatures and structures with electrically small features. The reduction in the number of unknowns is accomplished by a logical aggregation/grouping of the individual wire segments into equivalent continuous wires. A single composite basis function is applied over several wire segments. This decouples the number of unknowns from the number of geometrical segments. Aggregation of small features aims a reduction in the impedance matrix's condition number. The example of coil-loaded antenna has shown that the proposed novel algorithm achieves better accuracy with fewer unknowns than the traditional formulation of the method of moments.

1. INTRODUCTION

This work discusses aspects of and applies a novel impedance matrix compressing technique [1] to model a coil-loaded monopole antenna. The technique helps to reduce the number of variables required for modeling of structures with curvatures and structures with electrically small features. The presented realization of the technique assumes usage of piece-wise linear approximation of geometry. This approximation is seen as the core to inefficiency in modeling of the above-mentioned types of geometrical structures with a traditional method of moments (MoM). The reduction in the number of variables is accomplished by a logical aggregation/grouping of the individual straight wire segments into equivalent continuous wires with bends. This permits to apply a single basis function over several wire segments, and to decouple the number of unknowns from the number of geometrical segments. Aggregation of individual electrically small features also aims a reduction in the impedance matrix condition number [2]. Following the idea of aggregating domains of multiple segments, the basis functions used in the technique are herein referred to as multiple domain basis functions (MDBF). This name is also consistent with the terminology used in [11], and is an extension to it.

The technique is applied to the method of moments, under the thin wire approximation. The method used in this paper borrows the matrix form of expressions from [3] and develops it further, as to use a Galerkin approach [1, 10]. The method has many similarities with the macro and characteristic basis functions and related methods [4–6]. However, unlike this work, none of the references uses piecewise-linearly interpolated piecewise sinusoidal basis functions as the macro basis functions. In addition, in the main proposed domain of application, i.e., for smoothly bent structures, the technique proposed here requires fewer computations in comparison to the characteristic basis function, as the maximum electrical size covered by a single MDBF can usually be predicted and no solution of the localized systems is required to compose the set of new basis functions.

Section 2 of the paper describes the theoretical basis for the method. In the next section, the model of the coil-loaded monopole is analyzed. The validation of this numerical model is also presented there. Section 4 compares the developed approach to the traditional method of moments and provides a discussion on the results.

2. THEORY OF THE MULTIPLE-DOMAIN BASIS FUNCTIONS

The approach discussed in this paper involves the multiple domain basis functions (MDBF) [1], where a composite basis function aggregates one to several traditional basis functions. The technique is applied under the Galerkin procedure of the method of moments (MoM) as per [1]. This enhances robustness compared to the procedure derived in [3], where rooftop and pulse functions were used for both expansion and testing. The thin wire approximation [11] is used in the modeling. It may however be noted that the method is seen as equally applicable to the flat triangles [8], quadrilaterals or volumetric elements.

Mathematically, the procedure of obtaining the solution is as follows. It is assumed that the MoM procedure results in the set of linear algebraic equations $\mathbf{Z} \cdot \mathbf{I} = \mathbf{V}$, where \mathbf{Z} is the square impedance matrix, \mathbf{I} is the column vector of unknowns, and \mathbf{V} is the column vector describing excitations.

In applying the MDBFs, it is assumed that a relationship between a longer vector of original (old) unknowns \mathbf{I} and the shorter vector with new unknowns $\tilde{\mathbf{I}}$ exists, and may be written in a matrix form as $\mathbf{I} = \mathbf{M}\tilde{\mathbf{I}}$. Herein, \mathbf{M} denotes a matrix grouping/aggregating basis functions. Each row of this matrix contains weights defining which new basis functions are involved in the formation of the respective old basis functions, and with what weights.

The expression relating the old unknowns to the new ones may be substituted into the original system of linear equations $\mathbf{Z}\mathbf{I} = \mathbf{V}$. The resultant system $\mathbf{Z}\mathbf{M}\tilde{\mathbf{I}} = \mathbf{V}$ is then left-multiplied by the transposed transformation matrix \mathbf{M} to obtain the new system of linear equations: $\underbrace{\mathbf{M}^T\mathbf{Z}\mathbf{M}}_{\tilde{\mathbf{Z}}}\tilde{\mathbf{I}} = \underbrace{\mathbf{M}^T\mathbf{V}}_{\tilde{\mathbf{V}}}$.

This system may be rewritten in a short form as $\tilde{\mathbf{Z}}\tilde{\mathbf{I}} = \tilde{\mathbf{V}}$. Once this new system is solved and the new unknowns $\tilde{\mathbf{I}}$ obtained, the original unknowns may be computed from $\mathbf{I} = \mathbf{M}\tilde{\mathbf{I}}$.

The results of a MDBF based approach may be made equal to the results of a traditional MoM with the same original expansion functions, if the matrix \mathbf{M} is an identity matrix.

The validity of the technique may be limited when there is a strong feature present in a nearby current distribution which cannot be modeled with the chosen shape of the aggregating basis functions (such as piecewise linear or sinusoidal), like in [7]. This restriction is also characteristic for the global basis functions, where it leads to a poor convergence rate. A possible remedy to this problem within the proposed technique is to estimate the strength of the interactions from the values of the elements of the original impedance matrix in advance, and use this information to form/adjust the boundaries of the new MDBF basis functions. Such a solution can also apply to the macro and characteristic basis functions.

3. NUMERICAL MODEL DESCRIPTION

This paper discusses one of the examples considered in [9]. The choice was based on the complexity of the structure, and availability of measured reference data.

The geometry of the antenna is shown in Figure 1. The drawing shows the two straight wire segments joined by a helical coil. Both straight segments as well as the coil are modeled by straight thin wire sub-segments. The monopole is fed with a 1-Volt delta gap generator described in detail in [10, 11]. The generator was attached to the zero-radius end of a short wire, as seen in Figure 1. This aims to reduce the fringe capacitance problem [11] and improves accuracy of modeling.

The geometrical parameters for this example are as follows: number of turns = 8, length of the lower straight segment $L_{a1} = 15.02$ cm, length of the upper straight segment $L_{a2} = 6.68$ cm, length of the coil $L_c = 3.3$ cm, wire radius for all wires $a = 0.15$ cm, inner radius of the coil $a_c = 0.8$ cm. Unless stated otherwise, the frequency is 300 MHz. The modeled antenna includes two straight wire segments and a coil placed between them. The coil was modeled using from 3 up to 128 piecewise linear straight wire segments per turn.

Figure 1(b) shows a loaded dipole that was used to investigate some of the properties of the method. This dipole is equivalent to the monopole shown in Figure 1(a) and is made up of two such monopoles. It must be noted that the reason for using the dipole instead of monopole was of purely practical manner related to the easiest and quickest way of obtaining the impedance matrix's elements.

The numerical model was first verified using a traditional MoM with low and higher-order polynomial basis functions [10], where the low order functions are piecewise linear basis functions. This stage included a comparison of the frequency dependence and current distribution profiles against the experimental data from [9]. Figure 2 illustrates a part of the validation and displays an excellent match between the measurements and the numerical model built. The model validation procedures have confirmed quality of the numerical meshes and resultant models. The validation has also indicated the correctness of the original impedance matrix, critical for applying the method used in this paper.

The original fine mesh was aggregated using three different high level meshing approaches, namely algorithms *A*, *B*, and *C*, first published in [12]. The algorithm *A* iteratively, segment by segment, tries to aggregate the wire segments into new larger domains. This algorithm maximizes the size of each new domain, which sometimes may lead to undesirably small non-aggregated

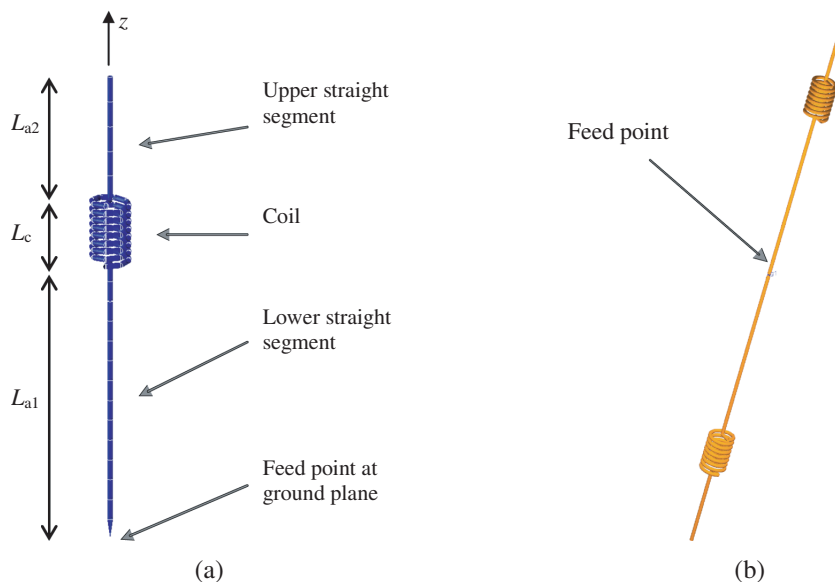


Figure 1: Geometry of the monopole loaded with coil (a), and (b) an equivalent dipole-like structure. Note: (i) The scales for the drawings in (a) and (b) are not the same. (ii) The radius of wire ends touching the feed point is zero for both monopole and dipole (may be especially difficult to see in Figure 1(b) due to the scale).

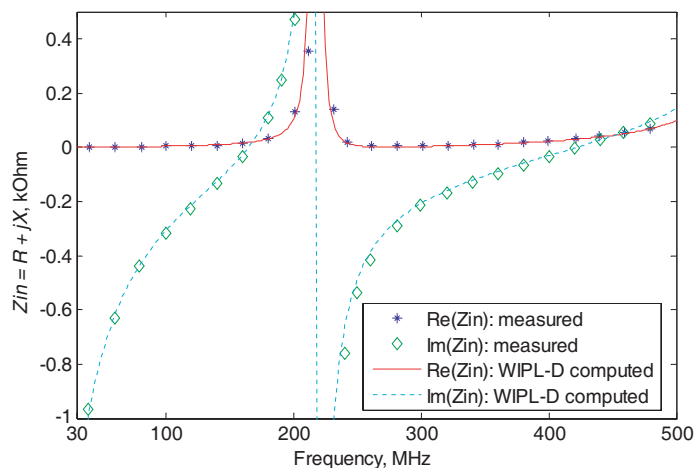


Figure 2: Input impedance of the coil loaded monopole antenna versus frequency, as computed by WIPL-D and measured in [9]. WIPL-D simulation was set to have a basis function based on a 2nd degree polynomial applied to each individual segment. WIPL-D model used eight straight wire segments per one turn of the coil.

segments. The algorithm *B* improves on this by considering two segments at a time. It reduces the probability of generating small non-aggregated segments but does not eliminate it fully. The algorithm *C* uses the global knowledge and tries to generate maximally equal new domains. The results are shown in the next section.

4. RESULTS

A set of simulations with various values of the meshing parameters and meshing algorithms were performed in order to generate the convergence curves (of error versus the total number of unknowns).

An example is shown in Figure 3. Each curve denoted with PWL or PWS was obtained by repeating the same simulation scenario, and permitting a different number of unknowns at each simulation. The error in the current at the feed point was taken as the measure of accuracy. The results of a direct MoM solution at the finest mesh were used as the reference. The convergence plots

show that the proposed novel algorithm converges quicker than the traditional MoM based on the piecewise linear basis functions (denoted with δ (Ymono) and δ (Ydip) for monopole and dipole models, respectively), especially if only few unknowns are available or permitted for modeling. When the number of unknowns is small, an order of magnitude improvement in the accuracy of the solution has been observed.

The plot shown in Figure 3 corresponds to a mesh obtained by the chain-splitting algorithm propagating its solution from the feed point towards the free ends. The curves (especially ones marked with dots) experience multiple dips as the number of unknowns is increased, in the region with the number of unknowns greater than 10. This phenomenon is not present (the curve is much smoother) for the chain-splitting algorithm propagating its solution from the free ends towards the feed point (this plot is not shown but may be found in [13]).

In addition, it was observed from Figure 4 that the condition number of the compressed solution for a dipole are ten-fold lower than that of the traditional solution “d1” for the same dipole, although it closely matches the condition number observed for a monopole model (except for the very high number of unknowns).

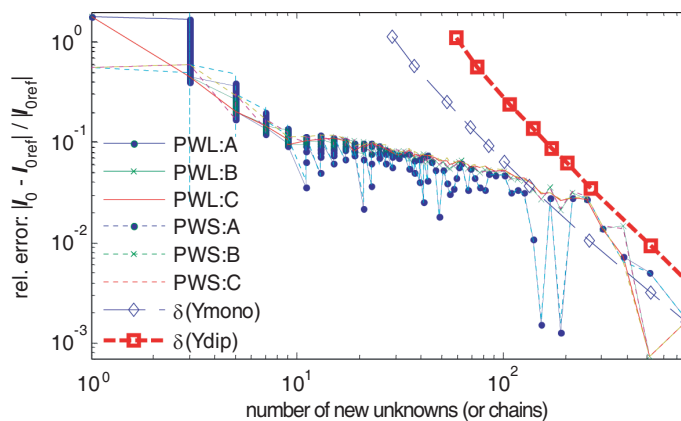


Figure 3: Convergence of error with growth in the total number of used variables (defined by the maximum permitted electrical length of a grouped chain of wire segments). The plot corresponds to the chain-splitting algorithm propagating its solution from the feed point towards the free ends. The notations PWL and PWS stand for piecewise linear and sinusoidal basis functions. The letters *A*, *B* or *C* following, denote the type of splitting algorithm applied [12]. The legend entries “ δ ” denote the convergence rate for the antennas modeled with a traditional direct MoM [10].

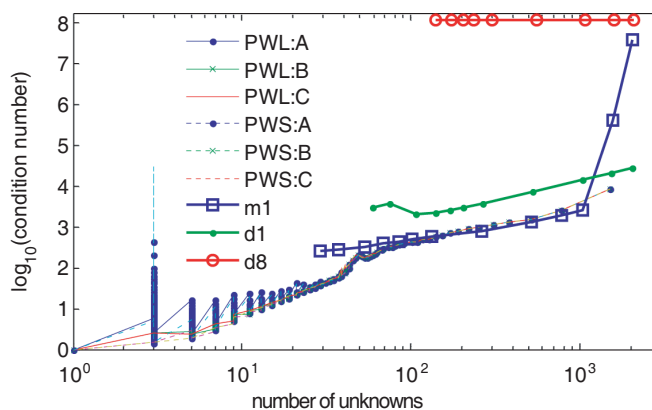


Figure 4: Condition number of the impedance matrix versus the number of unknowns when modeling a coil-loaded monopole and dipole. The notations PWL and PWS stand for piecewise linear and sinusoidal (basis functions used). The letters *A*, *B* and *C* denote the chain-splitting algorithm applied. The first six entries in the legend describe condition number of the new compressed impedance matrix for the respective meshing scenarios. The last three entries in the legend describe the condition number for direct solutions by the MoM. The first symbol in the notations *mN* or *dN* stand for monopole/dipole and the second symbol (digit) stands for the WIPL-D’s “current expansion” option. There is a slight monotonic increase in the condition number for d8, which is not readily visible due to the scale of the plot.

5. CONCLUSION

A novel method for effective modeling of curved structures and structures with electrically small features has been described. The method aggregates several basis functions into a composite basis function based on a linear interpolation between the original basis functions, and can thus permit a reduction in the number of unknowns with no sacrifice in accuracy. The method has been implemented over the framework of the method of moments

The method has been applied to an example of a coil-loaded antenna. Both piecewise-linear and piecewise-sinusoidal linearly-interpolated profiles of composite basis functions have been applied. The results confirm that the proposed method achieves better accuracy with fewer unknowns than the traditional method of moments.

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