

Intra-cavity flat-top beam generation

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ABSTRACT

In this paper we present the analytical and numerical analyses of two new resonator systems for generating flat-top-like beams. Both approaches lead to closed form expressions for the required cavity optics, but differ substantially in the design technique, with the first based on reverse propagation of a flattened Gaussian beam, and the second a metamorphosis of a Gaussian field into a flat-top beam. We show that both have good convergence properties, and result in the desired stable mode.

Keywords: intra-cavity beam shaping, flat-top beams, super-Gaussian beams.

1. INTRODUCTION

There are many applications where a laser beam with an intensity profile that is as flat as possible is desirable, particularly in laser materials processing. The methods of producing such flat-top beams (FTBs) can be divided into two classes, namely extra- and intra-cavity beam shaping. Extra-cavity (external) beam shaping can be achieved by manipulating the output beam from a laser with suitably chosen amplitude and/or phase elements, and has been extensively reviewed to date [1]. Unfortunately amplitude beam shaping results in unavoidable losses, while reshaping the beam by phase-only elements suffers from sensitivity to environmental perturbations, and is very dependent on the incoming field parameters. The second method of producing such beam intensity profiles, intra-cavity beam shaping, is based on generating a FTB directly as the cavity output mode. There are obvious advantages to this, not the least of which is the potential for higher energy extraction from the laser due to a larger mode volume, as well as an output field that can be changed in size by conventional imaging without the need for special optics in the delivery path. Unfortunately such laser beams are not solutions to the eigenmode equations of laser resonators with spherical curvature mirrors, and thus cannot be achieved (at least not as a single mode) from conventional resonator designs.

The key problem is how to calculate the required non-spherical curvature mirrors of the resonator in order to obtain a desired output field. One method to do this is to reverse propagate the desired field at the output coupler side of the resonator to the opposite mirror, and then calculate a suitable mirror surface that will create a conjugate field to propagate back. This will ensure that the desired field is resonant. This method was first proposed by Belanger and Pare [2–4], and we will refer to it as the *reverse propagation technique*. It was shown that the intra-cavity element could be defined such that a particular field distribution would be the lowest loss mode, opening the way to intra-cavity beam shaping by so-called graded-phase mirrors. This principle has been applied to solid state lasers [5], and extended by inclusion of an additional internal phase plate for improving the discrimination of undesired higher order modes [6]. However, in general this approach does not lead to closed form solutions for the required mirror phases.

In this paper we report on two resonator systems for producing flat-top beams, and show that in both cases closed form expressions may be found for the mirror surfaces. We outline a new method for generating a flat-top beam as the output mode of an optical resonator, based on an adaptation of well-known external laser beam shaping techniques. This is the first time such techniques have been employed inside a laser resonator. A major advantage of this approach is that simple expressions can be obtained for calculating the mirror surfaces. This approach is compared to the reverse propagating technique for calculating suitable graded-phase mirrors, and is

shown to have faster convergence to the fundamental flat-top beam mode, albeit with higher losses. Moreover, we show that by employing an approximation to flat-top beams in the form of Flattened Gaussian Beams (defined later), a closed form solution can be obtained for the mirror surface profiles even in the reverse propagating technique.

Throughout this paper a concept resonator with the following parameters is used to illustrate the two approaches to flat-top generation: wavelength of $\lambda = 1064$ nm; optical path length between the mirrors of $L = 300$ mm and an output flat-top beam of width $w_{FTB} = 4$ mm. These parameters have been chosen by way of example only, but can be considered realistic for experimental verification. The round trip modal build up and losses were studied numerically using the Fox-Li approach [7], by applying a fast matrix method to simplify the calculations and improve accuracy for an allowable computation time. This approach is outline in Appendix A.

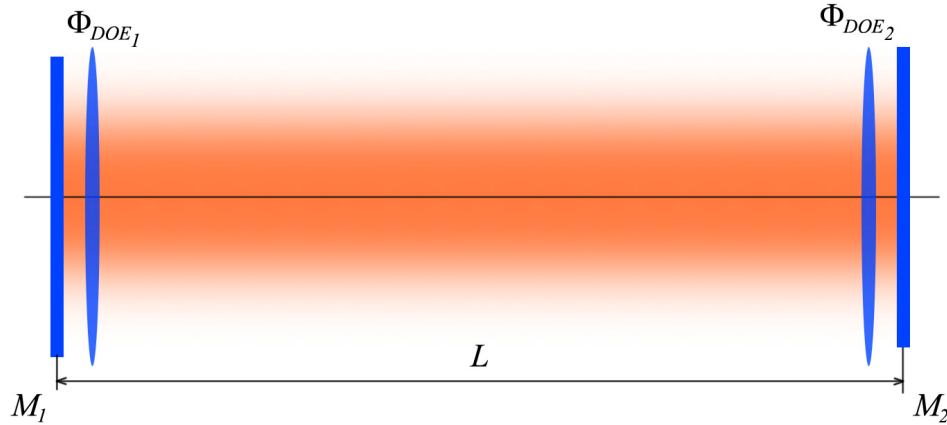


Fig. 1. A schematic of the resonator to be modeled: with output coupling at M_2 . Mirrors M_1 and M_2 are elements with non-spherical curvature.

The theory outlined in this paper, as well as the numerical simulations thereof, are restricted to the problem of one dimensional laser beam shaping, simply to keep the mathematical analysis as simple as possible. The two dimensional beam shaping problem has the same conceptual base, and all the results here may readily be extended to additional dimensions.

2. REVERSE PROPAGATION TECHNIQUE

For the benefit of the reader we briefly outline the reverse propagation technique, first proposed by Belanger and Pare [2, 3], since it will be used as a point of comparison for a new method proposed later in this paper.

Consider some arbitrary field that may be written in the form:

$$u(x) = \psi(x) \exp[-ik\phi(x)], \quad (1)$$

where $k = 2\pi/\lambda$ is the wavenumber, λ is the wavelength, and $\psi(x)$ and $\phi(x)$ are the amplitude and phase of the electric field respectively. The action of a DOE in the form of a phase-only mirror (graded-phase mirror) is to transform the phase $\phi_{in}(x)$ of an incoming field to a new phase $\phi_{out}(x)$ of an outgoing field according to:

$$\phi_{out}(x) = \phi_{in}(x) - 2\phi_{DOE}(x). \quad (2)$$

The salient point here is that this transformation takes place in a lossless manner, i.e., the amplitude is unchanged, $\psi_{in}(x) = \psi_{out}(x)$. In particular, one can show [2] that if the phase mirror is not spherical, then the change in phase also depends on the incoming field distribution $\psi_{in}(x)$. Thus it is expected that such a phase-only mirror will discriminate against those modes that do not have the correct distribution $\psi_{in}(x)$. By invoking the requirement that the mode must reproduce itself after one round trip, and considering the impact of the

graded-phase mirror on the curvature of the wavefront, it has been shown that the resulting restriction on the phase of the DOE mirror is given by [2]:

$$\int_{-\infty}^{\infty} x \left(\frac{\partial \phi_{in}}{\partial x} \right) |u_{in}^2(x)| dx = \int_{-\infty}^{\infty} x \left(\frac{\partial \phi_{DOE}}{\partial x} \right) |u_{in}^2(x)| dx, \quad (3)$$

from which we conclude that the phase of the resonator eigenmode is the same as the phase of the DOE mirror, apart from a constant:

$$\phi_{DOE}(x) = \phi_{in}(x) - \phi_{in}(0). \quad (4)$$

Combining Eqs. (2) and (4), and ignoring the constant phase offset, we see that

$$\phi_{out}(x) = -\phi_{in}(x). \quad (5)$$

Therefore the reflected beam $u_{out}(x)$ is the phase-conjugate of the incoming beam, $u_{out}(x) = u_{in}^*(x)$. In this resonator only a particular beam distribution is phase conjugated by the DOE mirror, so that the eigenmode of the resonator satisfies the criteria that its wavefront matches the phase of each mirror in the cavity.

If we describe the desired field at the output coupler end (mirror M_2) as u_2 , then reverse propagating the field to the DOE mirror (M_1) using the Huygen's integral in the Kirchhoff-Fresnel approximation yields the field at mirror M_1 as

$$u_1(x_1, L) = \sqrt{\frac{i}{\lambda L}} \int_{-\infty}^{\infty} u_2(x_2) \exp\left(-\frac{i\pi}{\lambda L} (x_1^2 - 2x_1x_2 + x_2^2)\right) dx_2, \quad (6)$$

where L is the optical path length of the resonator. If after reflection off mirror M_1 the field u_1 is to reproduce u_2 at the output coupler, then the required phase for the DOE mirror (M_1) must be given by

$$\phi_1 = \text{phase}[u_1^*(x, L)]. \quad (7)$$

One can also argue heuristically and reach the same conclusion as follows: one of the main properties of a fundamental mode of optical resonator is that the path of propagation both in the forward and in the reverse direction must repeat on one another. Consequently, in order to obtain a fundamental mode of a desired intensity profile we have to find a way to force the electromagnetic wave to follow exactly the same path in the forward and the reverse propagation directions. The conjugate of an incoming wave will produce an outgoing wave with exactly this property. Consequently we require that:

$$\exp(i\phi_{out}(x)) = \exp(-i\phi_{in}(x)) = \exp(-i2\phi_{DOE}(x)) \exp(i\phi_{in}(x)), \quad (8a)$$

and hence

$$\phi_{DOE}(x) = \phi_{in}(x), \quad (8b)$$

which is consistent with Eqs. (4) and (5). This is the basis by which custom resonators may be designed. In the following section we outline how this method may be applied to the generation of flat-top-like beams.

3. FLATTENED GAUSSIAN BEAM RESONATOR

The limitation in the approach outlined above is that the required mirror surface, as given by Eq. (7), is the solution to an integral problem (Eq. (6)) for which there is often not a closed form expression. Here we outline a suitable approximation to flat-top beams that leads to an analytical expression for the mirror surface.

3.1. Flattened Gaussian Beams

The exact definition of a flat-top beam (FTB) is one with constant field amplitude in some well defined region, and zero amplitude elsewhere:

$$u_{FTB}(x) = \begin{cases} u_0, & |x| < w \\ 0, & |x| \geq w \end{cases}, \quad (9)$$

where w is the radial width of the beam, and u_0 is a constant. Such a field does not result in a closed form solution to the problem of how the field appears after propagation through some distance z . However, there are many classes of flat-top-like beams that exhibit very similar propagation properties to true flat-top beams, where the rate of divergence (and profile shape change) may be controlled by a scale parameter closely coupled to the steepness of the edges and the flatness of the intensity profile at the centre of the beam [8]. Such classes of beams have been extensively studied both theoretically and experimentally [8–11]. One such class is the so-called Flattened Gaussian Beam (FGB), with a field distribution given by [11]:

$$u_{FGB}(x, z) = \sqrt{\frac{ik}{2z}} \exp\left[-\frac{ikz}{2}\right] \exp\left[-\frac{ikx^2}{2z}\right] \exp\left[-\frac{\left(\frac{kx}{z}\right)^2}{4\left(\frac{N+1}{w^2} + \frac{ik}{2z}\right)}\right] \\ \times \sum_{m=0}^N \left(\frac{1}{4}\right)^m \frac{1}{m!} \left(\frac{N+1}{w^2}\right)^m \left(\frac{N+1}{w^2} + \frac{ik}{2z}\right)^{-m} H_{2m}\left(\frac{\frac{kx}{z}}{2\sqrt{\frac{N+1}{w^2} + \frac{ik}{2z}}}\right) \quad (10)$$

where H_{2m} is the Hermite polynomial of order $2m$, and all other terms have their usual meaning. Such a field represents a suitably weighted linear combination of Hermite modes, such that the resulting intensity approximates a flat-top beam. The linear combination of fields with known propagation properties in turn allows the resulting field's propagation characteristics to be known analytically too, such as Rayleigh range, beam quality factor etc. The advantage of this profile as an approximation to a flat-top beam over that of others is that Eq. (10) offers an analytical expression for its profile at any propagation distance z . The order parameter associated with the field, given by the summation index N , allows the approximation to true flat-top beams to be exact when $N \rightarrow \infty$. In general as the order parameter increases, so the effective Rayleigh range decreases and the beam quality factor increases, resulting in a rapidly changing profile during free space propagation.

To design a resonator for such a beam, one simply follows the procedure outlined in section 2:

1. Select the desired field at the output coupler as $u_2 \equiv u_{FGB}(x, 0)$;
2. Reverse propagate this field using Eq. (10) to find the field at the opposite mirror, $u_1 = u_{FGB}(x, L)$;
3. The desired phase of mirror M_1 is then given by $\phi_1 = \text{phase}[u_1^*]$.

This approach may be used to calculate a suitable intra-cavity DOE that generates a FGB approximation to a flat-top beam as the resonant mode of the cavity. Following this procedure, we calculated the required mirror surface, shown in Fig. 2, to generate an $N = 20$ FGB, with $w_{FGB} = 4$ mm, as the output mode of the cavity. Since we wish the wavefront to be planar at the output coupler side, mirror M_1 has a planar surface.

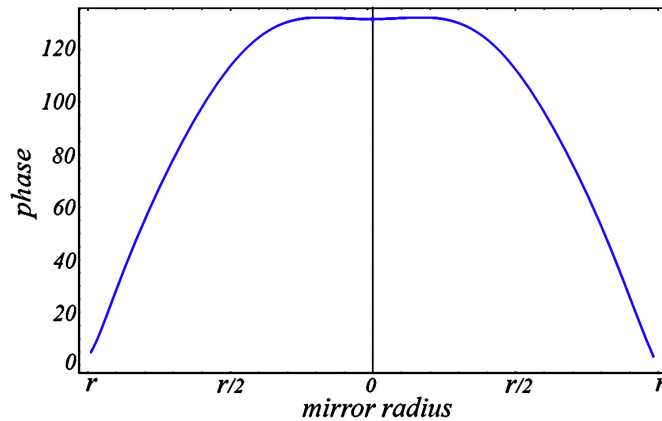


Fig. 2. Calculated phase profile required for mirror M_1 . The requirement for M_2 is that it is a planar mirror surface.

The resulting analysis of such a resonator is shown in the next section.

3.2. Simulation results

The calculated beam intensities at each mirror, for an $N = 20$ FGB, with $w_{FGB} = 4$ mm, are shown in Fig. 3 (a), together with the phase of the field at each mirror in Fig. 3 (b). The simulated results represented the field after stability using the Fox–Li approach, starting from random noise. The choice of $N = 20$ ensures a good quality flat-top beam, with reasonable Rayleigh length – i.e., the field does not change shape appreciably on propagating across the resonator length $L = 300$ mm. Such a field has a Fresnel number of ~ 50 .

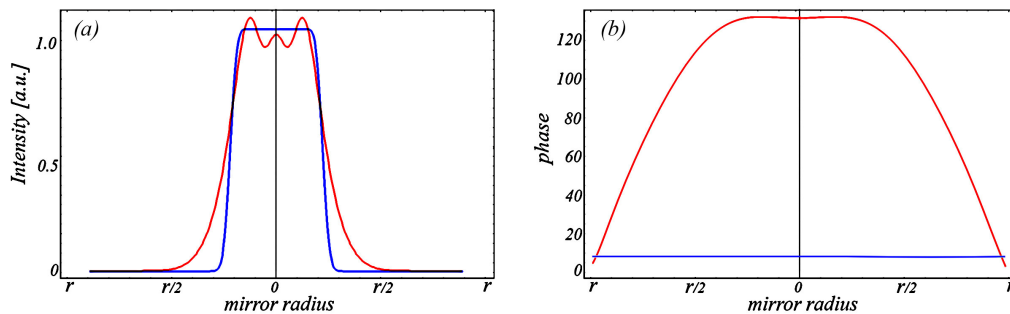


Fig. 3. The simulated field at mirror M_1 (red) and M_2 (blue): (a) intensity, showing a near perfect flat-top beam at M_2 , with slight change in flatness after propagating across the resonator to M_1 , (b) phase of the field, with a flat wavefront at M_2 as anticipated from the design.

It is clear that the approach outlined above correctly produces the desired FGB as the output mode of the resonator.

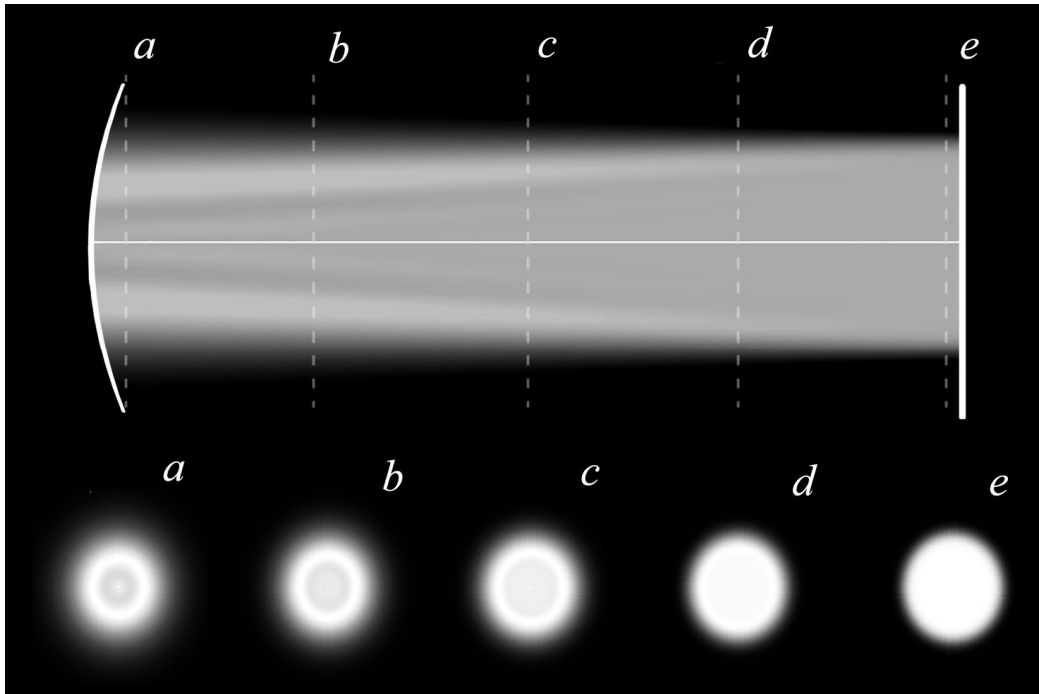


Fig. 4. The simulated field as it propagates across the resonator after stabilization, from M_1 (left) to M_2 (right). The perfect flat-top beam develops some intensity 'structure' as it propagates away from M_2 . This is in accordance with the propagation properties of such fields, and may be minimized by suitable choice of Rayleigh range of the field.

A cross-section through the resonator of the stabilised field is shown in Fig. 4, together with density plots of the field intensity at various planes in the resonator. The advantage of this order of FGB is that the beam is very close to an ideal flat-top, but with little change in the beam's cross-sectional intensity during propagation (in the absence of gain) across the resonator.

4. FLAT-TOP RESONATOR

Here we outline a new method for generating flat-top beams inside a laser resonator, based on an external lossless beam shaping technique converting a Gaussian input field to a flat-top output field [12, 13]. To the best of our knowledge these techniques have not been previously adapted or exploited for intra-cavity laser beam shaping.

4.1. Theory

Since we have *a priori* knowledge of how this resonator will be realised, consider a Gaussian field at mirror M_1 of the form $u_1(x) = \exp(-(x/w_g)^2)$, where w_g is the radius of the beam where the field is at $1/e$ of its peak value. If mirror M_1 is made up of a Fourier transforming lens and a transmission DOE and the resonator length is selected to match the focal length of the Fourier transforming lens ($L = f$), then the resulting field at mirror M_2 will be given by:

$$\begin{aligned}
 u_2(x_2, f) = & \sqrt{\frac{i}{\lambda f}} \int_{-\infty}^{\infty} u_1(x_1) \exp \left[-i \left(\phi_{DOE}(x_1) - \frac{ikx_1^2}{2f} \right) \right] \\
 & \times \exp \left(-\frac{i\pi}{\lambda f} (x_1^2 - 2x_1x_2 + x_2^2) \right) dx_1
 \end{aligned} \tag{11}$$

We may apply the method of stationary phase to find an analytical solution for the phase function of the DOE, ϕ_{DOE} , such that the field u_2 is a perfect flat-top beam, of width w_{FTB} . It has been shown that this may be expressed as [12]:

$$\phi_{DOE}(x) = \beta \left\{ \frac{\sqrt{\pi}}{2} \frac{\sqrt{2x}}{w_g} \operatorname{erf}\left(\frac{\sqrt{2x}}{w_g}\right) + \frac{1}{2} \exp\left(-\left[\frac{\sqrt{2x}}{w_g}\right]^2\right) - \frac{1}{2} \right\}, \quad (12)$$

where a dimensionless parameter β has been introduced, defined as

$$\beta = \frac{2\pi w_g w_{FTB}}{f\lambda} \quad (13)$$

This parameter has particular significance: at high values ($\beta > 30$) the geometrical approximations hold valid, and a perfect flat-top beam may be produced with relative ease. At very low values ($\beta < 10$), the geometrical approximations fail and the quality of the flat-top beam becomes less perfect. There is a fundamental lower limit for β at which the beam shaping problem is intractable [12]. A full discussion of how this parameter affects the resonator mode is beyond the scope of this paper, and is deferred to another occasion [14]. Since the flat-top beam is generated only at the Fourier plane of the lens, the effective phase profile of mirror M_1 mimicking both the lens and this element is given by:

$$\phi_1(x) = \frac{i}{2} \left(\phi_{DOE}(x) - \frac{kx^2}{2f} \right), \quad (14)$$

where the factor one-half takes into account the double pass through the mirror in an unfolded resonator, and the second term is the required Fourier transforming lens (or curvature of the mirror). In addition to an exact function for the first mirror's phase, we may use the stationary phase method to extract a closed form solution for the phase of mirror M_2 as

$$\phi_2(x) = - \left[\frac{k}{2f} x^2 + \frac{1}{2} \beta \exp(-\xi^2(x)) \right], \quad (15a)$$

$$\xi(x) = \operatorname{Inv} \left\{ \operatorname{erf} \left(\frac{2x}{w_{FTB} \sqrt{\pi}} \right) \right\} \quad (15b)$$

where

Here $\operatorname{Inv}\{\cdot\}$ is the inverse function. Such a mirror will reproduce our Gaussian field at mirror M_1 , as desired. The two required mirrors to generate a flat-top beam of width $w_{FTB} = 4$ mm are shown in Fig. 5.

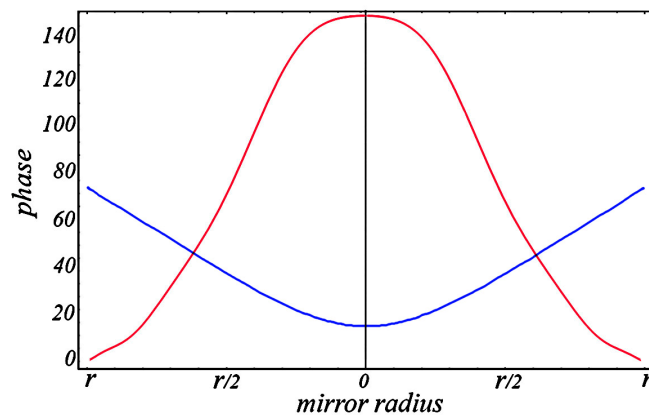


Fig. 5. The calculated required mirror phases to achieve the flat-top output mode for mirrors M_1 (blue) and M_2 (red).

An important aspect of the field in this resonator is its metamorphosis from a Gaussian beam at mirror M_1 , into a flat-top beam at mirror M_2 ; thus while we present the resonator concept here in terms of the generation of a flat-

top beam, there are obvious advantages in exploiting the same concept for the phase-only selection of a Gaussian output mode [14].

4.2 Simulation results

The calculated beam intensities at each mirror, using $\beta \sim 79$ with $w_{FTB} = 4$ mm and $w_g = 1$ mm, are shown in Fig. 6 (a), together with the phase of the field at each mirror in Fig. 6 (b). The simulated results represented the field after stability using the Fox–Li approach, starting from random noise. In this case a Gaussian beam is produced at M_1 and a flat-top beam at M_2 . While the Gaussian beam just prior to mirror M_1 has a flat wavefront, the surface of mirror M_1 is not planar.

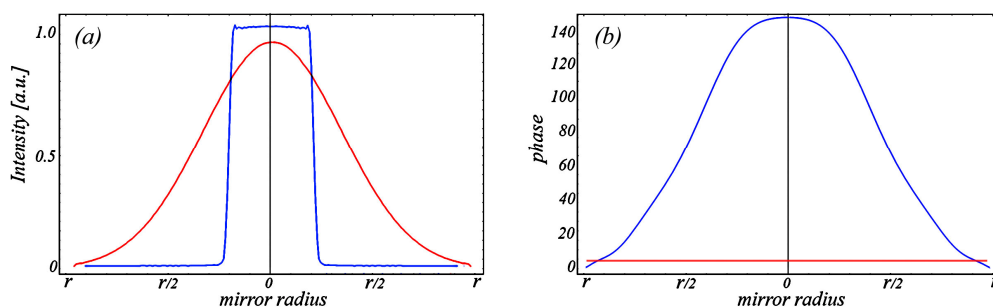


Fig. 6. The simulated field at mirror M_1 (red) and M_2 (blue): (a) intensity, showing a near perfect flat-top beam at M_2 , changing into a perfect Gaussian after propagating across the resonator to M_1 , (b) phase of the field, with a flat wavefront at M_1 as anticipated from the design.

A cross-section through the resonator of the stabilised field is shown in Fig. 7, together with density plots of the field intensity at various planes in the resonator.

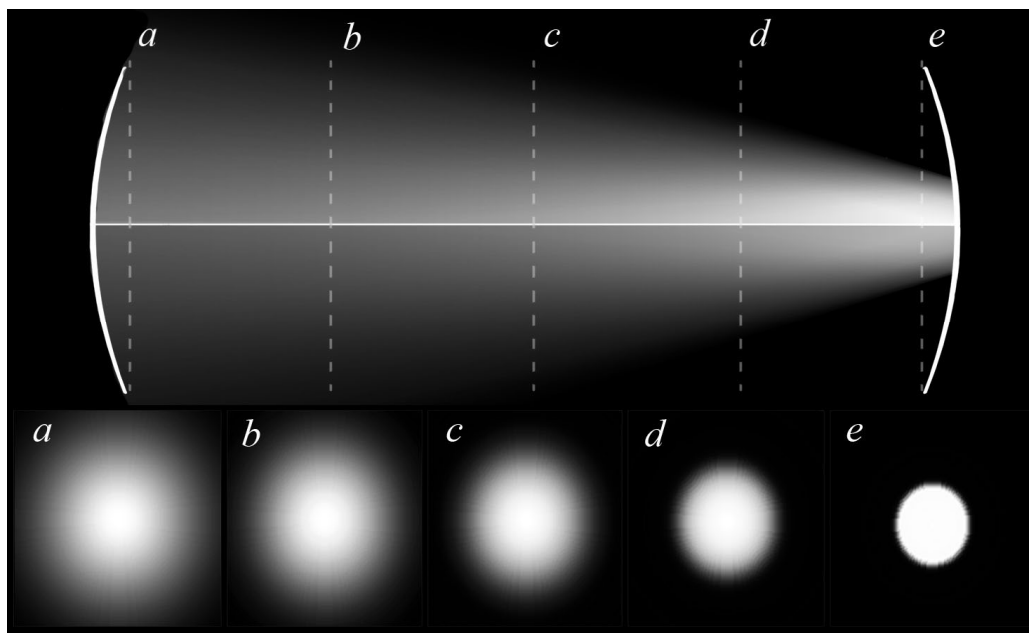


Fig. 7. The simulated field as it propagates across the resonator after stabilization, from M_1 (left) to M_2 (right). The perfect Gaussian beam (a) gradually changes into a perfect flat-top beam (e) on one pass through the resonator. In this design the field also decreases in size, as noted from the size of the grey scale images.

Because of the transformation during propagation from a Gaussian to a flat-top beam, the region of constant intensity is limited to near mirror M_2 . This impacts on energy that may be extracted from such a resonator since the gain volume would be somewhere between a single mode Gaussian and a single mode flat-top beam. The output coupler in this resonator has a phase profile etched into it, which together with the phase of the field at

M_2 , requires that a suitable DOE external to the cavity be used to convert the phase of the flat-top beam into a planar wavefront, should this be required.

5. CONCLUSION

We have presented two methods of creating flat-top beams as the output mode of a laser resonator where both approaches lead to analytical expressions for the required mirror surfaces. The first approach was to use an approximation to flat-top beams and apply phase conjugating mirrors at either end of the resonator. We showed that this leads to simple expressions for the mirror surfaces. The second approach is an extension of the external beam shaping technique where a suitable diffractive optical element converts a Gaussian beam to a flat-top beam at the Fourier plane of a lens. This method shows fast convergence and relatively low round trip loss for the fundamental mode of the resonator. A particularly interesting feature of this latter resonator is its ability to generate a Gaussian field that is selected by phase-only intra-cavity elements.

APPENDIX A

The central idea to the so-called Matrix Method approach is to note that only the integrand of the two propagation integrals (one for each direction) is changing on each pass of the resonator, and not the kernel itself. Therefore, if the transformation of a field on passing through the resonator could be expressed as the product of two matrices – one representing the starting field and the other the transformation of that field – only the former would have to be calculated on each pass, and not the latter.

To illustrate the method, consider sub-dividing the two mirrors into N parts each with size $\Delta x = 2X_2/N$ for mirror M_2 and, $\Delta x = 2X_1/N$ for mirror M_1 , where X is the radius of the respective mirrors. If Δx is small enough, then the field across that segment of the mirror may be assumed to be constant. We can now divide the Fresnel integral into a sum of integrals over each segment of mirror. As each segment has constant amplitude (albeit a different constant), this term may be removed from the integral, which in the case of propagating from mirror M_2 to M_1 becomes:

$$\begin{aligned}
 u_1(x_1, L) &= \sqrt{\frac{i}{\lambda L}} \int_{-X_2}^{X_2} u_2(x_2) \exp\left(-\frac{i\pi}{\lambda L}(x_1^2 - 2x_1x_2 + x_2^2)\right) dx_2 \\
 &= \sum_{i=0}^N u_2(X_2 - i\Delta x) \int_{X_2 - i\Delta x}^{X_2 - (i+1)\Delta x} \sqrt{\frac{i}{\lambda L}} \exp\left(-\frac{i\pi}{\lambda L}(x_1^2 - 2x_1x_2 + x_2^2)\right) dx_2
 \end{aligned} \tag{A1}$$

Since the integrand in Eq. (A1) does not change with the changing field, we may express Eq. (A1) in matrix form as

$$\vec{u}_1 = T \vec{u}_2 \tag{A2}$$

where

$$\vec{u}_1 = \begin{pmatrix} u_1(X_1) \\ \vdots \\ u_1(X_1 - n\Delta x) \\ \vdots \\ u_1(-X_1) \end{pmatrix}, \tag{A3}$$

$$\vec{u}_2 = \begin{pmatrix} u_2(X_2) \\ \cdot \\ u_2(X_2 - n\Delta x) \\ \cdot \\ u_2(-X_2) \end{pmatrix}, \quad (\text{A4})$$

$$T = \begin{pmatrix} T_{11} & T_{12} & \cdot & \cdot & T_{1N} \\ T_{21} & T_{22} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ T_{N1} & \cdot & \cdot & \cdot & T_{NN} \end{pmatrix}, \quad (\text{A5})$$

and

$$T_{ij} = \int_{X_2 - i\Delta x}^{X_2 - (i+1)\Delta x} \sqrt{\frac{i}{\lambda L}} \exp\left(-\frac{i\pi}{\lambda L}(x_i^2 - 2x_i x_j + x_j^2)\right) dx_2 \quad (\text{A6})$$

This approach dramatically decreases the computation time, since the elements of the transfer matrix, T , need be calculated only once. If the mirror segments are sufficiently small we may further reduce the Riemann integrals in T as

$$T'_{ij} = \lim_{\Delta x \rightarrow 0} T_{ij} = \sqrt{\frac{i}{\lambda L}} \exp\left(-\frac{i\pi}{\lambda L}(x_i^2 - 2x_i x_j + x_j^2)\right) \Delta x, \quad (\text{A7})$$

and thus decreasing the computational time further. For a non-symmetrical cavity, as is the case in this study, one is required to calculate the forward and reverse propagation matrices separately. The method may also be extended to multi-element resonators by application of a suitable Collins integral [16] in Eq. (A1).

If we write the matrices for the forward and backward propagation directions inside the resonator as T_1 and T_2 respectively, then the characteristic integral equation for any resonator system can be presented in the terms of the matrix method as:

$$\lambda \vec{u}_1 = T_1 T_2 \vec{u}_1 \quad (\text{A8})$$

Equation (A8) has solutions if the determinant of $I\lambda - T_1 T_2$ is zero; consequently all eigenvectors of $T_1 T_2$ represent the possible resonator modes, while all eigenvalues represent the losses with phase shift for these corresponding modes.

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