INVESTIGATION OF LOCAL SPATIAL SPECTRA OF BESSEL LIGHT BEAMS

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ABSTRACT

The spectrum of spatial frequencies (SFS) of Bessel beams properties emerging at their spatial localization is investigated. The case, when limiting aperture has a circular shape and a center at any distance from the optical axis, was studied. The Bessel beam local SFS was discovered to be radically changing at shifting the localization zone. Namely, a full ring transforms into a light arcs pair and its angular dimensions monotonically decreases as shift growth. The axis orientation of the spectrum maximal intensity coincides with the diaphragm shifting direction. The numerical modeling results agree well with the experiments.

Key words: Bessel beams, spatial spectrum, Fourier transformation, axicon, annular and circular aperture.

1 INTRODUCTION

The most important feature of light beams is the structure of their spatial frequencies spectrum (SFS). SFS gives the deep insight into spatial-angular structure of light beams, and also allows one to interpret clearly different linear and non-linear processes with their participation. The known annular structure of BLB spatial spectrum has clear explanation, if to represent BLB as a set of plane waves, wave vectors of which lie on the surface of a cone. This model was proposed in the paper [1], in which for obtaining BLB there was used narrow circular diaphragm placed in the front focal plane of lens and illuminated by plane wave. The superposition of plane waves in such scheme is proposed to be generated by point sources distributed within the circular diaphragm. In a case when diaphragm is placed outside of lens focus [2], Bessel beam can be described as an image in lens of spherical waves superposition formed by some effective circular source in object plane of lens. In the scheme of BLB obtaining based on axicons, the appearance of superposition of plane waves can be studied when analyzing diffraction integral by the stationary phase method. At last, in the basis of rigorous solutions of Helmholtz equation, Bessel beam can be presented in a form of superpositions of two conical beams described by the zeroth and first order Hankel function corresponding to convergent and divergent cylindrical waves [3].

It is necessary to point out that annular structure of SFS of Bessel beams follows from the listed above models

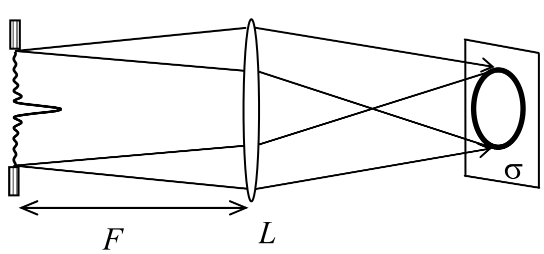

Figure 1. Optical Fourier transformation of Bessel beam.

that describe their inner structure. But the said above relates to the integral SFS, i.e. to the spectrum obtained for the

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whole beam. Namely, such spectrum is formed in standard experimental scheme using Fourier-transformation lens (Fig.1). In focal plane σ of lens *L* annular field has the radius $R_1 = F/\gamma$, where γ is cone angle of Bessel beam. From Fig. 1 it is seen that this scheme forms integral spatial spectrum, which is a characteristic of a beam in a whole. Integral spectrum appears experimentally in special non-localized physical processes, such as vector interaction at second harmonic generation or parametric generation of light [4, 5]. But obviously the majority of linear and non-linear processes with BLB are spatial localized in this or other way. Correspondingly, their realization depends on the structure of local SFS.

That is why the aim of this paper is to investigate the peculiarities of SFS of Bessel beams formed by the scheme of optical Fourier transformation with additional freely-localized limiting aperture.

2 SCHEME WITH FOURIER TRANSFORMATION OF ANNULAR FIELD

The case was examined when the limiting aperture has circular shape and its center is situated on the arbitrary distance from the beam optical axis. Optical scheme of the experiment, which was used to study local spectrum of Bessel beams, is shown in Fig.2.

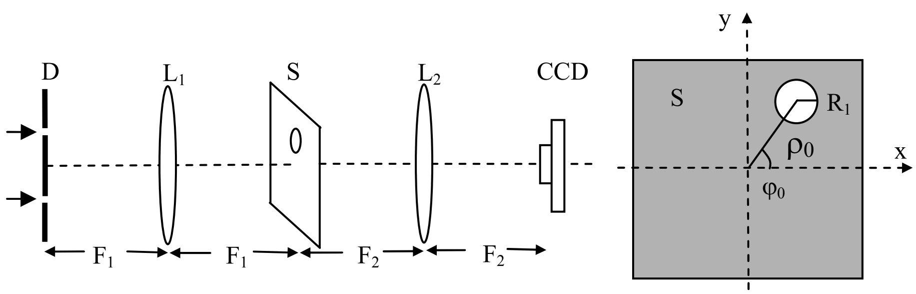


Figure 2. Optical scheme for measuring local spatial spectrum of Bessel beam formed by annular diaphragm. Here *D* is annular diaphragm, $L_{1,2}$ is Fourier transformation lenses, *S* is the screen with circular hole.

When calculating this scheme, the field $a(\vec{r}\,)$ in output plane can be presented in the form of Fourier transformation

$$a(\vec{r}\,) = -\frac{i}{\lambda F_2}\int a_1(\vec{r}_1)\sigma(\vec{r}_1)\exp\left(-\frac{ik_0\vec{r}_1\vec{r}}{F_2}\right)d^2r_1\,, \tag{1}$$

where $a_1(\vec{r}_1)$ is the field created by the diaphragm in focal plane of lens L_1, $\sigma(\vec{r}_1)$ is the transmission function of shifted diaphragm *S*. In its turn, the field $a_1(\vec{r}_1)$ is the Fourier transformation of the field in input plane, i.e.

$$a_1(\vec{r}_1) = -\frac{i}{\lambda F_1}\int \tau(\vec{r}_2)\exp\left(-\frac{ik_0\vec{r}_1\vec{r}_2}{F_1}\right)d^2r_2, \tag{2}$$

where $\tau(\vec{r}_2)$ is the transmission function of circular diaphragm *D*.

It follows from Eqs. (1) and (2) that

$$a(\vec{r}\,) = -\frac{i}{\lambda F_1}\int \tau(\vec{r}_1)\tilde{\sigma}\left(\vec{r} + F_2\vec{r}_1/F_1\right)d^2r_1\,, \tag{3}$$

where $\tilde{\sigma}(\vec{r}\,)$ is the Fourier transformation of the transmission function of the shifted circular hole.

Function $\tilde{\sigma}(\vec{r}\,)$ has the following known form:

$$\tilde{\sigma}(\vec{r}\,) = -\frac{ir_0}{r}J_1\left(\frac{k_0r_0r}{F_2}\right)\exp\left(-\frac{ik_0\vec{\rho}_0\vec{r}}{F_2}\right), \tag{4}$$

where $\vec{\rho}_0 = (\rho_0, \varphi_0)$ is the vector of diaphragm shifting (see Fig.1). Substituting Eq. (4) into Eq. (3), we obtain the following expression $a(\vec{r})$ for the field in the plane of CCD-camera:

$$a(\rho,\varphi) = \frac{-r_0}{\lambda F_1 F_2} \int \int_0^{2\pi} \tau(\rho_1) \frac{J_1\left(k_0 r_0 S(\rho,\rho_1,\varphi,\varphi_1)\right)}{S(\rho,\rho_1,\varphi,\varphi_1)} \exp\left[\frac{ik_0\rho_0[\rho_1\cos(\varphi_1-\varphi_0)+\rho\cos(\varphi-\varphi_0)]}{-F_1}\right]\rho_1 d\rho_1 d\varphi_1 , \quad (5)$$

where $S(\rho,\rho_1,\varphi,\varphi_1) = \sqrt{\rho^2/F_2^2 + \rho_1^2/F_1^2 + 2\rho_1\rho/F_1 F_2 \cos(\varphi-\varphi_1)}$, $J_1(x)$ is the first order Bessel function.

Neglecting in Eq.(5) the influence of circular diaphragm width we suppose for $\tau(\rho)$ the following approximation: $\tau(\rho) = R_{in}\delta(\rho - R_{in})$, where R_{in} is the radius of circular diaphragm. Then we have:

$$a(\rho,\varphi) = -\frac{r_0 R_{in}^2}{\lambda F_1 F_2} \int_0^{2\pi} \frac{J_1\left(k_0 r_0 S(\rho,R_{in},\varphi,\varphi_1)\right)}{S(\rho,R_{in},\varphi,\varphi_1)} \exp\left[\frac{ik_0\rho_0[R_{in}\cos(\varphi_1-\varphi_0)+\rho\cos(\varphi-\varphi_0)]}{-F_1}\right] d\varphi_1 \quad (6)$$

Circular diaphragm *D* with the diameter of 12 mm and width of the ring of 80mkm was placed in the front focal plane (F_1 = 50 cm) of the lens L_1. In neighboring of the back focal plane of lens the Bessel beam is known to be formed (Fig.3). Diaphragm *S* with circular hole having the radius r_0 of 185 mkm, the center of which is situated in the point with cylindrical coordinates (ρ_0, φ_0) (Fig.2), was oriented perpendicular to the optical axis of the scheme in zone of Bessel beam existing. Lens L_2 (F_2 = 20см) formed local Fourier-spectrum of Bessel beam, which two-dimensional intensity distribution was measured by CCD-matrix.

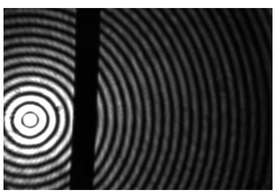


Figure 3. Fragment of Bessel beam formed in focal plane of lens L_1. For estimation of typical spatial sizes, the dark strip is shown with the width of 80 mkm.

Fig.3 shows the results of measurements of SFS obtained at various positions of circular diaphragm relatively optical axis.

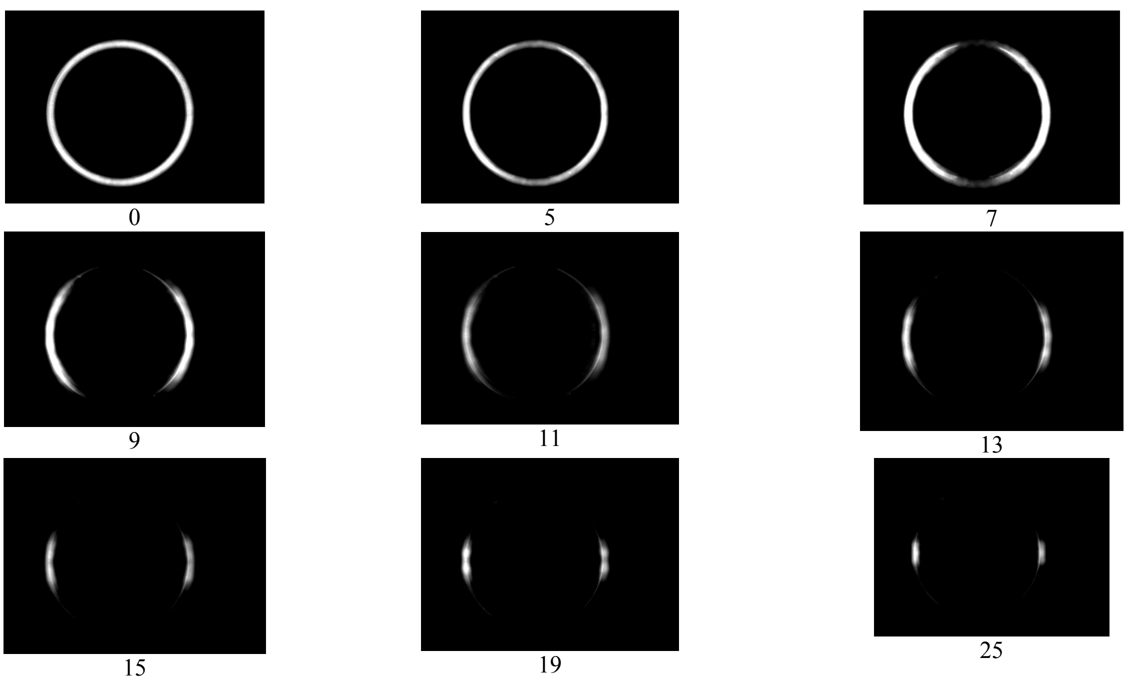


Figure 4. Dynamics of changes of local SFS of Bessel beam when moving off its center. The size of shifting is $\rho_0 = (30 \cdot m)$ mkm, where *m* is the number of the figure.

Figure depicts that local SFS of Bessel beam changes significantly at shifting of zone of its localization off optical axis. Full ring is transformed into a pair of arcs, which angular size reduces gradually. Axis orientation of spectrum maximal intensity at that coincides with the direction of circular diaphragm shifting. In experiment, which corresponds to Fig. 4, azimuthal angle φ_0 equals to zero.

Numerical studying the experimental scheme in Fig.2 has been made using equation (6). The results of intensity simulation in spectrum are given in Fig.5. As is seen, there is good correspondence with experimental measurements.

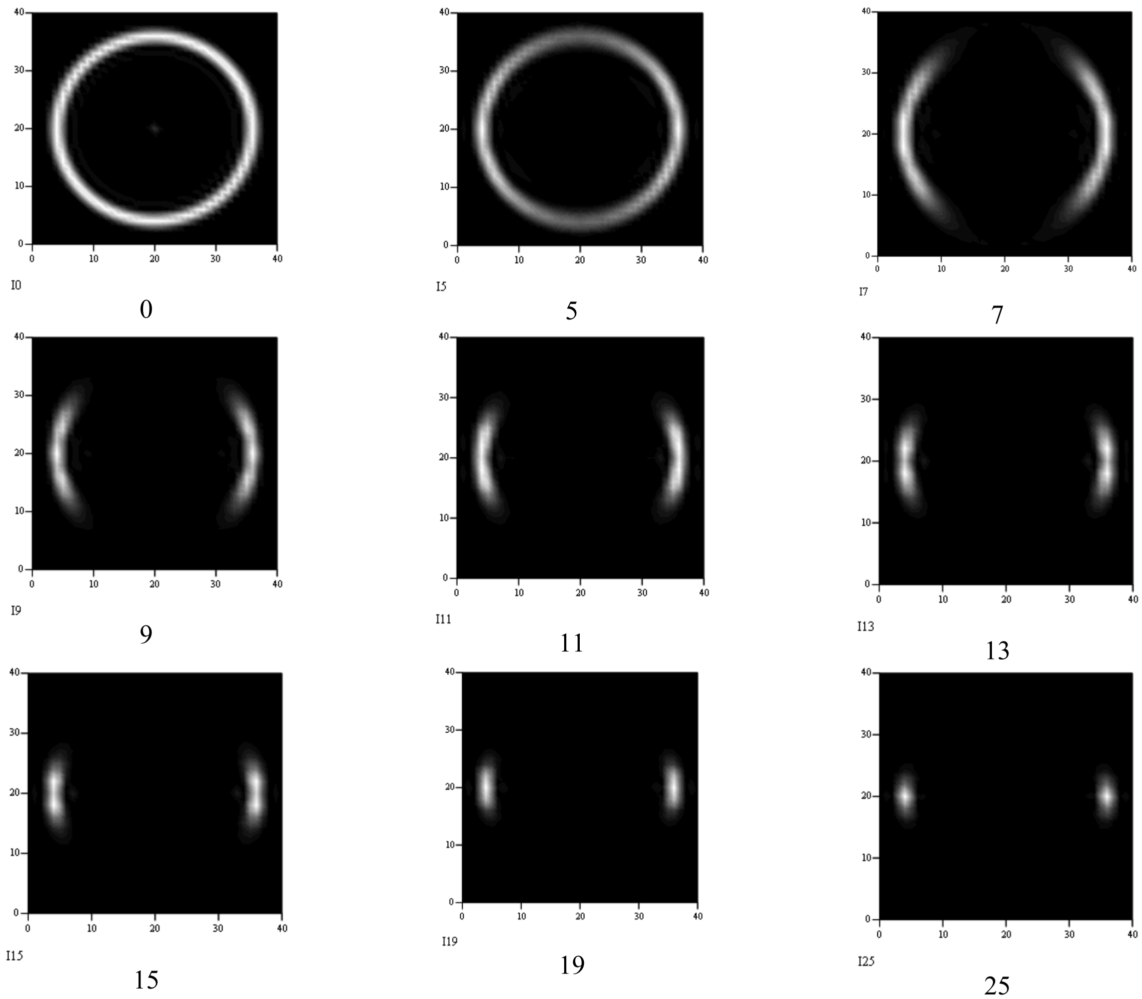
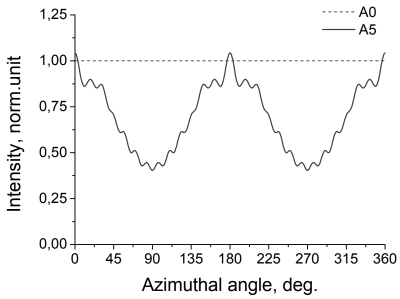
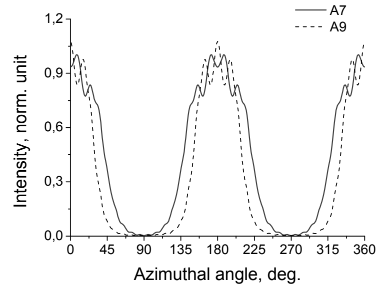


Figure 5. Theoretically investigated intensity distribution in local SFS of the Bessel beam. The value of shifting is the same as in Fig. 4.

Figs. 4 and 5 show that local BLB spectrum is not circular one, as a rule. The exclusions are cases, when the center of limiting diaphragm is situated on the axis of a beam or when the shift is so small that central lobe of a beam is within the limits of diaphragm. Consequently, circular spectrum is realized only in relatively narrow near-axial zone of a beam. Far from the center the spectrum takes the shape of a pair of arcs, here azimuthal orientation of arcs turns at diaphragm turning relatively beam axis. Accumulation of such turned local spectra gives the known integral circular spectrum.

The given results allow one to correct the known plane wave interpretation of Bessel beams. The matter concerns the representation of Bessel beams as composed of array of plane waves, wave vectors of which lay on the surface of the cone (see for example [1]). It is evident that such interpretation is not correct when applying to the optional beam area, except its central area. In off-axis area of beam, wave vectors of plane waves fill only a part of circular cone limited along the azimuthal coordinate. At enough shifting from the beam axis the form of SFS approaches to double-point one (see Figs. 5, 6) that corresponds to the approximation of two plane waves. More detailed description of this measurement can be fulfilled using graphic dependences of maximum intensity on the azimuthal angle (see Fig.6). Uniform intensity distribution of the field corresponds to non-shifted diaphragm. At the presence of shifting there appears the modulation with two maxima, which width decreases with the shift increasing. It is necessary to point out the presence on the graphs of relatively small modulation, which origin is still unclear.





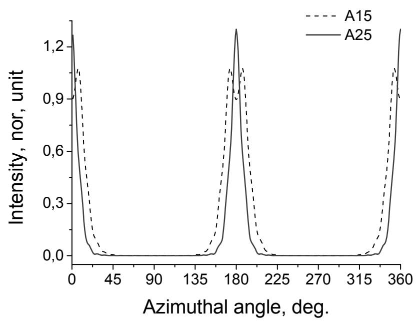


Figure 6. Dependence of maximum value of intensity of local spatial spectrum on the azimuthal angle. Indications A0, A5 and etc. correspond to images 0, 5, and etc. in Figs. 4 and 5.

3 SPATIAL SPECTRUM OF $J_0(q\rho)$ FUNCTION

Let us calculate the spatial spectrum of zeroth order Bessel $J_0(k_0\gamma\rho)$ function, irrespective of optical scheme of Bessel beam formation.

In a case when circular diaphragm is situated outside of beam axis, the spectrum is calculated according to the following formula

$$a(\rho,\varphi)=\int_{\rho_0-R}^{\rho_0+R} J_0(k_0\gamma\rho_1)D(\rho,\rho_1,\varphi)\rho_1 d\rho_1\ , \tag{7}$$

where

$$D(\rho,\rho_1,\varphi)=\int_{\varphi_0-\Delta\varphi(\rho_1)}^{\varphi_0+\Delta\varphi(\rho_1)} \exp\left(-\frac{ik_0\rho\rho_1}{F}\cos(\varphi-\varphi_1)\right)d\varphi_1\ , \tag{8}$$

$\Delta\varphi(\rho)=\arccos[(\rho^2+\rho_0^2-R^2)/2\rho_0\rho]$, R is radius of the diaphragm, (ρ_0 φ_0) is position of its center in cylindrical coordinates.

Fig. 7 shows the intensity distribution in spatial spectrum at different sizes and shifts of the diaphragm.

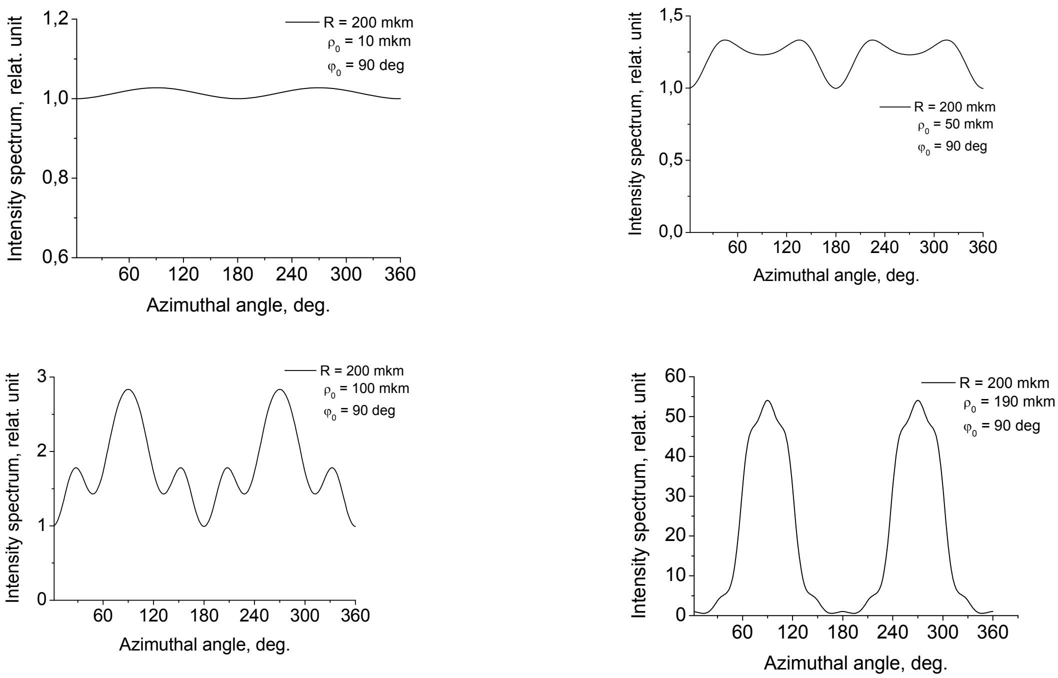
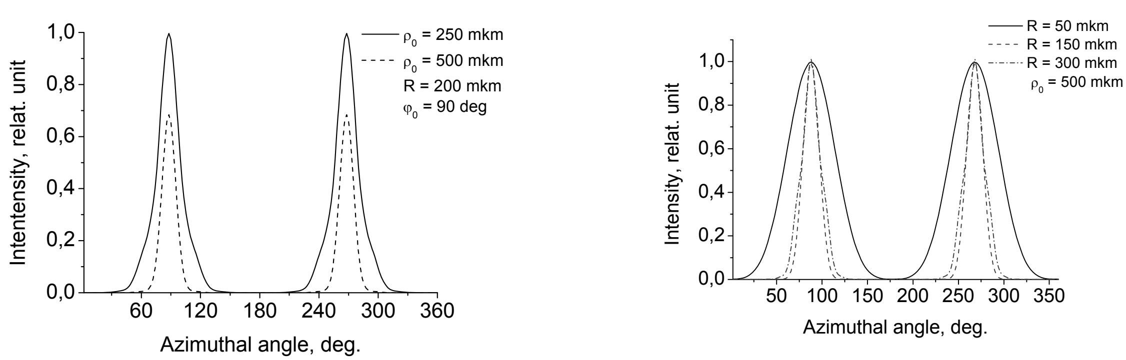


Figure 7. Dependence of intensity of local spectrum of Bessel function on azimuthal angle at different diaphragm radii. Parameters used: cone angle γ = 0.25 deg, radius of diaphragm R=200 mkm, F = 0.5m.

In a case when diaphragm shifting is $\rho_0 < R$, spectrum $a(\rho,\varphi)$ calculation was made according to the following formula

$$a(\rho,\varphi) = 2\pi \int\limits_0^{R-\rho_0} J_0(k_0\gamma\rho_1) J_0\left(\frac{k_0\rho\rho_1}{F}\right)\rho_1 d\rho_1 + \int\limits_{R-\rho_0}^{R+\rho_0} J_0(k_0\gamma\rho_1) D(\rho,\rho_1,\varphi)\rho_1 d\rho_1 \quad (9)$$

Fig.8 illustrates the property of spectrum change for this case.



(a) (b)

Figure 8. Dependence of the intensity of local spectrum of Bessel function on azimuthal angle at different values of shifting ρ_0 of annular aperture (a) and from its radius *R* (b). Cone angle is γ = 0.25 deg., F = 0.5m.

It is seen that for off-axial limiting aperture the spatial spectrum is localized at azimuth, at that the maximal intensity is realized, i.e. $\varphi = \varphi_0$ and $\varphi = \varphi_0 + \pi$. The spectrum width decreases at increasing the aperture ρ_0 shifting (Fig. 8b). The dependence of spectrum width on the diaphragm radius *R* is ambiguous (Fig. 8b). In the area of small *R* spectrum is relatively wide that is connected with the influence of diffraction. When increasing R the spectrum narrows at first, because the diffraction divergence of a beam decreases. Then spectrum widens that is determined by the approaching of the diaphragm edge to the beam center.

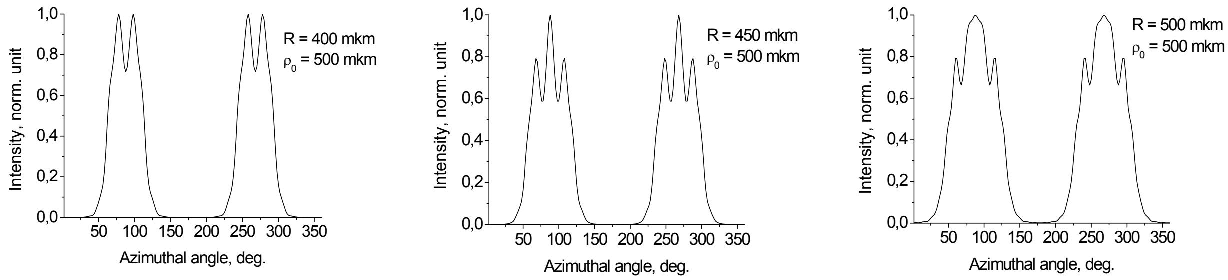


Figure 9. Spectrum transformation at approaching the edge of circular aperture to the beam center.

When approaching the diaphragm edge to the beam center the spatial spectrum begins to deform (Fig.9). In the area of maximum oscillating behavior appears with two and more maxima.

Thus, local spectra of Bessel function are not circular one in common case. The presence of two global maxima means that within the aperture the light field distributes in two primary directions. Azimuths of these directions, as it was pointed out above, are equal to $\varphi = \varphi_0$ and $\varphi = \varphi_0 + \pi$. When moving off the beam center two mentioned above waves approach to plane waves as their spatial spectrum gather in point. But in common case it is more correct to speak about the passing through the circular aperture of two fragments of conical waves. One of these waves is convergent; the second is divergent or reflected from the beam center [3].

4 SCHEME BASED ON AXICON

Analogical investigations were conducted for Bessel beams formed by the axicon. Diaphragm S (Fig.2) was placed in the area of maximal diameter of the formed BLB. Here also transformation was observed of circular spectrum in arc one while shifting

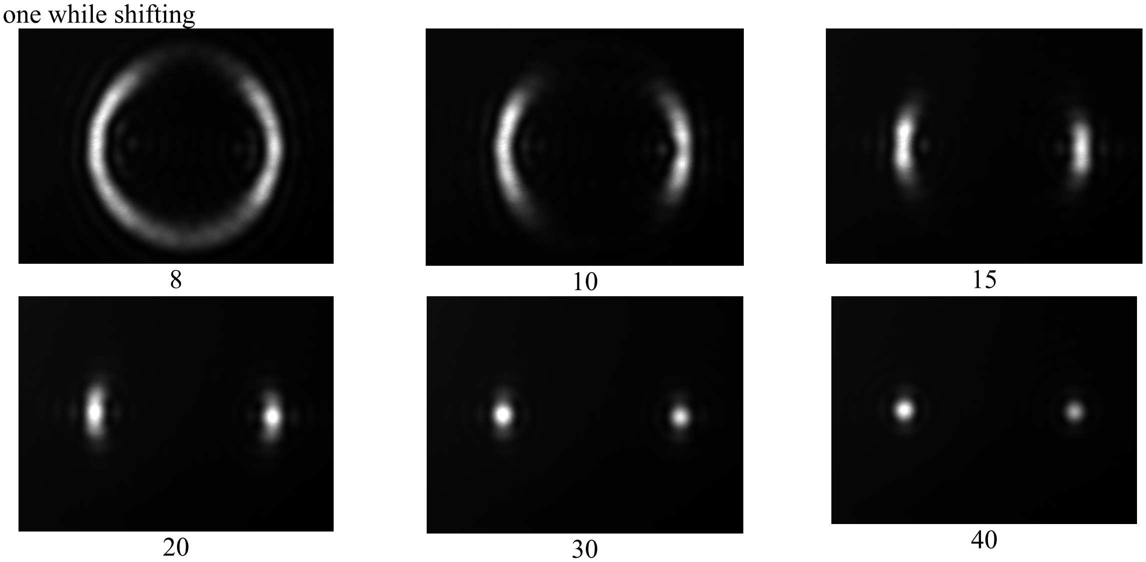


Figure 10. Experimental intensity distribution of local SFS of Bessel beam obtained by an axicon. The value of shifting equals the number under the figure multiplied by 30mkm.

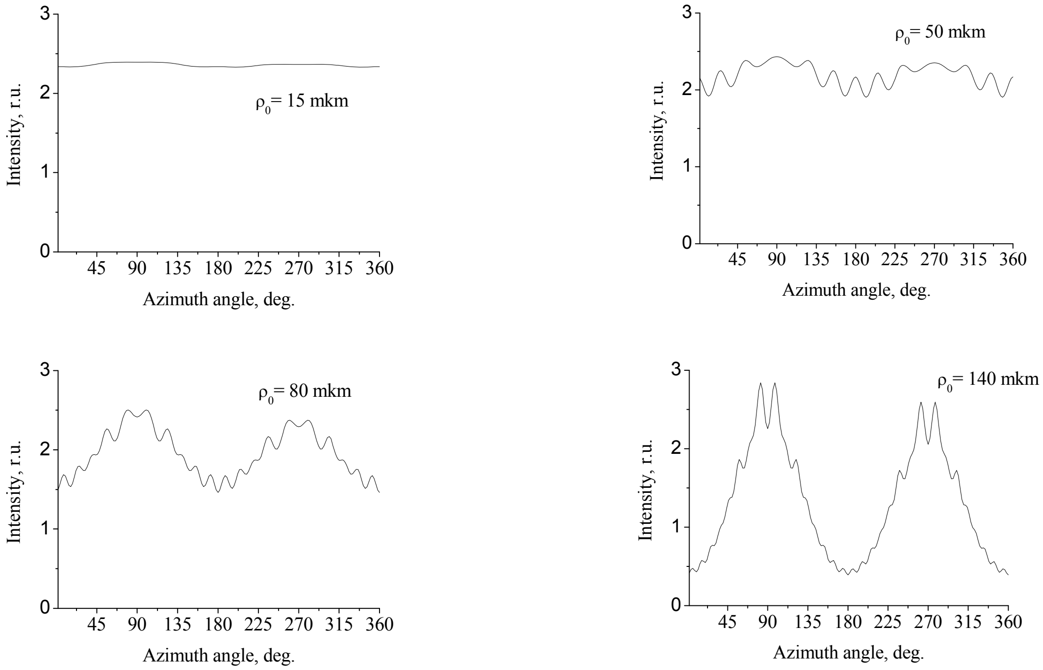


Figure 11. The calculated intensity dependences on azimuth angle of local BLB spectrum formed by axicon. The case when the beam axis is within the circular aperture. The parameters used: cone angle γ = 0.9 deg., focal distance of the lens f = 0.3m, aperture radius R = 185 mkm, half-width w= 6mm.

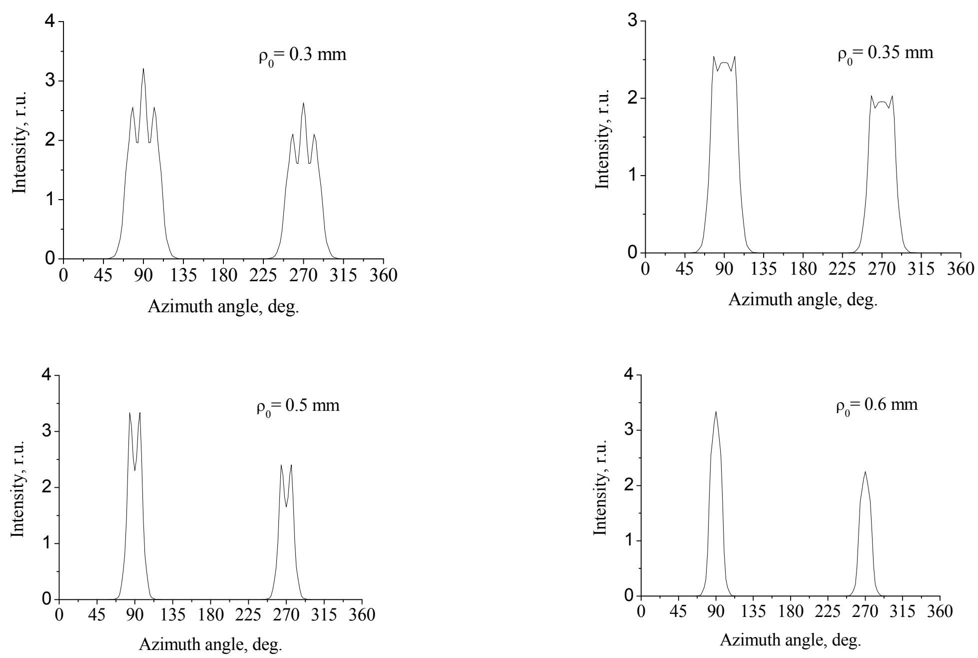


Figure 12. The calculated azimuthal intensity dependences of local BLB spectrum formed by axicon. The case when the position of the circular aperture is outside of the beam axis. The parameters used: cone angle γ = 0.9 deg., focal distance of the lens f = 0.3m, aperture radius R = 185 mkm, half-width w= 6mm.

circular aperture (see Figs. 10, 11 and 12). The difference here is in appearance of asymmetry of arc spectrum, which increases when shifting becomes larger. The asymmetry explanation is in that the formed by axicon beam differs slightly from Bessel one. Formally it is connected with the different deposit of two stationary points into diffraction integral that leads to some difference of convergent and divergent conical waves that form Bessel beam. Earlier, formed by the axicon field was shown to have the following form:

$$a(\rho,z)=\left[f_+(\rho,z)J_0(k_0\gamma\rho)-if_-(\rho,z)J_1(k_0\gamma\rho)\right], \qquad (10)$$

where $f_\pm(\rho,z)=\left(f_1(\rho,z)\pm f_2(\rho,z)\right)/2$, $f_{1,2}(\rho,z)=\sqrt{1\pm\frac{\rho}{\gamma z}}\exp\left(-\frac{(\gamma z\pm\rho)^2}{w^2}\right)$.

Spectrum simulation was made in assumption that the field behind the axicon is described by the Eq. (9). Here the Eqs. (7) and (9) were used with the substitution of $J_0(k_0\gamma\rho)$ into $a(\rho,z)$ from Eq. (10). The results of simulation are shown in Figs. 11 and 12. Here 1D graphs are given, which correspond to maximal value of intensity. The direction of shift of circular aperture is φ_0 = 90 deg. As is seen, for near-axial position of aperture (Fig.11) with the increasing ρ_0, gradual transformation takes place of homogenous annular field in two arcs, like described above. At off-axial position of aperture azimuthal heterogeneity of spectrum is expressed clearly. Here the asymmetry of intensity of two arcs is really observed. This asymmetry is seen to be more essential at BLB periphery (Fig.12) than in its central area (Fig.11). It corresponds accurately with the studied earlier field structure behind the axicon. Namely, the contribution of component containing BLB of first order increases when aperture moves off the beam axis.

5 CONCLUSIONS

Thus, local spatial spectra of Bessel beam fragments assigned by off-axial-shifted circular limiting diaphragm differs essentially from annular ones. This conclusion is made for cases, when initial BLB was formed in the scheme with annular diaphragm or axicon. This property of local spectra is necessary to be taken into account at interpretation of experimental results obtained with use of Bessel beams. The simplest example is the application of Bessel beam for the aims of profilometry of cylindrical surfaces by conical beams [6, 7]. In a case when tested cylindrical object is shifted relatively beam axis; then the measurement of local annular spectrum leads to azimuthal modulation of field intensity on cylinder surface. At rather large shifts a part of cylindrical surface is not illuminated at all. The second example is connected with the use of BLB for localization and manipulation of microparticles set. The efficiency of manipulation of microparticles here can essentially depend on the self-reconstruction of field, screened by these particles [8]. From considering the local spectra of BLB it follows that conditions of recovery of field behind a definite particle change, when occurs the transverse shift of this particle relatively beam axis. Properties of local spectra are necessary to be taken into account, particularly, when forming one-dimensional array of particles in Bessel beam optical tweezers [8].

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