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GYROSCOPIC EFFECTS IN VIBRATING FLUID-FILLED SPHERES SUBJECTED TO INERTIAL ROTATION

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ABSTRACT

In symmetric distributed structures subjected to vibration and an inertial rotation, the vibrating patterns turn in the direction of revolution at a rate proportional to the inertial angular rate. This effect has numerous applications in navigational instruments, such as hemispherical rotational sensor. It is also important for astrophysics and seismology to understand the dynamics of pulsating stars and earthquake series. The coefficient of proportionality between the inertial and vibrating pattern rates depends on the geometry of structure, mode number, et cetera, and plays a crucial role in this study. In this paper we consider gyroscopic effects in hollow solid spheres filled with an inviscid fluid. The dynamics of the sphere are considered in terms of linear elasticity. Two limiting cases of the fluid motion are considered: in the first case, we suppose that the fluid is fully involved in the rotation; in the second, the fluid does not rotate relative to the inertial reference frame. It is also assumed that the angular rate is constant and much smaller than the lowest eigenvalue of the system. Hence centrifugal effects, proportional to the square of the angular rate, are considered to be negligible. The effects of structure prestress due to gravitational forces are also neglected. Two types of non-axisymmetric modes of the system are considered, namely spheroidal and torsional. A numerical experimental observation is made that, for lower eigenvalues and lower circumferential wave numbers, the difference between the modulus of the rotational angular rates of the fluid-filled sphere and those of its vibrating patterns is small. However, this difference is large for higher modes and eigenvalues of the system.

1. INTRODUCTION

Let us consider a spherical body (S) with distributed parameters, either solid or fluid (Fig. 1). Suppose that the body is subjected to non-decaying vibrations on one of its natural modes as well as rotation with a small constant angular rate Ω relative to axis Oz in inertial space. By "smallness" of the angular rate of rotation Ω we mean that this rate is substantially smaller than the lowest eigenvalue of the system. Consequently, we will neglect centrifugal effects and all other terms proportional to Ω^2.

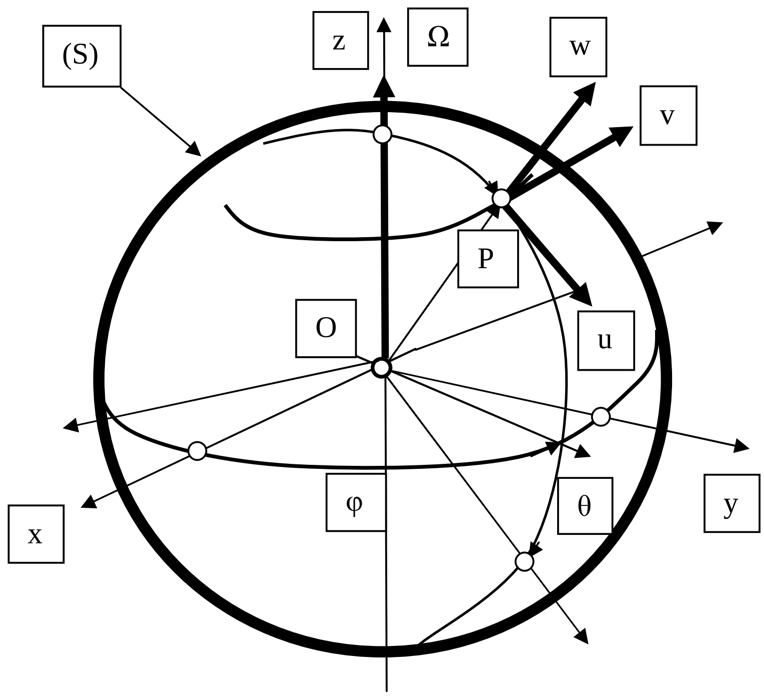

Figure 1. Coordinate systems for spherical body (*S*)

2. GYROSCOPIC EFFECTS IN DISTRIBUTED BODIES

Assume that $(u, v, w)^T$ is a vector of linear displacements of an arbitrary point P belonging to the body (*S*). The absolute linear velocity of this point is

$$\vec{V} = \begin{bmatrix} \dot{u} - \Omega v \cos\theta \\ \dot{v} + \Omega\left[u\cos\theta + (r+w)\sin\theta\right] \\ \dot{w} - \Omega v \sin\theta \end{bmatrix} \quad (1)$$

where r is the distance from the centre O to the point P of the body ([1]). The kinetic energy of the system of concentric spherical bodies is approximately:

$$K \approx \frac{1}{2}\sum_{i=1}^{N}\rho_i \int_0^{2\pi}\int_0^{\pi}\int_{a_{i-1}}^{a_i}\left\{\left(\dot{u}_i^2 + \dot{v}_i^2 + \dot{w}_i^2\right) + 2\Omega\left[\left(u_i\dot{v}_i - \dot{u}_i v_i\right)\cos\theta + \left(\dot{v}_i w_i - v_i\dot{w}_i\right)\sin\theta\right]\right\} r^2 \sin\theta dr d\theta d\varphi \quad (2)$$

where N is the number of concentric spherical bodies in the system ($i = 1, 2, \ldots, N$) and a_{i-1}, a_i are the inner and outer radii of the i^{th} body. We express the displacements u_i, v_i, w_i of the i^{th} body of the system as follows:

$$\begin{aligned} u_i(r,\theta,\varphi,t) &= U_i(r,\theta)\left[C(t)\cos(m\varphi) + S(t)\cos(m\varphi)\right] \\ v_i(r,\theta,\varphi,t) &= V_i(r,\theta)\left[C(t)\sin(m\varphi) - S(t)\cos(m\varphi)\right] \end{aligned} \quad (3)$$

$$w_i\left(r,\theta,\varphi,t\right)=W_i\left(r,\theta\right)\left[C\left(t\right)\cos\left(m\varphi\right)+S\left(t\right)\cos\left(m\varphi\right)\right]$$

where $U_i = U_i\left(r,\theta\right)$, $V_i = V_i\left(r,\theta\right)$, $W_i = W_i\left(r,\theta\right)$ are eigenfunctions of the system corresponding to the eigenvalue ω, which will be calculated later, and $m \in N$ is the circumferential wave number.

After substituting Equation (3) into Equation (2) we obtain an expression for the kinetic energy of the system $T = T(\dot{C},\dot{S},C,S)$. The system of equations for the mode under consideration is ([1])

$$\ddot{C}+2\eta\Omega\dot{S}+\omega^2C=0;\qquad \ddot{S}-2\eta\Omega\dot{C}+\omega^2S=0 \tag{4}$$

where η is the so-called “Bryan’s factor” ([2]) and is defined as follows:

$$-1\le\eta=\frac{2\cdot\sum_{i=1}^{N}\left\{\rho_i\cdot\int_0^{\pi}\int_{a_{i-1}}^{a_i}\left(U_i\cos\theta+W_i\sin\theta\right)V_ir^2\sin\theta drd\theta\right\}}{\sum_{i=1}^{N}\left\{\rho_i\cdot\int_0^{\pi}\int_{a_{i-1}}^{a_i}\left(U_i^2+V_i^2+W_i^2\right)r^2\sin\theta drd\theta\right\}}\le 1 \tag{5}$$

Bryan’s factor may be interpreted as follows: First, combine the two equations of system (4) by considering $X = C + iS$, where $i^2 = -1$ ([1]), [3]). Now apply the transformation

$$X\left(t\right)=Y\left(t\right)\cdot\exp(i\eta\Omega t) \tag{6}$$

and neglect $O\left(\Omega^2\right)$ terms. This yields the approximate relationship $\ddot{Y}+\omega^2Y=0$, which is the harmonic oscillator equation with two degrees of freedom. Hence, Equation (6) indicates that the vibrating pattern rotates with angular rate $\eta\Omega$ (in the rotating reference frame $Oxyz$). This rotation is in the direction of rotation of the system, if $\eta>0$, and in the opposite direction, if $\eta<0$.

3. EQUATIONS OF MOTION AND THEIR SOLUTIONS

Using [4] and its notation, the equations of motion of a solid body in spherical coordinate system are

$$\begin{aligned}
&\frac{\partial\sigma_{rr}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial\sigma_{r\varphi}}{\partial\varphi}+\frac{2\sigma_{rr}-\sigma_{\theta\theta}-\sigma_{\varphi\varphi}+\cot\theta\,\sigma_{r\theta}}{r}=\rho\frac{\partial^2u}{\partial t^2}\\
&\frac{\partial\sigma_{r\theta}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial\sigma_{\theta\varphi}}{\partial\varphi}+\frac{3\sigma_{r\theta}+\cot\theta\left(\sigma_{\theta\theta}-\sigma_{\varphi\varphi}\right)}{r}=\rho\frac{\partial^2v}{\partial t^2}\\
&\frac{\partial\sigma_{r\varphi}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\varphi}}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial\sigma_{\varphi\varphi}}{\partial\varphi}+\frac{3\sigma_{r\varphi}+2\cot\theta\,\sigma_{\theta\varphi}}{r}=\rho\frac{\partial^2w}{\partial t^2}
\end{aligned} \tag{7}$$

where stresses are given by

$$\sigma_{rr} = \lambda\left(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{\varphi\varphi}\right) + 2\mu\varepsilon_{rr}; \quad \sigma_{\theta\theta} = \lambda\left(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{\varphi\varphi}\right) + 2\mu\varepsilon_{\theta\theta}; \quad \sigma_{\varphi\varphi} = \lambda\left(\varepsilon_{rr} + \varepsilon_{\theta\theta} + \varepsilon_{\varphi\varphi}\right) + 2\mu\varepsilon_{\varphi\varphi};$$
$$\sigma_{\theta\varphi} = \mu\varepsilon_{\theta\varphi}; \qquad \sigma_{r\varphi} = \mu\varepsilon_{r\varphi}; \qquad \sigma_{r\theta} = \mu\varepsilon_{r\theta} \tag{8}$$

and strains are given by

$$\varepsilon_{rr} = \frac{\partial w}{\partial r}; \quad \varepsilon_{\theta\theta} = \frac{1}{r}\left(\frac{\partial u}{\partial \theta} + w\right); \quad \varepsilon_{\varphi\varphi} = \frac{1}{r}\left(\cot\theta\, u + \frac{1}{\sin\theta}\frac{\partial v}{\partial \varphi} + w\right); \tag{9}$$
$$\varepsilon_{\theta\varphi} = \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial u}{\partial \varphi} + \frac{\partial v}{\partial \theta} - \cot\theta\, v\right); \quad \varepsilon_{r\varphi} = \frac{\partial v}{\partial r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial w}{\partial \varphi} - v\right); \quad \varepsilon_{r\theta} = \frac{\partial u}{\partial r} + \frac{1}{r}\left(\frac{\partial w}{\partial \theta} - u\right)$$

By means of a change of variables $(u, v, w) \to (\Phi, \Psi, \mathrm{X})$, the system of Equation (7) becomes:

$$w = \left\{\frac{\partial \Phi}{\partial r} + \left[\frac{\partial^2 (r\mathrm{X})}{\partial r^2} + r\frac{\rho\omega^2}{\mu}\mathrm{X}\right]\right\} e^{i\omega t}; \tag{10}$$
$$u = \left\{\frac{1}{r}\frac{\partial}{\partial \theta}\left[\Phi + \frac{\partial (r\mathrm{X})}{\partial r}\right] + \frac{1}{a\sin\theta}\frac{\partial \Psi}{\partial \varphi}\right\} e^{i\omega t}; \quad v = \left\{\frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi}\left[\Phi + \frac{\partial (r\mathrm{X})}{\partial r}\right] - \frac{1}{a}\frac{\partial \Psi}{\partial \theta}\right\} e^{i\omega t}$$

[5] where a is a non-zero constant with the dimension of length (for example, it could be the radius of the first sphere) and $\Phi = \Phi(r, \theta, \varphi)$, $\mathrm{X} = \mathrm{X}(r, \theta, \varphi)$ and $\Psi = \Psi(r, \theta, \varphi)$ satisfy the Helmholtz equations:

$$\nabla^2 \Phi + k_1^2(\omega)\Phi = 0, \quad \nabla^2 \mathrm{X} + k_2^2(\omega)\mathrm{X} = 0, \quad \nabla^2 \Psi + k_2^2(\omega)\Psi = 0 \tag{11}$$

where $k_1^2(\omega) = \rho\omega^2 / (\lambda + 2\mu)$, $k_2^2(\omega) = \rho\omega^2 / \mu$ with ∇^2 the Laplace operator in spherical coordinates ([3]). The solutions to Equation (11) are:

$$\begin{aligned}
\Phi_{m,n}(r, \theta, \varphi, \omega) &= \left[B_1 j_n\left(k_1(\omega) r\right) + B_2 y_n\left(k_1(\omega) r\right)\right] P_n^m(\cos\theta)\cos(m\varphi) \\
\mathrm{X}_{m,n}(r, \theta, \varphi, \omega) &= \left[B_3 j_n\left(k_2(\omega) r\right) + B_4 y_n\left(k_2(\omega) r\right)\right] P_n^m(\cos\theta)\cos(m\varphi) \\
\Psi_{m,n}(r, \theta, \varphi, \omega) &= \left[B_5 j_n\left(k_2(\omega) r\right) + B_6 y_n\left(k_2(\omega) r\right)\right] P_n^m(\cos\theta)\sin(m\varphi)
\end{aligned} \tag{12}$$

where $B_1, B_2, \ldots, B_6$ are arbitrary constants (if the body contains the centre *O*, the constants $B_2 = B_4 = B_6 = 0$).

The motion of a compressible inviscid fluid is represented by the following wave equation:

$$\nabla^2 p + k_3^2(\omega)\, p = 0 \tag{13}$$

with $k_3^2(\omega) = E^{(f)} / \rho^{(f)}$, where $E^{(f)}$ is the bulk modulus of the fluid and $\rho^{(f)}$ the mass density of the fluid. The solution to this equation is:

$$p_{m,n}(r,\theta,\varphi,t) = \left\{ \left[B_7 j_n\left(k_3(\omega) r\right) + B_8 y_n\left(k_3(\omega) r\right) \right] P_n^m(\cos\theta) \cos(m\varphi) \right\} e^{i\omega t} \tag{14}$$

with $p = p_{m,n}(r,\theta,\varphi,t)$ the pressure in the fluid. Particle displacement of the fluid in the radial direction is:

$$w^{(f)} = \frac{1}{\rho^{(f)}\omega^2} \cdot \frac{\partial p}{\partial r} \tag{15}$$

4. BOUNDARY CONDITIONS AND EIGENFUNCTIONS

Boundary conditions of the system define the eigenvalues ω. It is possible to distinguish between *spheroidal* and *torsional modes*. For the *spheroidal mode* we assume that $\Psi \equiv 0$ ([4]) and the stress components of the solid are:

$$\begin{aligned}
\sigma_{rr} &= \left[\mu \frac{\partial^2 \Phi}{\partial r^2} - \lambda k_1^2(\omega) \Phi \right] + 2\mu \frac{\partial}{\partial r} \left[\frac{\partial^2 (r\mathrm{X})}{\partial r^2} + r k_2^2(\omega) \mathrm{X} \right] \\
\sigma_{r\theta} &= \frac{2\mu}{r} \frac{\partial}{\partial \theta} \left\{ \left(\frac{\partial \Phi}{\partial r} - \frac{\Phi}{r} \right) + \left[r \frac{\partial^2 \mathrm{X}}{\partial r^2} + \frac{\partial \mathrm{X}}{\partial r} + \left(\frac{r k_2^2(\omega)}{2} - \frac{1}{r} \right) \mathrm{X} \right] \right\} \\
\sigma_{r\varphi} &= \frac{2\mu}{r \sin\theta} \frac{\partial}{\partial \varphi} \left\{ \left(\frac{\partial \Phi}{\partial r} - \frac{\Phi}{r} \right) + \left[r \frac{\partial^2 \mathrm{X}}{\partial r^2} + \frac{\partial \mathrm{X}}{\partial r} + \left(\frac{r k_2^2(\omega)}{2} - \frac{1}{r} \right) \mathrm{X} \right] \right\}
\end{aligned} \tag{16}$$

For the *torsional mode* we suppose that $\Phi \equiv \mathrm{X} \equiv 0$ and corresponding stress components are:

$$\sigma_{r\theta} = \frac{\mu}{a \sin\theta} \frac{\partial}{\partial \varphi} \left(\frac{\partial \Psi}{\partial r} - \frac{\Psi}{r} \right) \text{ and } \sigma_{r\varphi} = -\frac{\mu}{a} \frac{\partial}{\partial \theta} \left(\frac{\partial \Psi}{\partial r} - \frac{\Psi}{r} \right) \tag{17}$$

Let us model a thick solid sphere filled with inviscid fluid. In this case, let $a_0 = 0,\ a_1 = a,\ a_2 = b$. Considering the spheroidal mode (because the torsional mode does not interact with an inviscid fluid), one can obtain the solutions:

$$\begin{aligned}
p = p_{m,n}(r,\theta,\varphi,t) &= \left[A_1 j_n\left(k_3(\omega) r\right) \right] P_n^m(\cos\theta) \cos(m\varphi) e^{i\omega t} \\
\Phi = \Phi_{m,n}(r,\theta,\varphi,\omega) &= \left[A_2 j_n\left(k_1(\omega) r\right) + A_3 y_n\left(k_1(\omega) r\right) \right] P_n^m(\cos\theta) \cos(m\varphi) \\
\mathrm{X} = \mathrm{X}_{m,n}(r,\theta,\varphi,\omega) &= \left[A_4 j_n\left(k_2(\omega) r\right) + A_5 y_n\left(k_2(\omega) r\right) \right] P_n^m(\cos\theta) \cos(m\varphi)
\end{aligned} \tag{18}$$

The following boundary conditions express the balance between the radial components of stress and pressure between the solid and fluid and the equality of their radial displacements at $r = a$. Furthermore, they mention the absence of stresses at the outer surface of the solid sphere ($r = b$):

$$r=a: \quad \left\{\left[\mu \frac{\partial^2 \Phi}{\partial r^2} - \lambda k_1^2(\omega)\Phi\right] + 2\mu \frac{\partial}{\partial r}\left[\frac{\partial^2 (rX)}{\partial r^2} + r k_2^2(\omega) X\right]\right\}_{r=a} = -p\big|_{r=a}$$

$$\left\{\frac{\partial \Phi}{\partial r} + \left[\frac{\partial^2 (rX)}{\partial r^2} + r k_2^2(\omega) X\right]\right\}_{r=a} = \frac{1}{\rho^{(f)}\omega^2}\left\{\frac{\partial p}{\partial r}\right\}_{r=a} \tag{19}$$

$$\left\{\left(\frac{\partial \Phi}{\partial r} - \frac{\Phi}{r}\right) + \left[r\frac{\partial^2 X}{\partial r^2} + \frac{\partial X}{\partial r} + \left(\frac{r k_2^2(\omega)}{2} - \frac{1}{r}\right) X\right]\right\}_{r=a} = 0$$

$$r=b \quad \left\{\left[\mu \frac{\partial^2 \Phi}{\partial r^2} - \lambda k_1^2(\omega)\Phi\right] + 2\mu \frac{\partial}{\partial r}\left[\frac{\partial^2 (rX)}{\partial r^2} + r k_2^2(\omega) X\right]\right\}_{r=b} = 0 \tag{20}$$

$$\left\{\left(\frac{\partial \Phi}{\partial r} - \frac{\Phi}{r}\right) + \left[r\frac{\partial^2 \mathrm{X}}{\partial r^2} + \frac{\partial \mathrm{X}}{\partial r} + \left(\frac{r k_2^2(\omega)}{2} - \frac{1}{r}\right) \mathrm{X}\right]\right\}_{r=b} = 0$$

By substituting Equation (18) into Equation (10) and simplifying, we obtain the following eigenfunctions (where superscript (*f*) indicates the quantities for the fluid):

$$\begin{aligned}
U(r,\theta) &= \frac{1}{r}\Big\{A_2 j_n(k_1 r) + A_3 y_n(k_1 r) + \\
&\quad A_4\left[(n+1) j_n(k_2 r) - k_2 r j_{n+1}(k_2 r)\right] + A_5\left[(n+1) y_n(k_2 r) - k_2 r y_{n+1}(k_2 r)\right]\Big\}\times \\
&\quad \left\{-(n+1)\cot\theta \cdot P_n^m(\cos\theta) + \frac{n-m+1}{\sin\ \theta} P_{n+1}^m(\cos\theta)\right\} \\
V(r,\theta) &= -\frac{m}{r\sin\theta}\Big\{A_2 j_n(k_1 r) + A_3 y_n(k_1 r) + A_4\left[(n+1) j_n(k_2 r) - k_2 r j_{n+1}(k_2 r)\right] \\
&\quad + A_5\left[(n+1) y_n(k_2 r) - k_2 r y_{n+1}(k_2 r)\right]\Big\} P_n^m(\cos\theta) \\
W(r,\theta) &= \left\{A_2\left[\frac{n}{r} j_n(k_1 r) - k_1 j_{n+1}(k_1 r)\right] + A_3\left[\frac{n}{r} y_n(k_1 r) - k_1 y_{n+1}(k_1 r)\right] + \right. \\
&\quad \left. A_4\left[\frac{n(n+1)}{r} j_n(k_2 r)\right] + A_5\left[\frac{n(n+1)}{r} y_n(k_2 r)\right]\right\} P_n^m(\cos\theta) \\
U^{(f)}(r,\theta) &= \frac{1}{r} A_1 j_n(k_3 r)\cdot\left\{-(n+1)\cot\theta\cdot P_n^m(\cos\theta) + \frac{n-m+1}{\sin\ \theta} P_{n+1}^m(\cos\theta)\right\}; \\
V^{(f)}(r,\theta) &= -\frac{m}{r\sin\theta} A_1 j_n(k_3 r) P_n^m(\cos\theta) \\
W^{(f)}(r,\theta) &= \left\{A_1\left[\frac{n}{r} j_n(k_3 r) - k_3 j_{n+1}(k_3 r)\right]\right\} P_n^m(\cos\theta)
\end{aligned} \tag{21}$$

4.1 Example

Let us consider the spheroidal vibrations of a thick spherical shell (with inner radius $a = 0.4\,m$, outer radius $b = 0.5\,m$), made from brass ($E = 100\,MPa$, $\rho = 8500\,{kg}/{m^3}$ and $\nu = 0.34$), and filled with water ($E^{(f)} = 2.2\,MPa$, $\rho^{(f)} = 1000\,{kg}/{m^3}$)) that is involved in rotation with a constant angular rate.

Suppressing the mode number subscripts *m*,*n*, the Bryan's factor for this structure is given by:

$$\eta = \frac{2\left\{\int_0^\pi \left[\int_0^a \rho^{(f)}\left(U^{(f)}\cos\theta + W^{(f)}\sin\theta\right)V^{(f)} r^2 dr + \int_a^b \rho\left(U\cos\theta + W\sin\theta\right)V\, r^2 dr\right]\sin\theta d\theta\right\}}{\int_0^\pi \left[\int_0^a \rho^{(f)}\left(U^{(f)2} + V^{(f)2} + W^{(f)2}\right) r^2 dr + \int_a^b \rho\left(U^2 + V^2 + W^2\right) r^2 dr\right]\sin\theta d\theta}$$

Calculations of eigenvalues and the corresponding Bryan's factors are given in Table 1, where ω_i indicates the eigenvalues and η_i the Bryan's factor:

Table 1. Eigenvalues and corresponding Bryan's factors

n	*m*	ω_1 (Hz) η_1	ω_2 (Hz) η_2	ω_3 (Hz) η_3	ω_4 (Hz) η_4	ω_5 (Hz) η_5
2	2	1791.171 -0.9107	1972.378 -0.7738	2093.701 -0.7952	2989.117 0.4319	4302.243 -0.0848
3	2	2664.052 -0.4775	2915.703 -0.547	3605.416 -0.5559	4101.496 0.2860	5065.809 -0.0648
3	3	2664.052 -0.7162	2915.703 -0.8204	3605.416 -0.8339	4101.496 0.4290	5065.809 -0.0973
4	2	3332.472 -0.3366	3981.479 -0.4397	5071.341 0.1735	5193.817 0.1635	5807.461 -0.0521
4	3	3332.472 -0.5048	3981.479 -0.6596	5071.341 0.2602	5193.817 0.2453	5807.461 -0.0781
4	4	3332.472 -0.6731	3981.479 -0.8795	5071.341 0.3470	5193.817 0.3270	5807.461 -0.1041

5. CONCLUSIONS

1. Gyroscopic effects in rotating symmetric distributed bodies were considered and the dependence of the rate of rotation of the vibrating pattern on inertial angular rate of the system was determined. This dependence is described by the so-called "Bryan's factor" which is calculated in spherical coordinates.
2. Solutions to the dynamic equations of elastic solid and fluid bodies composed of concentric spherical layers were obtained and boundary conditions were formulated for calculating eigenvalues and eigenfunctions for the system.

3. The results of the general theory were applied to an example of a rotating elastic, thick spherical shell filled with an inviscid compressible fluid. Eigenvalues and Bryan's factors were calculated and tabulated for various vibration modes. It was observed that negative Bryan's factors predominate in the table. However, no discernible pattern for the sign of the Bryan's factor is obvious from the table. Furthermore, for lower eigenvalues and lower circumferential wave numbers, the difference between the modulus of the rotational angular rates of the fluid-filled sphere and those of its vibrating patterns is small ($|\eta| \approx 1$). However, this difference is large for higher modes and eigenvalues of the system ($|\eta| \approx 0$).

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