SCATTERING OF OBLIQUELY INCIDENT STANDING WAVE BY A ROTATING TRANSVERSELY ISOTROPIC CYLINDER

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Abstract

It is known that vibrating patterns of an isotropic cylinder, subjected to inertial rotation over the symmetry axis, precess in the direction of rotation with a different angular rate. The proportionality coefficient, or the so-called Bryan factor, equal to the ratio of vibrating patterns rate to the inertial angular rate, depend on particular thickness modes as well as circumferential wave numbers. This effect is retained in the case of a transversely isotropic cylinder if the axis of anisotropy coincides with the axis of the cylinder. In the present paper we consider the case when the vibrating patterns of a rotating transversely isotropic cylinder are generated by an obliquely incident plane acoustic wave. In this case the capture effect exists, which demonstrate finite angular declinations of particular vibrating patterns at different angles depending on modes and circumferential numbers. The resulting scattered acoustic field is very specific, because the lower mode components, corresponding to the lower circumferential wave numbers are found to be turned over the cylinder's axis at higher angles compare to the higher modes. The angles of relative rotations of the modes are proportional to the abovementioned Bryan factors. These effects in the scattered patterns of the solid cylinder due to the inertial rotation are investigated.

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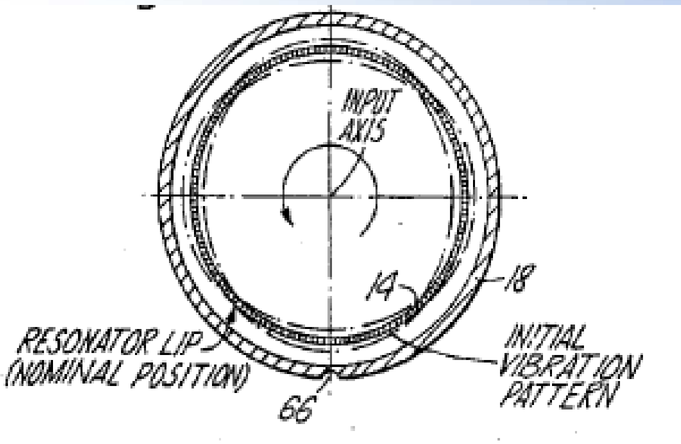
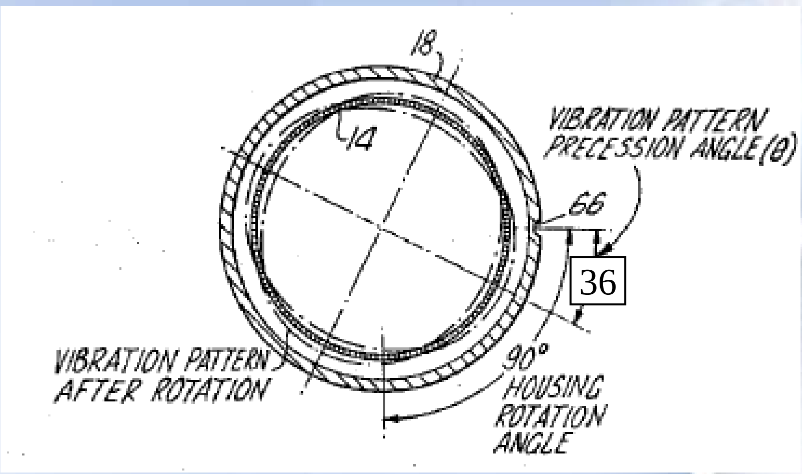
Bryan G.H. ***"On the beats in the vibrations of a revolving Cylinder or Bell"***, Proc. Cambridge Philos.Soc.Math.Phys. Sci., 1890, vol.7





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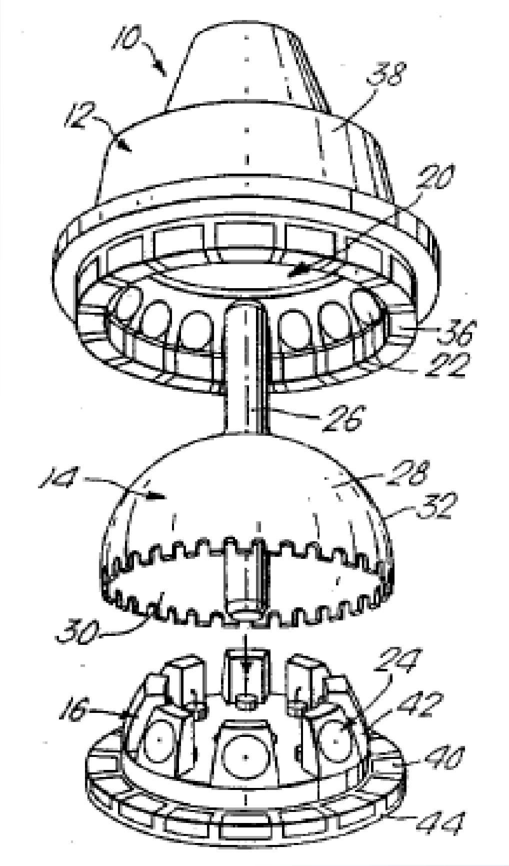

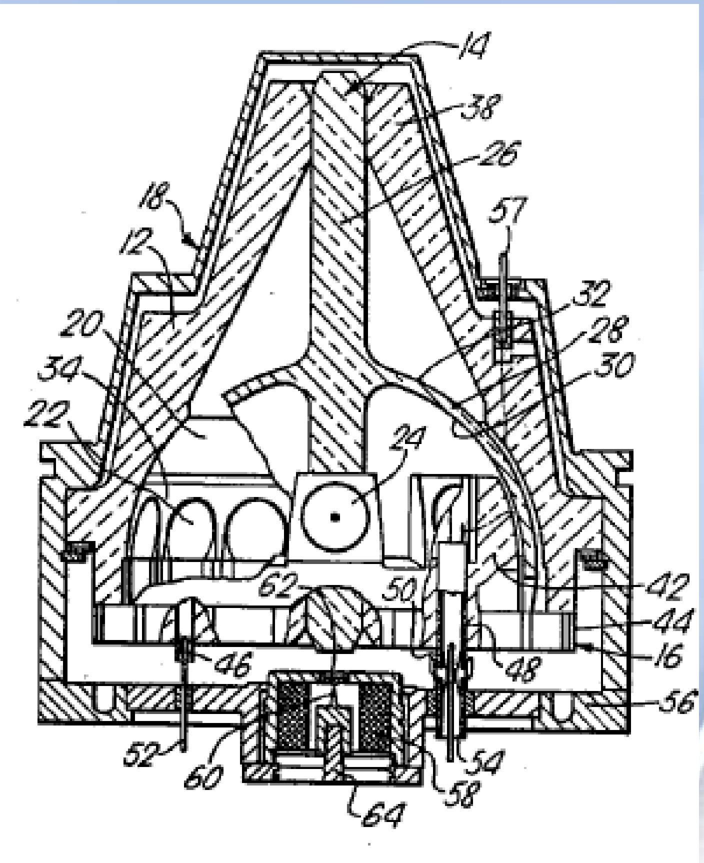

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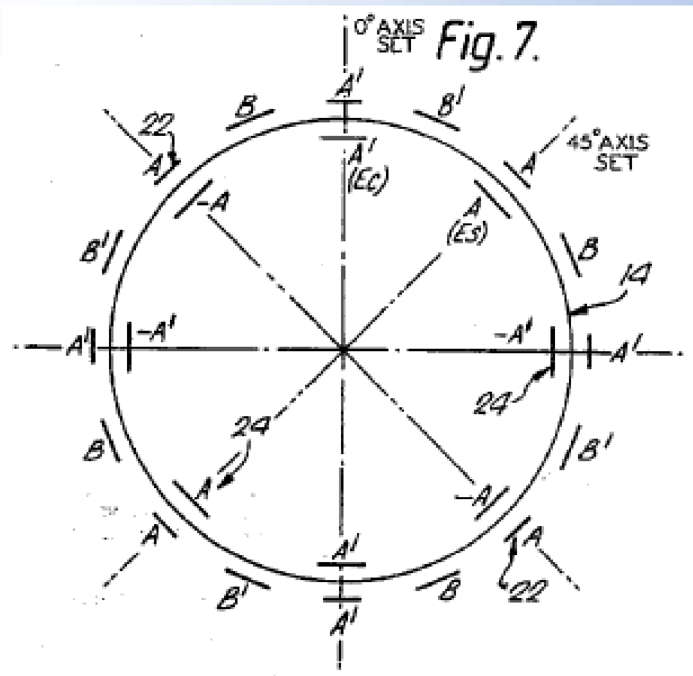

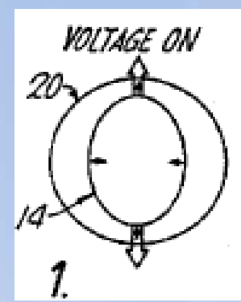

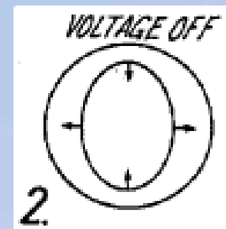

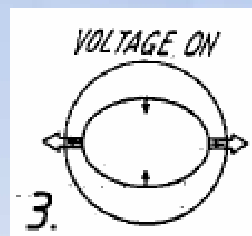

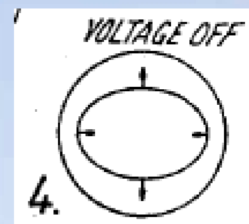

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EXPLANATION OF BRYAN's EFFECT

Equations of motion:

$$\begin{cases} \ddot{C} - 2\eta\,\Omega\,\dot{S} + \omega^2 C = 0 \\ \ddot{S} + 2\eta\,\Omega\dot{C} + \omega^2 S = 0 \end{cases}$$

$$Z = C + iS \qquad \ddot{Z} + 2i\eta\,\Omega\,\dot{Z} + \omega^2 Z = 0$$

$Z \to Y:$

$$Z(t) = Y(t)\cdot e^{i\alpha t} \qquad \alpha = const$$

$$\dot{Z} = \left(\dot{Y} + i\,\alpha Y\right)\cdot e^{i\alpha t}; \qquad \ddot{Z} = \left(\ddot{Y} + 2i\,\alpha\dot{Y} - \alpha^2 Y\right)\cdot e^{i\alpha t}$$

$$\ddot{Y} + 2i\left(\alpha + \eta\,\Omega\right)\dot{Y} + \left(\omega^2 - \alpha^2 - 2\,\alpha\,\eta\,\Omega\right)Y = 0$$

$$\alpha = -\eta\,\Omega$$

$$\ddot{Y} + \omega^2 Y \approx 0 \qquad Z(t) = Y(t)\cdot e^{-i\eta\Omega t}$$

$$\overset{\square}{\Omega} = -\eta\,\Omega \qquad \overset{\square}{\Omega} = (1-\eta)\,\Omega$$

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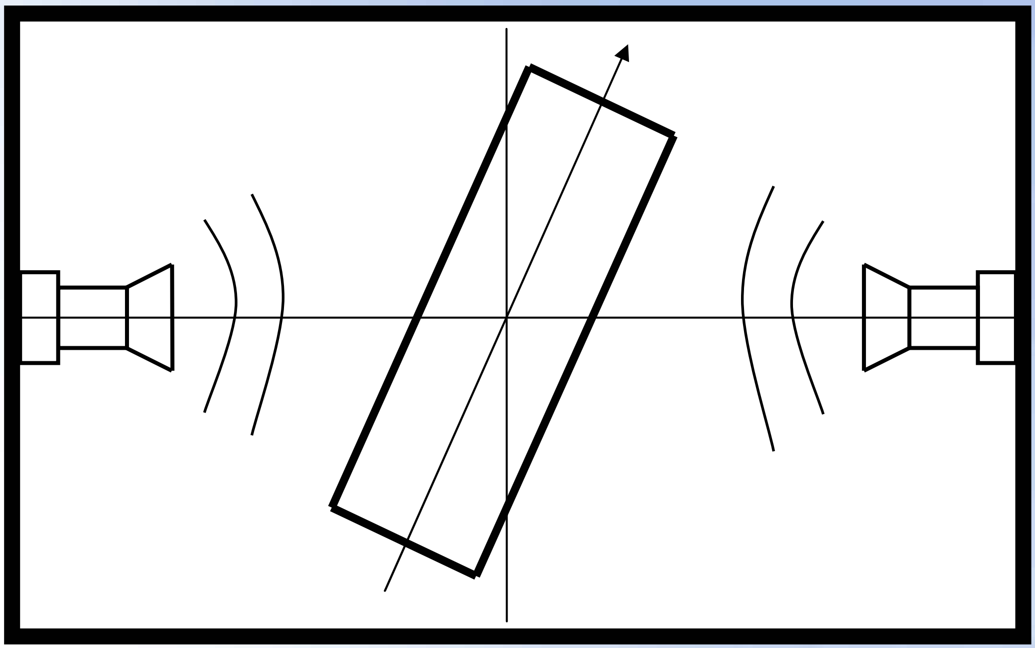
SONICATION OF THE CYLINDER



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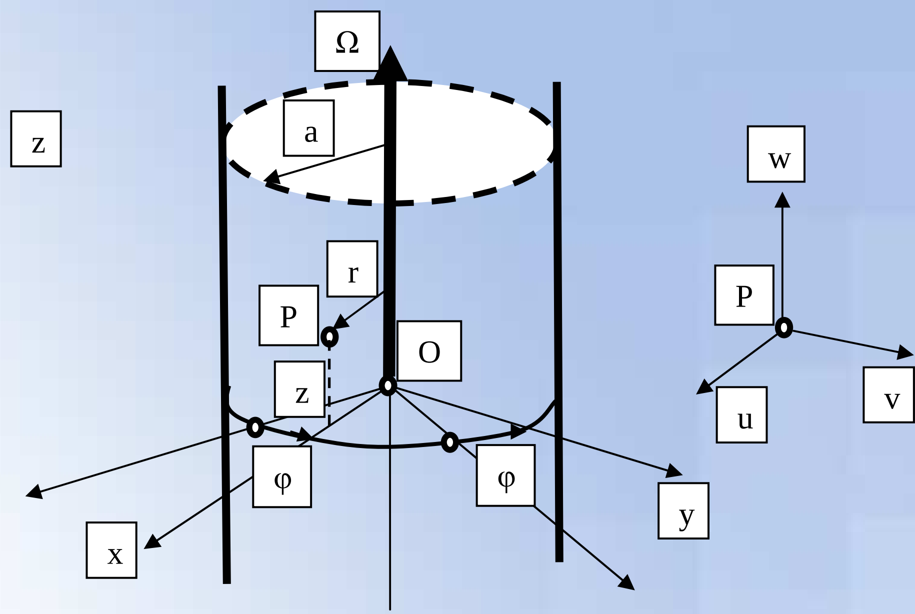

$$\frac{\partial\, \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial\, \sigma_{r\varphi}}{\partial \varphi} + \frac{\partial\, \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} = \rho\left(\frac{\partial^2 u}{\partial t^2} - 2\Omega\frac{\partial\, v}{\partial t}\right)$$

$$\frac{\partial\, \sigma_{r\varphi}}{\partial r} + \frac{1}{r}\frac{\partial\, \sigma_{\varphi\varphi}}{\partial \varphi} + \frac{\partial\, \sigma_{\varphi z}}{\partial z} + \frac{2\sigma_{r\varphi}}{r} = \rho\left(\frac{\partial^2 v}{\partial t^2} + 2\Omega\frac{\partial\, u}{\partial t}\right)$$

$$\frac{\partial\, \sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial\, \sigma_{\varphi z}}{\partial \varphi} + \frac{\partial\, \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = \rho\frac{\partial^2 w}{\partial t^2}$$

TRANSVERSELY ISOTROPIC CYLINDERS - 2

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\varphi}}{\partial \varphi} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\varphi\varphi}}{r} = \rho\left(\frac{\partial^2 u}{\partial t^2} - 2\Omega\frac{\partial v}{\partial t}\right)$$

$$\frac{\partial \sigma_{r\varphi}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\varphi\varphi}}{\partial \varphi} + \frac{\partial \sigma_{\varphi z}}{\partial z} + \frac{2\sigma_{r\varphi}}{r} = \rho\left(\frac{\partial^2 v}{\partial t^2} + 2\Omega\frac{\partial u}{\partial t}\right)$$

$$\frac{\partial \sigma_{rz}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\varphi z}}{\partial \varphi} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = \rho\frac{\partial^2 w}{\partial t^2}$$

$$\sigma_{rr} = c_{11}\varepsilon_{rr} + c_{12}\varepsilon_{\varphi\varphi} + c_{11}\varepsilon_{zz}; \quad \sigma_{\varphi\varphi} = c_{12}\varepsilon_{rr} + c_{11}\varepsilon_{\varphi\varphi} + c_{13}\varepsilon_{zz}; \quad \sigma_{zz} = c_{13}\left(\varepsilon_{rr} + \varepsilon_{\varphi\varphi}\right) + c_{33}\varepsilon_{zz};$$

$$\sigma_{z\varphi} = c_{44}\varepsilon_{z\varphi}; \qquad \sigma_{rz} = c_{44}\varepsilon_{rz}; \qquad \sigma_{r\varphi} = c_{66}\varepsilon_{r\varphi} \qquad \left(c_{66} = \frac{1}{2}(c_{11} - c_{12})\right)$$

$$\varepsilon_{rr} = \frac{\partial u}{\partial r}; \qquad \varepsilon_{\varphi\varphi} = \frac{1}{r}\left(u + \frac{\partial v}{\partial \varphi}\right); \qquad \varepsilon_{zz} = \frac{\partial w}{\partial z};$$

$$\varepsilon_{z\varphi} = \frac{\partial v}{\partial z} + \frac{1}{r}\frac{\partial w}{\partial \varphi}; \qquad \varepsilon_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}; \qquad \varepsilon_{r\varphi} = \frac{\partial u}{\partial r} + \frac{1}{r}\left(\frac{\partial u}{\partial \varphi} - v\right)$$

TRANSVERSELY ISOTROPIC CYLINDERS - 3

$$c_{11}\left(\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right)+\frac{c_{66}}{r^2}\frac{\partial^2 u}{\partial \varphi^2}+c_{44}\frac{\partial^2 u}{\partial r^2}+\frac{c_{11}-c_{66}}{r}\frac{\partial^2 v}{\partial r\,\partial \varphi}-\frac{c_{11}+c_{66}}{r^2}\frac{\partial v}{\partial \varphi}+\left(c_{13}+c_{44}\right)\frac{\partial^2 w}{\partial r\,\partial z}=\rho\left(\frac{\partial^2 u}{\partial t^2}-2\Omega\frac{\partial v}{\partial t}\right);$$

$$\frac{c_{11}-c_{66}}{r}\frac{\partial^2 u}{\partial r\,\partial \varphi}+\frac{c_{11}+c_{66}}{r^2}\frac{\partial u}{\partial \varphi}+c_{66}\left(\frac{\partial^2 v}{\partial r^2}+\frac{1}{r}\frac{\partial v}{\partial r}-\frac{v}{r^2}\right)+\frac{c_{11}}{r^2}\frac{\partial^2 v}{\partial \varphi^2}+c_{44}\frac{\partial^2 v}{\partial z^2}+\frac{c_{13}+c_{44}}{r}\frac{\partial^2 w}{\partial \varphi\,\partial z}=\rho\left(\frac{\partial^2 v}{\partial t^2}+2\Omega\frac{\partial u}{\partial t}\right);$$

$$\left(c_{13}+c_{44}\right)\left[\frac{\partial^2 u}{\partial r\,\partial z}+\frac{1}{r}\left(\frac{\partial u}{\partial z}+\frac{\partial^2 v}{\partial \varphi\,\partial z}\right)\right]+c_{44}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{r^2}\frac{\partial^2 w}{\partial \varphi^2}\right)+c_{33}\frac{\partial^2 w}{\partial z^2}=\rho\frac{\partial^2 w}{\partial t^2}$$

$$u=\sum_{m=0}^{\infty}\left[\frac{\partial \Phi_1^{(2m)}}{\partial r}+\frac{\partial \Phi_2^{(2m)}}{\partial r}+\frac{1}{r}\frac{\partial \mathrm{X}}{\partial \varphi}\right]\cos\left(k_t z\right)\cos\left(\nu t\right);$$

$$v=\sum_{m=0}^{\infty}\left[\frac{1}{r}\left(\frac{\partial \Phi_1^{(2m)}}{\partial \varphi}+\frac{\partial \Phi_2^{(2m)}}{\partial \varphi}\right)+\frac{\partial \mathrm{X}}{\partial r}\right]\cos\left(k_t z\right)\cos\left(\nu t\right);$$

$$w=\sum_{m=0}^{\infty}\left[\eta_1\Phi_1^{(2m)}+\eta_2\Phi_2^{(2m)}\right]\sin\left(k_t z\right)\cos\left(\nu t\right)$$

$$\nabla^2\mathrm{X}_1+\varsigma_1{}^2\mathrm{X}_1=0;\qquad \nabla^2\mathrm{X}_2+\varsigma_2{}^2\mathrm{X}_2=0;\qquad \nabla^2\Psi+\varsigma_3{}^2\Psi=0$$

TRANSVERSELY ISOTROPIC CYLINDERS - 4

$$\Delta X_1 + \varsigma_1^{\,2} X_1 = 0; \qquad \Delta X_2 + \varsigma_2^{\,2} X_2 = 0; \qquad \Delta\Psi + \varsigma_3^{\,2}\Psi = 0$$

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial}{\partial \varphi^2}$$

$$\Phi_i^{(2m)} = \Phi_i^{(2m)}(r,\varphi) = A_i^{(2m)} J_{2m}(\varsigma_i r)\cos(2m\varphi) \qquad X = X(r,\varphi) = A_3^{(2m)} J_{2m}(\varsigma_3 r)\sin(2m\varphi)$$

$$(i = 1,2)$$

$$\varsigma_{1,2} = \sqrt{\frac{B \pm \sqrt{B^2 - 4AC}}{2A}}; \quad \zeta_3 = \sqrt{\frac{\rho\omega^2 - \hat{k}^2 c_{44}}{c_{66}}}$$

$$A = c_{11}c_{44}; \quad B = (c_{11} + c_{44})\rho\omega^2 + \left(c_{13}^{\,2} + 2c_{13}c_{44} - c_{11}c_{33}\right)\hat{k}^2; \quad C = \left(\rho\omega^2 - \hat{k}^2 c_{33}\right)\left(\rho\omega^2 - \hat{k}^2 c_{44}\right)$$

$$X_1(r,\varphi) = R_1(r)\cdot\Phi_1(\varphi); \quad X_2(r,\varphi) = R_2(r)\cdot\Phi_2(\varphi); \quad \Psi(r,\varphi) = R_3(r)\cdot\Phi_3(\varphi)$$

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OBLIQUELY INCIDENT ACOUSTIC STANDING WAVE

$$P^{(ac)} = P_0 \cos(k_t z)\cos(k_n r \cos\varphi)\cos(\nu t)$$

$$k_t = k \sin\alpha \qquad k_n = k\cos\alpha \qquad k = \nu / c^{(ac)}$$

$$P^{(sc.ac)} = \sum_{m=0}^{\infty} \left\{ \left[P_0 (-1)^m \varepsilon_m J_{2m}(k_n r) + C_{2m} \cdot H_{2m}^{(2)}(k_n r) \right] \cdot \cos(2m\varphi) \right\} \cos(k_t z)\cos(\nu\, t)$$

Boundary Conditions:

$$\left[\sigma_{rr} + P^{(sc.ac)}\right]_{r=a} = 0; \quad \left[u - u^{(ac)}\right]_{r=a} = 0; \quad \left[\sigma_{r\varphi}\right]_{r=a} = \left[\sigma_{rz}\right]_{r=a} = 0$$

$$u^{(ac)} = \frac{1}{\rho^{(ac)}\omega^2} \frac{\partial\, P^{(sc.ac)}}{\partial r}$$

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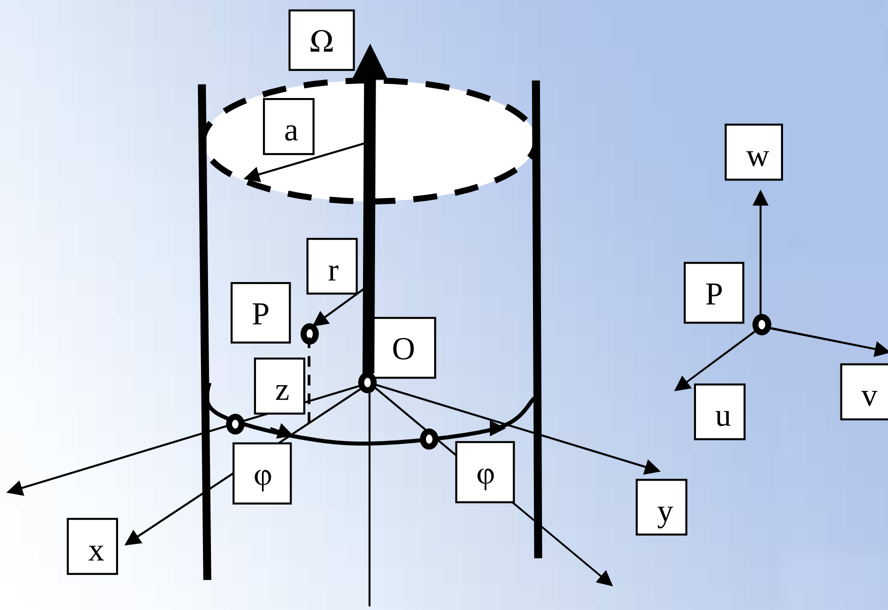
INFLUENCE OF INERTIAL ROTATION



$$\vec{V}_P^{(2m)} = \begin{bmatrix} \dot{u}_{2m} - \Omega \cdot v_{2m} \\ \dot{v}_{2m} + \Omega \cdot \left(r + u_{2m}\right) \\ \dot{w}_{2m} \end{bmatrix}$$

$$u_{2m} = u_{2m}(r,\varphi,z,t) = U_{2m}(r)\cdot\left[c_{2m}(t)\cos 2m\varphi + s_{2m}(t)\sin 2m\varphi\right]\cdot\cos(k_t z);$$

$$v_{2m} = v_{2m}(r,\varphi,z,t) = V_{2m}(r)\cdot\left[c_{2m}(t)\sin 2m\varphi - s_{2m}(t)\cos 2m\varphi\right]\cdot\cos(k_t z);$$

$$w_{2m} = w_{2m}(r,\varphi,z,t) = W_{2m}(r)\cdot\left[c_{2m}(t)\cos 2m\varphi + s_{2m}(t)\sin 2m\varphi\right]\cdot\sin(k_t z);$$

$$U_{2m}(r) = A_1^{(2m)}\left[\frac{dJ_{2m}(\varsigma_1 r)}{dr}\right] + A_2^{(2m)}\left[\frac{dJ_{2m}(\varsigma_2 r)}{dr}\right] + A_3^{(2m)}\left[\frac{2m}{r}J_{2m}(\varsigma_3 r)\right];$$

$$V_{2m}(r) = A_1^{(2m)}\left[-\frac{2m}{r}J_{2m}(\varsigma_1 r)\right] + A_2^{(2m)}\left[-\frac{2m}{r}J_{2m}(\varsigma_2 r)\right] + A_3^{(2m)}\left[-\frac{dJ_{2m}(\varsigma_3 r)}{dr}\right];$$

$$W_{2m}(r) = A_1^{(2m)}\left[\eta_1 J_{2m}(\varsigma_1 r)\right] + A_2^{(2m)}\left[\eta_2 J_{2m}(\varsigma_2 r)\right] + A_3^{(2m)}\cdot[0] \qquad \left(W_{2m}(r) = 0, \quad if \quad k_t = 0\right)$$

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LAGRANGIAN OF THE SYSTEM

$$L_{2m} = T_{2m} - P_{2m} + A_{2m}$$

$$T_{2m} = \frac{\rho}{2 \cdot 2\pi} \cdot \int_0^{2\pi} \int_0^a \left| \vec{V}_P^{(2m)} \right|^2 \cdot r\, dr\, d\varphi; \quad P_{2m} = \frac{1}{2} \cdot I_2^{(2m)} \cdot \left[c_{2m}^2(t) + s_{2m}^2(t) \right];$$

$$A_{2m} = \frac{a \cdot U_{2m}{}^*(a)}{2} \cdot \left[c_{2m} P_{2m,c}^{(sc.ac)} + s_{2m} P_{2m,s}^{(sc.ac)} \right]_{r=a}$$

$$P = \frac{1}{2} \cdot \int_0^{2\pi} \int_0^a \left[\sigma_{rr} \varepsilon_{rr}{}^* + \sigma_{\varphi\varphi} \varepsilon_{\varphi\varphi}{}^* + \sigma_{zz} \varepsilon_{zz}{}^* + \sigma_{\varphi z} \varepsilon_{\varphi z}{}^* + \sigma_{rz} \varepsilon_{rz}{}^* + \sigma_{r\varphi} \varepsilon_{r\varphi}{}^* \right] \cdot r\, dr\, d\varphi =$$

$$= \frac{1}{2} \cdot \int_0^{2\pi} \int_0^a \left\{ \begin{array}{l} c_{11} \left(\varepsilon_{rr} \varepsilon_{rr}{}^* + \varepsilon_{\varphi\varphi} \varepsilon_{\varphi\varphi}{}^* \right) + c_{33} \left(\varepsilon_{zz} \varepsilon_{zz}{}^* \right) + c_{44} \left(\varepsilon_{\varphi z} \varepsilon_{\varphi z}{}^* + \varepsilon_{rz} \varepsilon_{rz}{}^* \right) + c_{66} \left(\varepsilon_{r\varphi} \varepsilon_{r\varphi}{}^* \right) \\ + c_{12} \left(\varepsilon_{rr} \varepsilon_{\varphi\varphi}{}^* + \varepsilon_{rr}{}^* \varepsilon_{\varphi\varphi} \right) + c_{13} \left[\left(\varepsilon_{rr} \varepsilon_{zz}{}^* + \varepsilon_{rr}{}^* \varepsilon_{zz} \right) + \left(\varepsilon_{\varphi\varphi} \varepsilon_{zz}{}^* + \varepsilon_{\varphi\varphi}{}^* \varepsilon_{zz} \right) \right] \end{array} \right\} \cdot r\, dr\, d\varphi$$

$$L_{2m} = L_{2m} \left(\dot{c}_{2m},, \dot{s}_{2m} c_{2m}, s_{2m} \right) = \frac{1}{2} \left\{ \begin{array}{c} I_0^{(2m)} \cdot \left(\dot{c}_{2m}^2 + \dot{s}_{2m}^2 \right) + 2\Omega I_1^{(2m)} \cdot \left(\dot{c}_{2m} s_{2m} - c_{2m} \dot{s}_{2m} \right) \\ -I_2^{(2m)} \cdot \left(c_{2m}^2 + s_{2m}^2 \right) + \frac{a \cdot U_{2m}{}^*(a)}{2} \cdot \left[c_{2m} P_{2m,c}^{(sc.ac)} + s_{2m} P_{2m,s}^{(sc.ac)} \right]_{r=a} \end{array} \right\}$$

$$I_0^{(2m)} = \frac{\rho}{2} \cdot \int_0^a \left[U_{2m} U_{2m}{}^* + V_{2m} V_{2m}{}^* + W_{2m} W_{2m}{}^* \right] \cdot r\, dr; \qquad I_1^{(2m)} = \frac{\rho}{2} \cdot \int_0^a \left[U_{2m} V_{2m}{}^* + U_{2m}{}^* V_{2m} \right] \cdot r\, dr;$$

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SOLUTIONS OF EQUATIONS OF MOTION - 1

$$\frac{d}{dt}\left(\frac{\partial L_{2m}}{\partial \dot{c}_{2m}}\right)-\frac{\partial L_{2m}}{\partial c_{2m}}=0; \qquad \frac{d}{dt}\left(\frac{\partial L_{2m}}{\partial \dot{s}_{2m}}\right)-\frac{\partial L_{2m}}{\partial s_{2m}}=0$$

$$\ddot{c}_{2m}+2\eta_{2m}\Omega\dot{s}_{2m}+\omega_{2m}^{2}c_{2m}=f_{2m}^{(c)}\cos(\nu t)$$

$$\ddot{s}_{2m}-2\eta_{2m}\Omega\dot{c}_{2m}+\omega_{2m}^{2}s_{2m}=0$$

$$\eta_{2m}=\frac{I_1^{(2m)}}{I_0^{(2m)}}; \qquad \omega_{2m}^2=\frac{I_2^{(2m)}}{I_0^{(2m)}}; \qquad f_{2m}^{(c)}=\frac{a\cdot\left[P_{2m,c}^{(sc.ac)}\cdot U_{2m}^{*}(r)\right]_{r=a}}{2I_0^{(2m)}}$$

$$c_{2m}(t)\approx f_{2m}\cdot\frac{1}{\omega_{2m}^2-\nu^2}\cdot\cos(\nu t); \qquad s_{2m}(t)\approx -f_{2m}\cdot\frac{2\eta_{2m}\nu\Omega}{\left(\omega_{2m}^2-\nu^2\right)^2}\cdot\sin(\nu t)$$

SOLUTIONS OF EQUATIONS OF MOTION - 2

$$\ddot{c}_{2m} + 2\eta_{2m}\Omega\dot{s}_{2m} + \omega_{2m}^2 c_{2m} = f_{2m}^{(c)}(t)$$

$$\ddot{s}_{2m} - 2\eta_{2m}\Omega\dot{c}_{2m} + \omega_{2m}^2 s_{2m} = f_{2m}^{(s)}(t)$$

$$\eta_{2m} = \frac{I_1^{(2m)}}{I_0^{(2m)}}; \quad \omega_{2m}^2 = \frac{I_2^{(2m)}}{I_0^{(2m)}}; \quad f_{2m} = \frac{a \cdot \left[P_{2m}^{(sc.ac)} \cdot U_{2m}{}^{*}(r) \right]_{r=a}}{2I_0^{(2m)}}$$

$$f_{2m}^{(c)}(t) = \frac{f_{2m}}{2}\left[\cos(\nu - 2m\Omega)t + \cos(\nu + 2m\Omega)t\right] \qquad f_{2m}^{(s)}(t) = \frac{f_{2m}}{2}\left[-\sin(\nu - 2m\Omega)t + \sin(\nu + 2m\Omega)t\right]$$

$$c_{2m}(t) \approx f_{2m} \cdot \frac{1}{\omega_{2m}^2 - \nu^2} \cdot \cos(\nu t)\cos(2m\Omega t) - f_{2m} \cdot \frac{2(2m - \eta_{2m})\nu\Omega}{\left(\omega_{2m}^2 - \nu^2\right)^2} \cdot \sin(\nu t)\sin(2m\Omega t);$$

$$s_{2m}(t) \approx f_{2m} \cdot \frac{1}{\omega_{2m}^2 - \nu^2} \cdot \cos(\nu t)\sin(2m\Omega t) + f_{2m} \cdot \frac{2(2m - \eta_{2m})\nu\Omega}{\left(\omega_{2m}^2 - \nu^2\right)^2} \cdot \sin(\nu t)\cos(2m\Omega t)$$

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CONCLUSIONS

- Expressions for Bryan's factors have been derived in the presence of acoustic scattered field, which characterize the coefficients of proportionality between angular rate of precession of a vibrating pattern and the inertial angular rate of an isotropic elastic body.
- The capture effects have been discussed, which demonstrated finite angular declinations of particular vibrating patterns at different angles depending on modes and circumferential numbers.
- The angles of relative rotations of the scattered modes are proportional to the abovementioned Bryan factors.