MODELLING AND IMPLEMENTATION
OF 1-3 PIEZOCOMPOSITE SIDE SCAN SONAR ARRAY

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Abstract: *A design and implementation of a 1-3 piezocomposite side scan transmit sonar array is considered. The theoretical approach to the array design is based on a new concept of unidimensional modeling of 1-3 piezocomposite transducers, developed by the authors. This concept corresponds to the generalization of the Smith-Auld unidimensional model and takes into consideration lateral motions of the piezoelectric pillars and polymer, which are supposed to be proportional to the strain in the axial direction. The main advance of the approach is that the electromechanical model is formulated in terms of variational approach. A new set of equivalent elastic, electrical and electromechanical constants of 1-3 piezocomposite is derived. Lamb modes of the 1-3 piezocomposites are investigated in term of the Certon-Patat membrane model by means of direct variational method application. The implementation of the array is proposed with generation of acoustic beam patterns at three different frequencies: 80 kHz, 300 kHz and 510 kHz. It is shown that the experimental data are in good agreement with the theoretical predictions.*

1. INTRODUCTION

Multi-frequency side scan transmit arrays broadly used in underwater acoustic measurements. A three frequency side scan array is designed in the Centre for Integrated Sensing Systems based on the technology of 1-3 piezocomposites. Thickness modes are designed using a new unidimensional model which is a generalization of the known Smith-Auld model[1]. Proper prediction of lateral resonances is important for increasing bandwidth and improving performance of transducers. Prediction of lateral resonances is based on Certon-Patat "membrane" theory[2]. A modification of this theory based on variational approach is presented. The implementation of the array is given with generation of acoustic beam patterns at frequencies: 80 kHz, 300 kHz and 510 kHz and electrical impedance for three apertures is measured.

2. UNIDIMENSIONAL MODEL OF 1-3 PIEZOCOMPOSITES

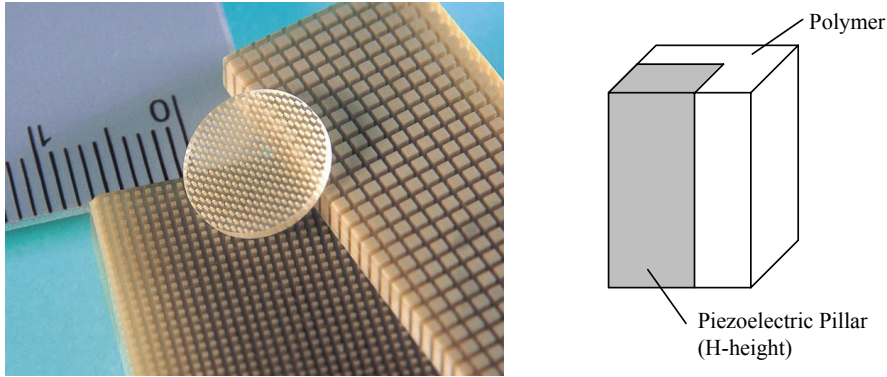Typical 1-3 piezocomposites are shown in Fig. 1.

Fig. 1.

The proposed unidimensional model is a generalization of the Smith-Auld model[1] and is based on the Rayleigh's theory of longitudinal oscillations of bars. Displacements and strains are supposed to be

$$u_i = -v_i x w_z'; \qquad v_i = -v_i y w_z'; \qquad (i = 1, 2)$$
$$S_1 = u_x'; \quad S_2 = v_y'; \quad S_3 = w_z'; \quad S_4 = S_5 = S_6 = 0 \tag{1}$$

where the equivalent Poisson's ratios and modules of elasticity are

$$v_1 = \frac{c_{13}^D h_{33} - c_{33}^D h_{31}}{(c_{11}^D + c_{12}^D) h_{33} - 2c_{13}^D h_{31}}; \qquad v_2 = \frac{E_2}{2G_2} - 1 = \frac{\lambda_2}{2(\lambda_2 + \mu_2)};$$
$$E_1 = c_{33}^D + 2v_1^2 (c_{11}^D + c_{12}^D) - 4v_1 c_{13}^D; \qquad E_2 = \frac{\mu_2(3\lambda_2 + 2\mu_2)}{\lambda_2 + \mu_2} \tag{2}$$

Lagrangian of the system is

$$L = L\left[\dot{w}, \dot{w}', w', w\left(\frac{H}{2}\right), w\left(-\frac{H}{2}\right), D_3, \lambda\right] =$$

$$\frac{1}{2}\int_{-H/2}^{H/2}\left(F_1\dot{w}^2+F_2\dot{w}'^2-F_3w'^2\right)dz+\frac{1}{2}F_4D_3^2+\lambda\left[F_5D_3-F_6w\left(H/2\right)+F_6w\left(-H/2\right)-V\right] \qquad (3)$$

where V – applied voltage, D_3 - electric displacement, λ - Lagrange multiplier and

$$F=\rho_1A_1+\rho_2A_2;\quad F_2=\rho_1v_1^2I_1+\rho_2v_2^2I_2;\quad F_3=\left(E_1A_1+E_2A_2\right)+8E_2\left[\frac{a}{\Delta}\left(v_1a+v_2\Delta\right)^2+\left[v_2\left(a+\Delta\right)\right]^2\right];$$

$$F_4=\beta_{33}^SA_1H;\qquad F_5=\beta_{33}^SH;\qquad F_6=h_{33}-2h_{31}v_1 \qquad (4)$$

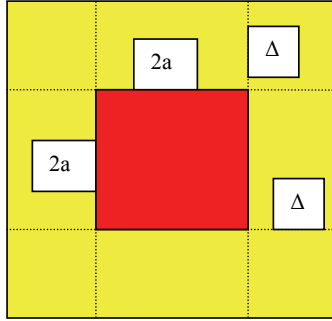Geometry of the cell is shown in Fig. 2.

Fig. 2.

Equations of motion are obtained from the Lagrangian (24) as follows:

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{w}}\right)-\frac{\partial^2}{\partial t\,\partial z}\left(\frac{\partial L}{\partial \dot{w}'}\right)+\frac{\partial}{\partial x}\left(\frac{\partial L}{\partial w'}\right)=F_1\ddot{w}-F_2\ddot{w}''-F_3w''=0;\qquad \frac{\partial L}{\partial D_3}=F_4D_3+\lambda F_5=0 \qquad (5)$$

Boundary conditions are:
- mechanical boundary conditions:

$$\left[\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{w}'}\right)-\frac{\partial L}{\partial w'}\right]_{z=\pm H/2}=\left[F_2\ddot{w}'+F_3w'\right]_{z=\pm H/2}=0 \qquad (6)$$

- electrical boundary conditions:

$$\frac{\partial L}{\partial \lambda}=F_5D_3-F_6w\left(H/2\right)+F_6w\left(-H/2\right)-V=0 \qquad (7)$$

Suppose that

$$w=\hat{W}(z)\cdot e^{i\omega t};\quad D_3=\hat{D}_3\cdot e^{i\omega t};\quad V=\hat{V}\cdot e^{i\omega t};\quad \lambda=\hat{\Lambda}\cdot e^{i\omega t} \qquad (8)$$

In this case the first equation (26) and boundary conditions (27) – (28) could be rewritten as:

$$\hat{W}''+\eta^2\hat{W}=0\quad\left(\eta=\eta(\omega)=\omega\cdot\sqrt{\frac{F_1}{F_3-\omega^2F_2}},\quad 0<\omega<\sqrt{\frac{F_3}{F_2}}\right);\qquad \hat{\Lambda}=-\frac{F_4}{F_5}\hat{D}_3; \qquad (9)$$

$$\left(F_3-\omega^2F_2\right)\left[\hat{W}'\right]_{z=\pm H/2}=0;\qquad F_6\hat{W}\left(-H/2\right)-F_6\hat{W}\left(H/2\right)+F_5\hat{D}_3=V \qquad (10)$$

First equation (30) has the following solution:

$$\hat{W}=\hat{W}(z)=\hat{C}\cdot\cos(\eta z)+\hat{S}\cdot\sin(\eta z) \qquad (11)$$

Unknown constants $\hat{C}$ and $\hat{S}$ as well as $\hat{D}_3$ could be found from the system of boundary conditions (10). Physical meaning of the Lagrange coefficient $\lambda=-A_1D_3\cdot e^{i\omega t}$ is a charge on both faces of the piezoelectric pillar, which maintain the electro-mechanical motion of the 1-3 piezocomposite.

3. LATERAL VIBRATIONS OF 1-3 PIEZOCOMPOSITES

The elementary cell in Certon and Patat's approach[2] is formed by a right-angled triangle and so the only symmetric solutions of lateral vibrations are considered (Fig. 3). If periods of the 1-3 piezocomposite are different in x- and y-directions asymmetric solutions are piezoelectrically coupled as well as the symmetric solutions. In the present paper the elementary cell is formed by a rectangle due to periodicity in x- and y-directions and hence, the symmetric and asymmetric solutions are considered.
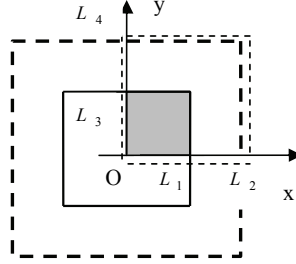

Fig. 3

Lateral modes, dependent on x and y, are considered separately. Our aim is to define lateral motions of 1-3 piezocomposites. It could be done by means of the following simplifications. Let us assume that:

- Lateral displacements are zero $u^{(j)} = v^{(j)} = 0$.
- Thickness motion is neglected, i.e. the thickness strain is zero $S_z^{(j)} = 0$: $w_z'^{(j)} = 0$.

The last hypothesis means that two types of structures could be considered:

- Semi-infinite medium at which the lateral displacements $w = w(x, y)$ represent the surface waves;
- Thin plate with boundary conditions $S_z^{(j)} = 0$ at $z = \pm H$. In this case the approximate condition $S_z^{(j)} \approx 0$ is true for all z: $-H \leq z \leq H$.

In the frames of these assumptions the "membrane" model of lateral vibrations of 1-3 piezocomposite could be described. In this case:

$$S_1^{(j)} = S_2^{(j)} = S_3^{(j)} = S_6^{(j)} = 0; \quad S_4^{(j)} = w_y'^{(j)}; \quad S_5^{(j)} = w_x'^{(j)} \tag{10}$$

To simplify the model we consider steady-state vibrations:

$$w^{(j)}(x, y, t) = W^{(j)}(x, y) \cdot e^{i\omega t} \tag{11}$$

and the model could be described by a simplified Lagrangian:

$$L = \frac{1}{2} \sum_{j=1}^{2} \int_{(A_j)} \left\{ \rho^{(j)} \omega^2 W^{(j)2} - c_{44}^{(j)} \left[W_x'^{(j)2} + W_y'^{(j)2} \right] \right\} dA_j \tag{12}$$

The Euler-Lagrange equations for this Lagrangian are:

$$\frac{\partial}{\partial x} \left(\frac{\partial L}{\partial W_x'^{(j)}} \right) + \frac{\partial}{\partial y} \left(\frac{\partial L}{\partial W_y'^{(j)}} \right) - \frac{\partial L}{\partial W^{(j)}} = 0, \quad (j = 1, 2) \tag{13}$$

or

$$\rho^{(1)} \omega^2 W^{(1)} = c_{44}^{(1)D} \left[W_{xx}''^{(1)} + W_{yy}''^{(1)} \right]; \quad \rho^{(2)} \omega^2 W^{(2)} = c_{44}^{(2)} \left[W_{xx}''^{(1)} + W_{yy}''^{(1)} \right] \tag{14}$$

Boundary conditions, which could be obtained from the Lagrangian (12) are:

$$x = L_1, \quad 0 \le y < L_3: \qquad W^{(1)} - W^{(2)} = 0; \quad c_{44}^{(1)D} W_x'^{(1)} - c_{44}^{(2)} W_x'^{(2)} = 0$$

$$y = L_3, \quad 0 \le x < L_1: \qquad W^{(1)} - W^{(2)} = 0; \quad c_{44}^{(1)D} W_y'^{(1)} - c_{44}^{(2)} W_y'^{(2)} = 0$$

$$x = 0, \quad 0 \le y < L_3: \qquad W_x'^{(1)} = 0 \qquad\qquad x = 0, \quad L_3 \le y \le L_4: \qquad W_x'^{(2)} = 0$$

$$y = 0, \quad 0 \le x < L_1: \qquad W_y'^{(1)} = 0 \qquad\qquad y = 0, \quad L_1 \le x \le L_2: \qquad W_y'^{(2)} = 0$$

$$x = L_2, \quad 0 \le y \le L_4: \qquad W_x'^{(2)} = 0 \qquad\qquad y = L_4, \quad 0 \le x \le L_2: \qquad W_y'^{(2)} = 0 \tag{15}$$

Using Rayleigh method, an approximate solution which automatically satisfied the last six equations of the system (15) is

$$W = W(x,y) \approx \sum_{m=0}^{M} \sum_{n=0}^{N} C_{m,n} \cos\left(\frac{m\pi}{L_2} x\right) \cdot \cos\left(\frac{n\pi}{L_4} y\right) \tag{16}$$

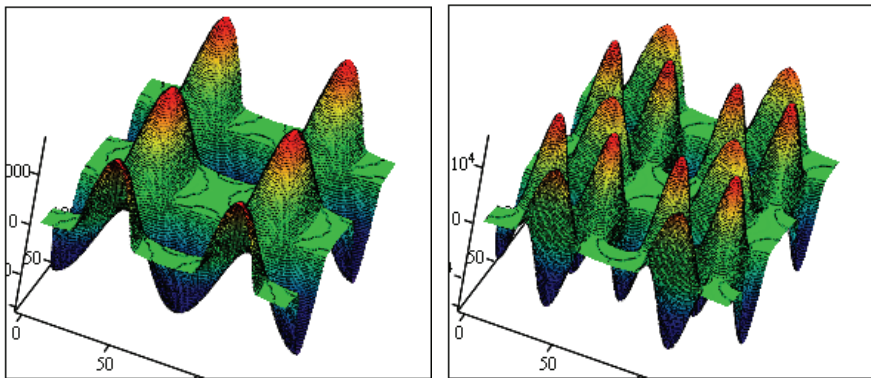There are *(M+1)(N+1)* unknown constants $C_{m,n}$ in this expressions.

In contrast to the approach, used by Certon and Patat [1, 2] who considered only symmetric vibrations, the elementary cell is formed by not a right-angled triangle but a rectangle due to periodicity in x- and y-directions (Fig. 3) and hence, the symmetric and asymmetric solutions are considered (Fig. 4, 5).
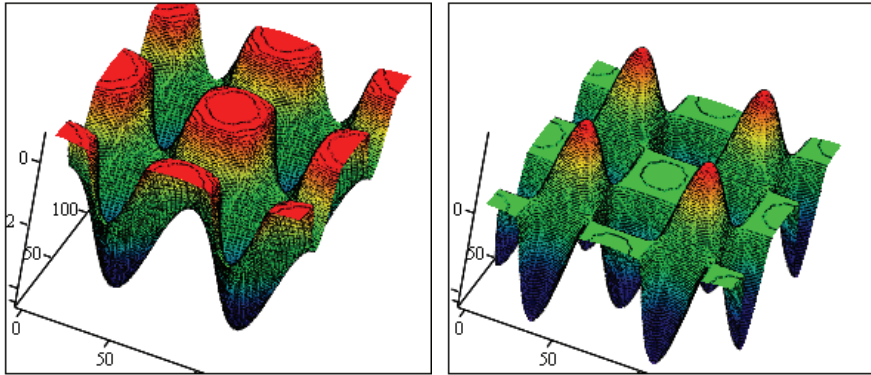
Fig. 4

Fig. 5

Symmetric eigenfunctions are shown in Fig. 4. They are well piezoelectrically coupled and hence, could be simply detected by means of electric measurements.

4. 1-3 PIEZOCOMPOSITE TRANSMIT SONAR ARRAY

The Transmit Sonar Array below was developed to operate as a single array in three frequency modes 80KHz, 300kHz and 510kHz while maintaining a constant vertical and horizontal beam width. This was achieved by using a 1-3 piezocomposite material and electrode patterning. Its implementation is shown in Fig. 6.

Front view of the multi-frequency array

Fig. 6.

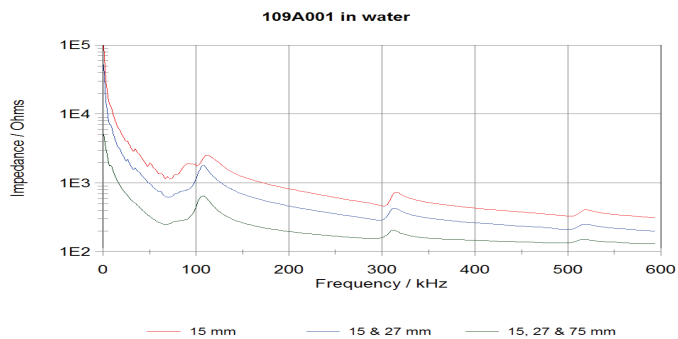Electrical impedances for three apertures are depicted in Fig. 7.

Fig. 7.

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