INTRODUCTION OF NEW ROAD PAVEMENT RESPONSE MODELLING SOFTWARE BY MEANS OF BENCHMARKING

J.W. Maina, E. Denneman* M. De Beer **

CSIR, P.O. Box 395 Pretoria, 0001, South Africa, jmaina@csir.co.za
*CSIR, P.O. Box 395 Pretoria, 0001, South Africa, edenneman@csir.co.za
**CSIR, P.O. Box 395 Pretoria, 0001, South Africa, mbeer@csir.co.za

ABSTRACT

Pavement response modelling based on the theory of linear elasticity was introduced in South Africa in the late 1960's and early 1970's. Since that time, continuous developments took place, and today it is possible to determine the stress/strain distribution of multi-layered road pavement problems in a matter of seconds. This paper provides an overview of historical developments, followed by the introduction of new software and the validation thereof. In the latest version of the Mechanistic Empirical Pavement Analysis and Design Software (*me*-PADS v1.1), the previous ELSYM5 analysis engine was replaced by General Analysis of Multi-layered Elastic Systems (GAMES) software. Among the advantages of GAMES are the ability to model multiple pavement layers and loads, the inclusion of layer interface slip and higher accuracy close to the pavement surface. In the present study the results generated using GAMES for a number of standard load cases are compared to those of a range of other packages used world wide. Newly developed Finite Element Method for Pavement Analysis (FEMPA) software, which is currently only available for use in a research environment, is also benchmarked against these other packages. The results show that both the GAMES and FEMPA packages can be used with confidence.

1. INTRODUCTION AND BACKGROUND

Development of analytical methods for resilient response of pavements may be traced back to Burmister (1943), who presented a method to determine stresses, strains and displacements in a two-layer elastic system based on axi-symmetric analysis of vertical load using stress function. Since then, Muki (1960) developed solutions for half-space under horizontal loading, while for a two layered system, Kimura's contributions are well known (e.g. Kimura, 1966). Compilations of the research works on half-space and two layered system can be found in references such as (Poulos and Davis, 1974). Sneddon (1951) presented detailed classical mechanics of elasticity using Fourier and Hankel transforms. Computer modelling and simulation of layered road pavements started to play a major role during the late 1960's (Freeme, 1975, 1977; Yoder and Witczak, 1975) and its application grew with the availability of fast modern day computers with larger memory and storage capacity (Haung, 1993).

Multi layer linear elastic systems are mathematically exact. However, most of the models, especially the earlier ones, are based on the following assumptions:

- One with infinite thickness or more layers with a finite thicknesses and an infinite bottom layer,
- Homogeneous and isotropic layer material properties,
- Layers are extend to infinite in horizontal directions,
- Full friction at layer interfaces,
- No surface shearing forces, and
- The materials are characterized by the Poisson's ratio and modulus of elasticity.

Because of the limitations imposed by some of the assumptions used in these analytical methods, road pavement structures with complex loading, boundary conditions and/or material behaviours can not be handled and for this reason numerical methods are recommended in certain cases. One of the methods used for numerical analysis is known as the matrix method. The popular Finite Element method (FEM) falls under this category and is based on discrete-element idealization. In the finite element analysis, pavement layers are considered to be solid continuum. The domain of the problem is divided into sub domains of which pavement layers are an example. These sub domains are discretized into a number of finite-sized elements. Finite elements are subsequently interconnected by nodes at their common edges and assembly of all these elements will represent the problem for general analysis. The pioneering work by Argyris in his matrix formulation of problems of structural mechanics provided the foundation for this development (Argyris, 1954). The work by Argyris became also the basis for practical and efficient way of organizing automatic computation.

The scope of this paper is to briefly summarise the local historical development of pavement analysis software and to introduce new software by way of benchmarking between existing modelling systems (methodologies). The aim of the paper is to present the results of benchmarking, and to recommend further use of these methodologies in pavement engineering.

1.1. <u>Developments with Multi-Layer Linear Elastic software in SA</u>

Multi-layer linear elastic software was introduced to South Africa by the CSIR in the late 1960's and early 1970's, (Freeme, 1975, 1977) and today most recognised road pavement consulting firms do have one or more of these systems available to the designer. One of the first packages available in SA was the Chevron multi layer program, refined by CSIR researchers into packages such as CHEV, MECDE1 (CHEV4, MECDE1, 1977; Maree, 1982) and CHEV15 (Coetzee, 1982).

During the mid 1980's the package called "ELSYM5" was introduced in South Africa by Dr Charles Freeme and soon became the most used package in research and normal pavement design over the years. Extensive Accelerated Pavement Testing (APT) using the fleet of three Heavy Vehicle Simulator (HVS) in SA (Walker et al, 1977; Paterson and Maree, 1978; Freeme et al, 1978) resulted in more than a dozen of thesis and dissertations based on forward and backward calculation using ELSYM5 and CHEV15 software.

In the late 1980's MichPave was introduced to South Africa and used in at least one PhD thesis by Wolff, (Wolff, 1990) and in a paper by De Beer (1992). The axi-symmetric MichPave software is a combination of finite element method and multi-layer theory developed by researchers at the University of Michigan in the 1990's. This facilitates the analysis of granular and soil layers as non-linear, based on the rather well known "K-Theta" models, which was also used by Maree (1982). MichPave was one of the first software codes to run on a PC with reasonable speed in the research and development, as well as for practical applications for everyday pavement design. For practical use and application of the above software packages, the reader is referred to the full reference list and suggested additional reading on published research works in the South African road pavement field.

During early 1990's Prof Lynne Irwin of USA introduced an update on ELSYM5, with some improvements on the integration algorithms of close to the tyre contact area. In the late 1990's a metric version of this package (ELSYM5M, 1995) was released to industry as well as tertiary institutions and is still used today (together with CHEV15) as the main software for mechanistic layered road pavement design and analysis. At the turn of the century ELSYM5 was integrated into a package called Mechanistic-Empirical Pavement Design and Analysis Software (*me*-PADS) by Theyse and Muthen, 2000.

The Rubicon pavement design and analysis software was developed in 2001 by the South African based company Modelling and Analysis Systems (MAS, 2007). The Rubicon Toolbox covers a range of functionalities including linear elastic response calculations performed using the WESLEA "engine".

Finally, it should be stated that with the introduction of Falling Weight Deflectometer (FWD) technology[1] in SA by Horak (1988), back-calculation was (and still is today) performed by a software packages such as ELMOD, MODCOMP and MODULUS, which are basically linear elastic multi-layer, very similar to ELSYM5 (RR 93/269, 1997). Some of these packages are, based on Odemark's "Equivalent Layer Theory, (ELT)", and introduced in the FWD technology by Ullidtz, mainly because of speed of back-calculation (Ullidtz, 1998, Huang, 1993).

In 2004/5 Maina et al, developed software called GAMES based on analytical/closed-form solutions, which has three distinct and major differences compared to any of the forerunners mentioned above, i.e.:

- Improved definition of the applied loading from 1 dimension to 3 dimensions (i.e. 1-D to 3-D);
- Possibility for modelling interlayer friction properties between any two layers;
- Higher accuracy close to the pavement surface as shown by Matsui et al (2005) in a direct comparison to BISAR, which is widely used as a benchmark. Accuracy in this area has become increasingly important in the light of new knowledge from Stress-In-Motion (SIM) measurements of complex stress distributions at the tyre-pavement contact area (De Beer et al, 1997).

The intention is to integrate GAMES as the basic "engine" for forward and back-calculation in road pavement design and analysis in South Africa owing to the ease of use, accuracy and speed of solution. GAMES has already been integrated into the latest version of *me*-PADS software.

1.2. Developments with Finite Element Method (FEM)

FEM in the research environment is nothing new, and was used as far back as 1970's by Pretorius in his PhD study (Pretorius, 1970), as well as Otte (1987), and Maree (1980). A major drawback of FEM and associated dynamic analysis of pavements since then was,

---

[1] It is important to note that during the period 1990 to 1994 the FWD technology was referred to as Impulse Deflectometer (IDM). See Horak (1988) and LaCante (1992).

and still is, that it is time consuming and needs relatively large computing memory and is very time consuming which adversely affects its everyday use. This type of analysis was only available to large consulting firms. As mentioned earlier, it is important to realise that FEM can also be used to solve multi-layered pavement problems with non-linear material (constitutive) engineering properties, non-uniform loading distributions (1-D to 3-D), as well as dynamic modelling of vehicle/pavement interaction (Lourens, 1991). Assumptions of modelling the tyre loading as circular uniform patterns of stress are not necessary with FEM.

In FEM, the equilibrium equations for each nodal point in the structural system are derived in terms of unknown nodal displacements and a solution set of the equilibrium equations constitutes a solution to the structural system. With the aid of experimental data, material properties for each element can be approximated fairly accurately and the element assemblage can represent complex bodies containing many different layers and material properties. Moreover, boundary conditions related to displacements and stresses may be specified at any point within the finite element system. Equilibrium equations for FEM result in a symmetric positive definite matrix that may be stored in a banded form and solved with minimum computer storage and time.

CSIR has embarked on the development of a finite element package Finite Element Method for Pavement Analysis (FEMPA) for applications in pavement engineering with the objective of improving accuracy of pavement structural analysis by taking into consideration:

- Characteristics of real-life pavement loadings;
- Geometry of pavement systems;
- Mechanical properties and behaviour of pavement materials, and;
- Responses of pavement systems to vehicular loading.

Preliminary results from FEMPA have shown good accuracy and efficiency of the finite element solver for static analysis cases. It is expected that successful development of this software will provide pavement engineers with a tool that closely simulates actual pavement structural behaviour for proper design, performance evaluation as well as timely identification of potential failures in the pavement systems. FEMPA is capable of analysing 2-D (plane strain, plane stress, axi-symmetric) as well as 3-D element shapes. To ensure that the results obtained from *me*-PADSs based on GAMES as well as the results from FEMPA are reliable, the present paper compares their output against results from a range of other available software packages.

2. COMPARING RESPONSE MODELS

The present paper uses results published in the Advanced Models for Analytical Design of European Pavement Structures (AMADEUS, 2000) report for benchmarking. As part of the AMADEUS study a number of popular response models were compared against the standard pavement structure shown in Table 1. Four packages, that can facilitate layer interface slip situations, were further compared against the pavement structure shown in Table 2. It should be noted that, most of the pavement analysis software such as the ones used in the AMADEUS study were developed using analytical/closed-form solutions. Their accuracy are, therefore, judged by how close their output are to the known values such as uniformly distributed contact stresses or stresses at the edge of the load and if the response distribution within the pavement system agrees with the theory.

Table 1. Pavement system used to compare various packages (Amadeus 2000)

	Material	Interface Condition	Layer Thickness (mm)	Modulus E* (MPa)	Poisson's ratio (ν)
1	DBM	-	260	5000	0.35
	Interface	Full friction	0	-	-
2	Granular	-	500	200	0.40
	Interface	Full friction	0	-	-
3	Soil	-	∝	50	0.45

Table 2. Pavement system with layer interface slip

	Material	Interface Condition	Layer Thickness (mm)	Modulus E* (MPa)	Poisson's ratio (ν)
1	DBM	-	260	5000	0.35
	Interface	Smooth	0	-	-
2	Granular	-	500	200	0.40
	Interface	Full friction	0	-	-
3	Soil	-	∝	50	0.45

Two loading conditions were considered for each of the two pavement structures. The first loading condition is a 50 kN wheel load at coordinates X = 0 mm, Y = 0 mm, with a tyre inflation pressure of 0.7 MPa, uniformly distributed over a circular contact area with a 150.8 mm radius.

The second load case involved a dual wheel, with the wheel centres at coordinates X = 0 mm, Y = 0 mm and X = 340 mm, Y = 0 mm. A 25 kN load per wheel was used, the tyre pressure was kept at 0.7 MPa, resulting in a circular contact area of 106.6 mm in radius. Results for stresses and strains were determined at points of interest shown in Table 3.

Table 3. Points of interest for computation of stresses and strains within the pavement system (Amadeus, 2000)

Parameter	Case I: Single wheel						Case II: Dual wheel					
	Centre of load			Edge of load			Centre of one wheel			Midpoint of dual wheel		
	X	Y	Z	X	Y	Z	X	Y	Z	X	Y	Z
σ_{z1}=vertical stress on the road surface	0	0	0	151	0	0	0	0	0	140	0	0
ε_{t1}= horizontal strain at the bottom of the asphalt	0	0	259.9	151	0	259.9	0	0	259.9	140	0	259.9
ε_{z2}= vertical strain on the surface of the granular material	0	0	260.1	151	0	260.1	0	0	260.1	140	0	260.1
ε_{z3}=vertical strain on the surface of the soil	0	0	760.1	151	0	760.1	0	0	760.1	140	0	760.1

The results for the first case in which the stresses and strains are determined under a single wheel, on a structure with full friction at the layer interfaces are shown Table 4. The results reported in AMADEUS (2000) are shown in the top part of the table, while the results produced during the course of present study are shown in shaded cells of the bottom part. The results indicate that all software, be it of local or international origin, produces similar results. The accuracy of the FEM results depends on the fineness of the mesh and the number of integration points per element. Although the mesh used for both the 2-D and 3-D FEMPA analysis was considered to be quite coarse, the results are accurate. It can be seen from the table that the results from GAMES are in better agreement with international results than its ELSYM5M predecessor in *me*-PADS. Based on recent private communication with Prof Lynne Irwin, who introduced an update of ELSYM5 in the early 1990's, the main reason for the differences could be due to the use of different versions of this software.

Table 4. Results single wheel without layer interface slip

	Centre of Load				Edge of Load			
Software	**σ_{z1} (MPa)**	**ε_{t1} (με)**	**ε_{z2} (με)**	**ε_{z3} (με)**	**σ_{z1} (MPa)**	**ε_{t1} (με)**	**ε_{z2} (με)**	**ε_{z3} (με)**
APAS	-0.700	100.4	-251.8	-185.1	0.000	61.8	-192.0	-177.5
AXIDIN	-0.723	116	-212.0	-163.0	-0.386	68.1	-167.0	-156.0
MICHPAVE	-0.700	91.6	-238.9	-129.0	0.000	38.0	-177.0	-119.0
BISAR	-0.700	100.5	-251.7	-185.0	-0.350	61.9	-192.2	-177.5
CIRCLY	-0.700	93.96	-246.7	-185.1	-0.350	62.9	-193.1	-177.5
ELSYM5	-0.700	99.7	-250.1	-176.0	-0.342	61.1	-190.7	-168.3
KENLAYER	-0.817	100.5	-251.6	-185.3	-0.319	62.0	-192.2	-177.0
NOAH	-0.700	100.5	-251.6	-185.0	0.000	61.9	-192.1	-177.4
VAGDIM	-0.700	100.5	-251.6	-185.1	-0.327	61.9	-192.0	-177.4
VESYS	-0.700	99.4			-0.327	61.5		
WESLEA	-0.700	100.4	-251.5	-185.0	0.000	61.9	-192.0	-177.4
CHEVRON 15	-0.700	99.7	-250.2	-176.0	0.000	63.1	-194	-169
Rubicon Toolbox	-0.700	100	-252	-176	0.000	61.8	-192	-177
me-PADS (GAMES)	-0.700	100.5	-251.6	-185.1	-0.345	61.9	-192.2	-177.5
me-PADS (ELSYM5M)	-0.700	97.9	-249.2	-169.2	-0.327	59.4	-189.6	-161.4
FEMPA 2D	-0.700	99.1	-250.4	-181.6	-0.0002	60.8	-190.8	-173.9
FEMPA 3D	-0.701	100.9	-247.4	-181.4	-0.179	59.9	-190.3	-173.4

With the inclusion of GAMES, it is now possible to model layer interface slip using *me*-PADS. The results produced by the GAMES interface slip model currently included in *me*-PADS compare well to the results for a single wheel situation obtained with other software as shown in Table 5. In a direct comparison with BISAR, the slip models used in GAMES were shown to be more accurate (Matsui et al, 2005). It has, however, also been shown that accuracy of GAMES can be increased by using a different slip model than is incorporated in *me*-PADS at the moment (Maina et al, 2007). This model will be included in the next version of *me*-PADS. The Rubicon Toolbox software assumes full friction between layers.

Table 5. Results single wheel including layer interface slip

	Centre of Load				Edge of Load			
Software	**σ_{z1} (MPa)**	**ε_{t1} (με)**	**ε_{z2} (με)**	**ε_{z3} (με)**	**σ_{z1} (MPa)**	**ε_{t1} (με)**	**ε_{z2} (με)**	**ε_{z3} (με)**
BISAR	-0.7000	120	-1	-217	-0.3500	78	10	-205
KENLAYER	-0.7000	120	-1	-216	0.0000	78	10	-205
NOAH	-0.7000	110	-111	-204	-0.7000	69	91	-194
WESLEA	-0.7000	122	-1	-43	0.0000	81	8	-41
me-PADS (GAMES)	-0.7000	119	-11	-217	0.0000	77	1	-205

Table 6 shows the results for the dual wheel case with full friction between layers. Again the locally developed software compares well to internationally used packages. Dual wheel situations can only be analyzed using 3-D FEMPA and not with the 2-D variant.

Table 6. Results dual wheel without layer interface slip

	Centre of one wheel				Midpoint of dual wheel			
Software	σ_{z1} (MPa)	ε_{t1} (με)	ε_{z2} (με)	ε_{z3} (με)	σ_{z1} (MPa)	ε_{t1} (με)	ε_{z2} (με)	ε_{z3} (με)
APAS	-0.7000	60	-186	-170	0.0000	51	-183	-177
BISAR	-0.7000	N/A	-186	-170	0.0000	N/A	-182	-177
CIRCLY	-0.7000	N/A	-181	-170	0.0000	N/A	-186	-177
ELSYM5	-0.7013	N/A	-185	-168	0.0000	N/A	-184	-170
KENLAYER	-1.4660	85	-186	-170	-0.0045	89	-183	-177
NOAH	-0.7000	N/A	-186	-170	0.0000	N/A	-183	-177
WESLEA	-0.7000	N/A	-186	-170	0.0000	N/A	-182	-177
CHEVRON 15	-0.7000	84	-185	-168	0.0000	88	-182	-169
Rubicon Toolbox	-0.7000	85	-186	-170	0.0000	89	-184	-177
me-PADS (GAMES)	-0.7000	85	-186	-170	0.0000	89	-183	-177
me-PADS (ELSYM5M)	-0.7013	81	-183	-161	-0.0016	87	-180	-168
FEMPA 3D	-0.7056	81	-177	-158	-0.0032	85	-176	-166

For the dual wheel load case with interlayer slip, GAMES again compares well with other software as shown in Table 7.

Table 7. Results dual wheel with layer interface slip

	Centre of one wheel				Midpoint of dual wheel			
Software	σ_{z1} (MPa)	ε_{t1} (με)	ε_{z2} (με))	ε_{z3} (με)	σ_{z1} (MPa)	ε_{t1} (με)	ε_{z2} (με)	ε_{z3} (με)
BISAR	-0.7000	N/A	9	-193	0.0000	N/A	12	-204
KENLAYER	-0.5885	103	9	-194	-0.0045	107	12	-204
NOAH	-0.7000	93	-85	-184	0.0000	N/A	-87	-195
WESLEA	-0.7000	N/A	8	-40	0.0000	N/A	11	-41
me-PADS (GAMES)	-0.7000	101	-3	-194	0.0000	106	-1	-205

3. CONCLUSIONS

The results presented in this paper show that the GAMES software integrated in *me*-PADS as well as the newly developed FEMPA software produces accurate results. It needs to be noted that the accuracy of the results obtained from FEMPA depend on element size and number and position of integration points. With the inclusion of GAMES, *me*-PADS v1.1 now provides engineers with the capability to model interlayer slip.

4. RECOMMENDATIONS

From this work, it is recommended that the engineer be given the opportunity to use either multi-layer linear elastic theory or finite element analysis to solve pavement problems using *me*-PADS. It is further recommended that the slip model included in *me*-PADS be updated to as per the findings by Maina et al (2007) to further improve accuracy.

5. ACKNOWLEGEMENTS

The permission of the Acting Director of CSIR Built Environment to publish this paper is acknowledged.

6. REFERENCES

AMADEUS, (2000). Advanced models for analytical design of European pavement structures. Report number RO-97-SC.2137 prepared for the European Commission.

Argyris, J.H., (1954). Energy Theorems and Structural Analysis. *Aircraft Engineering, Vol. 26*, pp. 347-356 (Oct.), 383-387, 394 (Nov.)

Burmister, D. M., (1943). The Theory of Stresses and Displacements in Layered Systems and Applications to the Design of Airport Runways, *Proc. of the 23rd Annual Meeting of Highway Research Board*.

CHEV4 (1977). Stresses and Strains in Layered Systems, ISBN 0 788 1037 8, Manual P4, Computer Information Centre for Transportation, NITRR, CSIR, Pretoria, South Africa.

Coetzee, C. H., (1982). CHEVRON Program For Calculating Stresses, Strains and Displacement in a Layered System Having a Maximum of Fifteen Layers. Unpublished Internal Technical Note TC/27/82, NITRR, July 1982.

De Beer, M., (1990). Aspects of the design and behaviour of Roads Structures incorporating Lightly Cementitious Layers. Ph. D Dissertation, Faculty of Engineering, University of Pretoria, Pretoria, South Africa, 1990.

De Beer, M., (1992). The effects of non-linear material parameters on the shape of the deflection basin for flexible pavements: a parametric study. 12th Annual Transportation Convention (ATC '92), Session 4B: Pavement Engineering, Paper 15, 22 pp. University of Pretoria, 29 June to 3 July 1992. ISBN 0958323747.

De Beer, M., Fisher C. and Jooste F. J., (1997). Determination of pneumatic tyre/pavement interface contact stresses under moving loads and some effects on pavements with thin asphalt surfacing layers. Eight (8th) International Conference on Asphalt Pavements (8th ICAP '97), August 10-14, 1997, Seattle, Washington, USA. Vol. 1, pp 179-227. ISBN 8790145356.

ELSYM5M (1995). Analysis of Elastic Layered Systems under normal wheel loads. Volume 1: PC (Version 2.0 metric) User Manual DPVT-M27, CICTRAN Software, Division of Roads and Transport Technology, CSIR, ISBN 0-7988-5448-0, February 1995.

Freeme, C. R., (1975). Computer Programs to Aid the Structural Design of Highway Pavements and Airfields, Symposium on Computer Applications, Pretoria, February 1975, Pretoria, South Africa.

Freeme, C. R., (1977). Computer Applications, CSIR Special Report PAD 36, 1977. CSIR Pretoria, South Africa.

Horak, E. (1988). Aspects of deflection basin parameters used in a mechanistic rehabilitation design procedure for flexible pavements in South Africa. PhD. Civil Engineering thesis, Civil Engineering Department, Faculty of Engineering, University of Pretoria, Pretoria, South Africa, January 1988.

Huang, Y. H., (1993). Pavement Analysis and Design. ISBN 0-13-655275-7, Prentice Hall, Inc, New Jersey 07632, USA.

Kimura, T. (1966). Pavement stress due to horizontal load, *JSCE, No. 133*, pp. 21-28.

Lacante S. C. (1992). Comparative Study of Deflection Basins Measured on Road Structures with various Non-Destructive Measuring Devices. Master Diploma in Technology, Faculty of Civil Engineering, Technikon Pretoria, May 1992.

Lourens, J.P., (1991). Non-Linear Dynamic Analysis and Design of Road Pavements. RDAC Research Report RR 90/030/1, January 1991.

Maree, J.H., (1982). Aspects of the Design and Behaviour of Pavements with Crushed Stone Bases". D(Eng) Dissertation (in Afrikaans), Faculty of Engineering, University of Pretoria, Pretoria, South Africa, 1982.

Maina, J.W., De Beer, M., Matsui, K., (2007). Effects of interface slip on the response and performance of elastic multi-layered flexible airport pavement systems. Proceedings of the fifth international conference on maintenance and rehabilitation of pavements and technological control, Utah, USA.

MAS (2007). Modelling and Analysis Systems, http://www.modsys1.com

Matsui, K., Maina, J.W., Fujinami, K., (2005). Development of analysis of multi-layered elastic systems towards mechanistic-empirical design of pavement structures. China-Japan, 3rd workshop on pavement technologies, Nanjing, China.

MECDE1 (1977). Stresses and Strains in Layered Pavements under Dual Wheel Loads, MECDE1, NITRR, Manual P5, ISBN 0 7988 1107 2, CSIR, 1977.

Muki, R., (1960). Asymmetric Problems of the Theory of Elasticity for Semi-infinite Solid and a Thick Plate. *Progress in Solid Mechanics (Edited by I.N. Sneddon and R. Hill)* 1, North-Holland Publishing Co., Amsterdam, pp. 400-439.

Otte, E., (1978). A structural design procedure for cement - treated layers in pavements. DSc thesis, University of Pretoria, South Africa.

Paterson, W. D. O and Maree H. J., (1978). An Interim Mechanistic Method for Structural Design of Road Pavements. Technical Report RP/5/78, NITRR.

Paterson, W.D.O., (1978). Observations of Pavement Behaviour under Heavy Vehicle Simulator 1971-1976. Technical Report RP/7/77, NITRR.

Poulos, H. G. and Davis, E. H., (1974). Elastic Solutions for Soil and Rock Mechanics. John Wiley & Sons.

RR 93/269: (1997). A Short Course on Rehabilitation Design of Flexible Pavements in South Africa, Chapter 7,. Department of Transport, Pretoria, South Africa.

Pretorius, P. C., (1970). Design considerations for pavements containing soil-cement bases. Ph.D thesis, University of California, Berkeley, California.

Sneddon, I. N. (1951). Fourier Transforms. McGraw-Hill Book Company.

Theyse H. L., and Muthen, M., (2000). Pavement Analysis and Design Software (Pads) Based on the South African Mechanistic-Empirical Design Method. 19th South African Transportation Conference (SATC 2000), July 2000.

Ullidtz, P. (1998). Modeling Flexible Pavement Response and Performance. 1st Edition 1998, ISBN 87-502-0805-5, Technical University of Denmark, Printed in Denmark 1998 by Narayana Press, Gylling.

Walker, R. N., Paterson, W. D. O., Freeme C. R., and Marais, C. P., (1977). The South African Mechanistic Pavement Design Procedure. 4th International Conference in Structural Design of Asphalt Pavements, Ann Arbor, Michigan, USA, 22 to 26 August 1977.

Wolff H., (1990). The Elasto-Plastic behaviour of Granular Pavement Layers in South Africa. November 1992. Ph.D Faculty of Engineering, Department of Civil Engineering, University of Pretoria, November 1992.

Yoder, E. J. and Witczak, M. W. (1975). Principles of Pavement Design, 2nd Edition. John Wiley & Sons, Inc.

7. SUGGESTED ADDITIONAL READING:

Freeme, C. R., De Beer, M. and Viljoen, A. W., (1987). The Behaviour and Mechanistic Design of Asphalt Pavements. Proceedings of the 6th International Conference in Structural Design of Asphalt Pavements, Ann Arbor, Michigan, USA, 13 - 17 July 1987.

Theyse H. L., De Beer M., Rust F. C., (1996). Overview of the South African Mechanistic Pavement Design Analysis Method. Preprint Paper Number 96-1294 presented at the 75th Annual Transportation Research Board Meeting, January 7 - 11, 1996, Washington, D.C.

Paterson W. D. O., and Van Vuuren D. J., (1974). Diagnosis of working strains in a pavement using deflection profiles. Proceedings of the 7th Conference of ARRB, Adelaide, 1974, vol 7, No. 6, pp 129-147 (NITRR Reprint , RR 224, 1974).

Van Vuuren, D. J, Otte, E., and Paterson, W. D. O. (1974). The Structural Design of Flexible Pavements in South Africa. Proc. 2nd Conference on Asphalt Pavements in South Africa (CAPSA), Durban, 1974.