Mobility and Kinematics Analysis of a Novel 5-DOF Hybrid Manipulator for Reconditioning of Mold and Die Tools: Part 2

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***Abstract*- Two novel hybrid manipulators that can be used for reconditioning of mould and die tools are presented in this paper. These designs have been developed using the knowledge of the different manufacturing process and defects that occur in mould and die tools. The similarities in the two novel hybrid manipulators are: they all used serial kinematics chains that implement parallelograms for linear translations. The designs differ in the manner the parallelograms are used to achieve linear translations. Moreover, rotary motions are achieved differently in these designs.**

The mechanical architectures and kinematics capabilities of the novel manipulators are presented. Finally, the kinematics capabilities of the developed manipulators are compared with those of selected 5 axis milling machine.

Key words: Manipulator configuration, workspace utilization factor, machine volume, orientation space index, and orientation angle.

I. INTRODUCTION

Most industrial robots are serial manipulators that have low stiffness required for dynamically demanding tasks (such as metal removal process, high accuracy applications, etc. during reconditioning of mould and die tools). Serial manipulators comprise of mechanical links, which are connected to at most two other links. The stationery link attached to the ground is called the base, the last link attaching a tool is called the end-effector. No closed loops are formed from the base to the end-effector. Thus, serial kinematics chain is achieved, where the previous link has to support the mass of the links that follow it, including the load on the end-effector or tool. Serial kinematics manipulators are highly kinematical reconfigurable machines that are dexterous in orientating the tool with the work-piece. This characteristic of serial manipulators is good for applications that required high dexterity such as reconditioning of mold and dies.

However, serial manipulators exhibit low dynamic performance due to the fact that the closer the link is next to the base, the more mass it must support which comes from the links far away from the base. Moreover, most serial manipulators normally referred to as robots, use only revolute joints in long kinematics chains, which results in lower stiffness. This characteristic of serial manipulators renders them useless for applications that required high dynamic performance such as reconditioning of mold and dies.

On the other hand, machine tools are stiff machine, but have low kinematics reconfigurability. Machine tools normally use a combination of revolute and prismatic joints, divided appropriately into two short kinematics chains; tool carrying kinematics chain and work-piece holding kinematics chain (this is more like two robot cooperating to achieve a manufacturing process). This set-up results in high rigidity, but low kinematics reconfigurability.

This paper, presents novel hybrid manipulators that combine the good kinematical reconfigurability of serial manipulators with high dynamic performance of machine tools. Kinematics capability and the workspace are considered useful measures of manipulator's capabilities [1], [2], [3], and have been used in this paper to illustrate the comparison between the novel manipulators with those of five axis milling machines. Different methods for the analyzing of kinematics and workspace of manipulators do exist and can be distinguished between discretization methods, geometric method and analysis method [4], [5]. The geometric method of analysis which has been adapted by most researchers [6], [7], [8], is being used in the analysis on this paper.

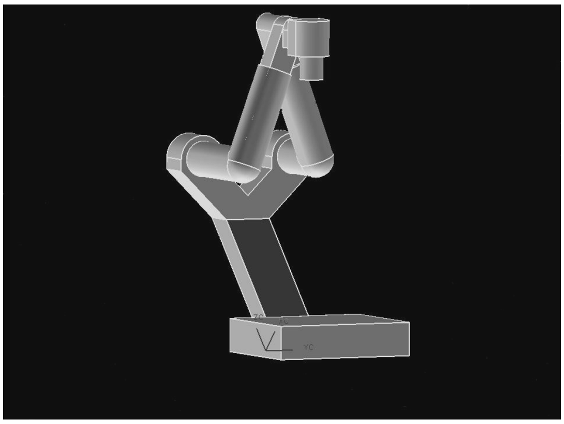
The next sections of the paper are organized as follows. In section two, each of the two manipulators is described, their inverse and forward kinematics is defined and then the theoretical workspace is determined. In section three, the comparison between the novel manipulators and a selected five axis milling machine is discussed and lastly a conclusion is given in section four.

II. DESCRIPTION AND ANALYSIS OF THE MANIPULATORS

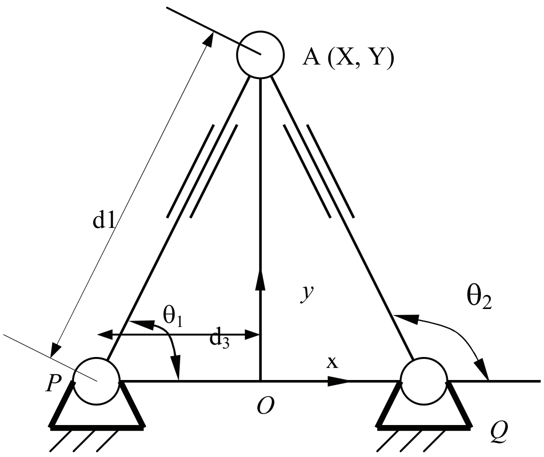
A. Planar 2RPR Hybrid Manipulator

The manipulator is of a hybrid kinematics structure as shown in Figure 1a. It consists of a fixed base, 2RPR parallelogram with extensible limbs and the output. The output is connected to the fixed base through the parallelogram structure, which is made up of two limbs. Each limb is connected to the base by an actuated revolute joint, and consists of two links connected by an actuated prismatic joint, providing a translation and rotation movement for the output.

Figure 1b shows the kinematics model of the mechanism. The two actuated joints at the base are denoted *P* and *Q* and the output is denoted A. The origin of the fixed reference coordinate frame is located at point O, with the Y-axis pointing along the direction of OA, and the X-axis perpendicular to OA. The length of the two limbs has dimension d_i (i = 1, 2) and length *OP* dimension $x_p = x_Q = d_3$



(a)



(b)

Figure 1. (a) 3D Model (b) Parametric description of the RPR mechanism

The position of the output point *A* can be written in terms of the two limbs as

$$x_A = -x_P + d_1 C\theta 1 \ , \quad (1)$$

$$y_A = y_P + d_1 S\theta 1 \quad (2)$$

for limb 1 and for limb 2 as:

$$x_A = -x_Q + d_2 C\theta_2 \quad (3)$$

$$y_A = y_Q + d_2 S\theta_2 \quad (4)$$

where x_A and y_A are the coordinate of point A on the *o-yx* reference system.

Inverse Kinematics

In inverse kinematics, the position of A (x_A, y_A) is defined, d_1 and d_2 for the translation movement should be determined as well as the joint angles θ_1 and θ_2 for the rotation movement.

This is accomplished by eliminating the joint passive variables, squaring and adding equations (1) and (2) yields

$$x_A^2 + y_A^2 + x_P^2 + 2x_A x_P = d_1^2 \ , \quad (5)$$

$$x_A^2 + y_A^2 + x_Q^2 - 2x_A x_Q = d_2^2 \quad (6)$$

And

$$\theta_1 = Sin^{-1}\left[\sqrt{\frac{y_A^2}{x_A^2 + y_A^2 + x_P^2 + 2x_A x_P}}\right] \quad (7)$$

$$\theta_2 = Sin^{-1}\left[\sqrt{\frac{y_A^2}{x_A^2 + y_A^2 + x_Q^2 - 2x_A x_Q}}\right] \quad (8)$$

Direct Kinematics

This is defined where the joint angles θ_1 and θ_2 and the limb lengths d_1 and d_2 are known and the position of A (x_A, y_A) is to be found.

Subtracting equation (6) from (5) yields:

$$x_A = \frac{d_1^2 - d_2^2}{2x_P x_Q} \quad (9)$$

$$y_A = \sqrt{d_1^2 - x_P^2 - \left(d_1^2 - d_2^2 \Big/ 2x_P x_Q\right)^2 - 2x_p\left(d_1^2 - d_2^2 \Big/ 2x_P x_q\right)} \quad (10)$$

Workspace

The workspace to be determined here is the region that the output point *A* can reach with the prismatic actuated displacement d_1 and d_2 changing from their minimum to maximum values and *θi* changing from *0* to *Π.*

Equation (5) can be viewed as a circle equation with radius d_1 centered at (*-d3,0*) for the first limb. Since each limb has a prismatic joint *di,* which can range from *do* (minimum) to *dm* (maximum), the workspace of the limb is the region bounded by the two circles with the following equation:

$$(x_A + d_3)^2 + y_A^2 = d_{1o}^2 \tag{11}$$

$$(x_A + d_3)^2 + y_A = d_{1m}^2 \tag{12}$$

On the second limb the workspace is bounded by circles with equation:

$$(x_A - d_3)^2 + y_A^2 = d_{2o}^2 \tag{13}$$

$$(xA - d3)^2 + y_A^2 = d_{2m}^2 \tag{14}$$

The theoretical workspace of the RPR mechanism is the region obtained at the intersection of the two limb workspaces as shown on fig 1c.

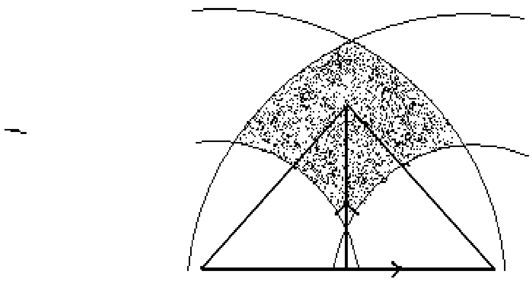


Figure 1c: Theoretical Workspace of RPR mechanism shown on the shaded region

B. Planar 2RRR Hybrid Manipulator

The manipulator shown in figure 2a has a RRR parallel mechanism. The output is connected to the base through a parallelogram, which consists of two limbs, of which each is made out of two links and three revolute joints. The two limbs connect at two points: at the base point and the output point by revolute joints. The actuated joint is the revolute joint at the base connecting to the two limbs. This mechanism can rotate the output point about the x-axis and move it in a translation. The geometric parameters of the manipulator are a_i , b_i , θ_i , φ_i (*i*=1,2). *A* is the output point at the common joint of the two limbs and can be described in *o-yx* frame as

$$x_{A1} = a_1 C\theta_1 + b_1 C(\theta_1 + \varphi_1) \tag{15}$$

$$y_{A1} = a_1 S\theta_1 + b_1 S(\theta_1 + \varphi_1) \tag{16}$$

$$x_{A2} = a_2 C\theta_2 + b_2 C(\theta_2 + \varphi_2) \tag{17}$$

$$y_{A2} = a_2 S\theta_2 + b_2 S(\theta_2 + \varphi_2) \tag{18}$$

Since φ_i (*i*=1, 2) is a passive joint angle, it is eliminated from the above equation to give:

$$x_A^2 + y_A^2 - 2x_A a_1 C\theta_1 - 2y_A a_1 S\theta_1 + a_1^2 - b_1^2 = 0 \tag{19}$$

$$x_A^2 + y_A^2 - 2x_A a_2 C\theta_2 - 2y_A a_2 S\theta_2 + a_2^2 - b_2^2 = 0 \tag{20}$$

Inverse Kinematics

Given position of point A (x_A, y_A), the input joint angles θ_1 and θ_2 are to be determined. This is defined by equations (19) and (20), which are arranged in the form:

$$e_1 S\theta_i + e_2 C\theta_i + e_3 = 0 \quad \text{where } (i=1,2) \tag{21}$$

Solving equation (17) yields:

$$\theta_i = \tan^{-1} \frac{-e \pm \sqrt{e_1^2 + e_2^2 - e_3^2}}{e_3 - e_2} \tag{22}$$

Where:

$$e_1 = -2y_A a_i$$
$$e_2 = -2x_A a_i$$
$$e_3 = x_A^2 + y_A^2 + a_i^2 - b_i^2$$

Direct Kinematics

The input joint angles θ_1, θ_2 are known and the position of A(x_A, y_A) can be obtained by subtracting equation (20) from (19), which gives:

$$x = ey \tag{23}$$

Where

$e = \dfrac{a_2 S\theta_2 + a_1 S\theta_1}{a_1 C\theta_1 + a_2 C\theta_2}$. Substituting equation (23) onto (19) yields:

$$my^2 + ny + k = 0 \tag{24}$$

Where

$$m = e^2 + 1$$
$$n = -2(ea_1 C\theta_1 + a_1 S\theta_1)$$
$$k = a_1^2 - b_1^2$$

Solution to equation (24) gives

$$y_A = \frac{-n \pm 1\sqrt{n2 - 4mk}}{2m} \quad (25)$$

Workspace

The workspace of the RRR planar parallel mechanism is the region that the output A can reach with every change in θi from 0 to Π. From equation (19) it can been seen that if θi is known the workspace of the first limb is a circle centered at a point ($a_1 C\theta_1$, $a_1 S\theta_1$) with radius b_1.

When θ_i changes from 0 to 2Π, the center point is located at (0,0) with radius of a_1. The workspace of the first limb is the region enveloped by the two circles with equations:

$$\text{CL1O:} \quad x_A^2 + y_A^2 = (b_1 - a_1)^2 \quad (26)$$

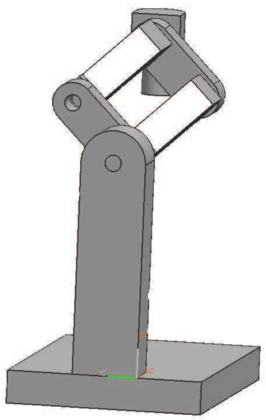
$$\text{CL11:} \quad x_A^2 + y_A^2 = (b_1 + a_1)^2 \quad (27)$$

For the second limb the workspace is a region enveloped by circles:

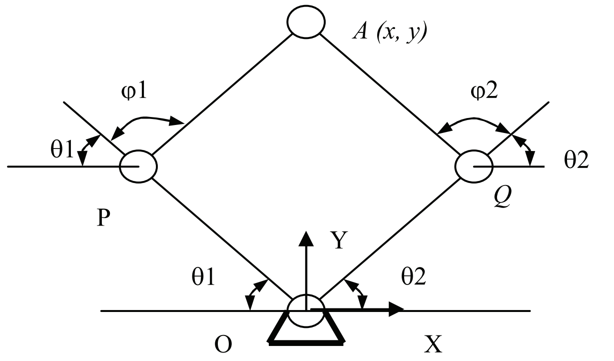
$$\text{CL2O:} \quad x_A^2 + y_A^2 = (b_2 - a_2)^2 \quad (28)$$

$$\text{CL21:} \quad x_A^2 + y_A^2 = (b_2 + a_2)^2 \quad (29)$$

Since (b_1-a_1) = (b_2-a_2) and (b_1+a_1) = (b_2+a_2) equations (26) and (28) are equal same as for equations (27) and (29) , which can be said the workspace of the mechanism is the region bounded by the two circles centered at point o with radii (b_1-a_1) and(b_1+a_1) as shown figure 2c.



(a)



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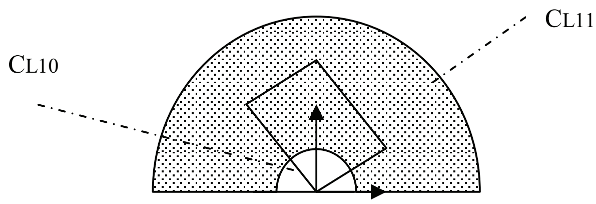

(c)

Figure 2: (a) 3D Model, (b) Parametric description of the 2RRR mechanism, (c) Theoretical Workspace shown on the shaded region

III. DISCUSSION

Two novel manipulators presented on this paper have been designed to incorporate the advantage of the serial manipulators, which are highly kinematical reconfigurable and of machine tools, which have high stiffness.

In this section the manipulators are compared to the five-axis milling machines. The comparison is based on the kinematics capabilities of both manipulators and machine tools. The configuration of the manipulators has been designed using the idea of machine tools, using two short kinematics chains to carry the tool and the workpiece. The tool part is carried by 2 degrees of freedom (rotation about the x-axis and translation along the z-axis) and the remaining three are attached to the workpiece part (two translations along both the x and y axis and rotation about the z-axis).

The workspace of the five axis milling machines is defined by the workspace of the tool and the workspace of the workpiece, based on this definition some quantitative parameters have been defined which are useful for comparison, selection and design of different types of machines [9].

Workspace utilization factor: is the ratio of the Boolean intersection of the workpiece workspace and tool workspace and the union of the workpiece workspace and the tool workspace.

Orientation space index, which is defined as the ratio of the volume of the largest spherical dome that can be machined with the machine using full range of the rotary axes, divided by the machine tool workspace.

Orientation angle index is defined as the ratio of the product of the max range of the two rotary axes divided by 360x180 multiplied by α_{12} /90, with α_{12} being the angle between the two rotary axes.

Machinable volume is the total volume that can be removed from the workpiece.

Five axis milling machine (Fig 3) have been selected for comparison with the novel hybrid manipulators. The selected machine has the same range of travel as the two manipulators and the comparison is based on the mentioned quantitative parameters.

Table 1 summarizes the specifications of the milling machine and that of the manipulators, and Table 2 gives the comparison of the features.

From the results in table 2 it can be seen that for the five axis milling machine, very small machinable volume can be obtained and also a very small percentage of the workspace can use the full range of the rotary axes the main reason being this types of machine are very low in kinematics reconfigurability. Comparing with the two novel manipulators it can be observed that much more machinable material can be obtained and the higher orientation space index indicate that more percentage of their workspaces can use the full range of the rotary axes.

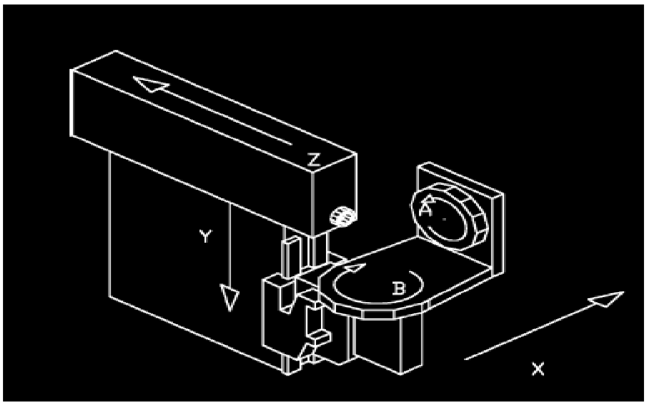

Figure3: Five axis Milling Machine

TABLE I

MACHINE SPECIFICATIONS:

Machine Type	Five-axis Milling Machine	Manipulators
Table Diameter	320mm	320mm
X Y Z Range	600x450x500	600x450x500
Range of Rotational Axes	B-axis: -105 to +105°	A-axis: 180°
	A-axis: 360°	C-axis : 360°

TABLE II

WORKSPACE COMPARISON:

	Five-axis Milling Machine	RPR Manipulator	RRR Manipulator
W_R	0.25	0.63	0.89
OS_I	0.17	0.619	0.66
OA_I	1.17	0.78	1.0
Machinable Material	84.0 dm3	174 dm3	315 dm3
WS TOOL U WS WORK	334.5 dm3	278 dm3	354 dm3

IV. CONCLUSION

Two novel hybrid manipulators to be used in the repairing and reconditioning of molds and dies are presented on this paper. The manipulators have been designed using serial kinematics chains that implement parallelograms for linear transformation. Their kinematics capabilities are analysed and workspace determined, then compared to that of five-axis milling machines.

ACKNOWLEDGMENT

The author would like to acknowledge the support of the Tshwane University of Technology and The Department of Material Science and Manufacturing, CSIR.

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