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Performance evaluation of high rate space–time trellis-coded modulation using Gauss–Chebyshev quadrature technique

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Abstract: The performance analysis of high rate space–time trellis-coded modulation (HR-STTCM) using the Gauss–Chebyshev quadrature technique is presented. HR-STTCM is an example of space–time codes that combine the idea used in trellis coded modulation (TCM) design that is signal set expansion and set partitioning into its construction. HR-STTCM construction is based on the concatenation of an outer TCM encoder and inner space–time block code. This paper evaluates the exact pairwise error probability of HR-STTCM based on the Gauss–Chebyshev quadrature formula. Comparison of numerical and simulation results shows that the proposed method is accurate. The method used is shown to be computationally simpler than those in the literature.

1 Introduction

In [1] high rate space–time trellis-coded modulation (HR-STTCM) was introduced as a space–time coding scheme that has higher coding advantage when compared with the earlier design of space–time trellis-coded modulation (TCM) [2–4]. The advantage of the construction in [1] is that the standard technique for designing good TCM codes [5], such as the classic set-partitioning concept, can be adopted to realise the HR-STTCM design with large coding gain.

A parameterised class of space–time codes was introduce in [6], that is, super-orthogonal space–time trellis codes (SOSTTC), which gives a systematic approach in the design of HR-STTCM.

The SOSTTC does not only provide a scheme that has an improvement in the coding gain when compared with the original space–time TCM schemes, but it answers the question of a systematic design for any rate, number of states and the maximisation of coding gain. This matrix expansion in HR-STTCM given in [1] corresponds to the angle multiplication in the SOSTTC. This means that the identity matrix multiplication with the original Alamouti code [7] corresponds to the angular multiplication in the SOSTTC.

For example, the multiplication of diag$[1, -1]$ with Alamouti code in HR-STTCM corresponds to $\theta = \pi$ in the orthogonal transmission matrix of the SOSTTC shown in (1) below.

$$A(x_1, x_2, \theta) = \begin{pmatrix} x_1 e^{j\theta} & x_2 \\ -x_2^* e^{j\theta} & x_1^* \end{pmatrix} \tag{1}$$

where $(^*)$ stands for conjugate and $x_i \in e^{j(2\pi a/m)}$, $i = 1, 2$.

When $\theta = 0$ (1) becomes the Alamouti code. For an m-PSK constellation with constellation signals represented by $e^{j(2\pi a/m)}$, $a = 0, 1, \ldots m-1$, one can pick $\theta = 2\pi a'/m$, where $a' = 0, 1, \ldots, m-1$.

Our interest in this paper is in the pairwise error probability (PEP) and the average bit error probability (BEP) in slow (quasi-static) fading channel of the HR-STTCM using the orthogonal transmission matrix shown in (1).

In [4], performance criteria for space–time codes were derived based on an upper bound on the PEP for both quasi-static and fast fading channels. Although the upper bound derived in [4] allows for considering all possible error events and gives a final expression for the average error probability terms, it is too loose for most signal-to-noise ratio ranges. Several research works [8–10] have been done to obtain a tighter bound for most space–time codes using various methods and based on these expressions an analytical estimate for the BEP can be computed, taking into account the dominant error events. A closed form expression for the exact PEP for space–time trellis code was derived in [9] based on the residual method using characteristic function [11], which has been used previously in the performance analysis of TCM. On the basis of this expression, an analytical estimate for the BEP was computed, taking into account the dominant error events. The derivation in [9] shows that the exact PEP is the upper bound derived in [4] modified by a correcting factor given by the second product term whose value depends on the poles of the characteristic function of the quadratic form of the complex Gaussian random variable.

In [12], the moment-generating function previously used for the analysis of uncoded and coded digital communication over fading channels using only a single transmitter is applied to provide a closed form expression of the PEP for space–time coded systems with multiple antennas. The method used in [12] has an additional advantage of allowing for direct evaluation of the transfer function upper bound on the average BEP. In [10], the moment-generating function-based approach was extended to analyse the PEP of the SOSTTC. It was shown that for slow and fast fading channels, it is possible to obtain a closed form expression for the PEP in terms of the element of the error signal difference matrix that characterises the super-orthogonal space–time block code.

A different approach to finding the exact expression of the PEP with less computational difficulty is presented in this paper. This approach is based on the Gauss–Chebyshev quadrature technique that has been used in the performance analysis of TCM [13]. This method combines both simplicity and accuracy in finding the closed form expression of the PEP.

The paper is organised as follows. In Section 2, we discuss the general transmission model of the HR-STTCM and the channel model. In Section 3, we describe the derivation of the PEP using the Gauss–Chebyshev quadrature technique and also give a numerical example. In Section 4, we use the PEP obtained in Section 3 to estimate the average BEP for slow fading channels. Section 5 concludes the paper with discussion on the results obtained from the numerical example of both the PEP and the average BEP.

2 System model

We consider a transmission system of n_t transmit antennas and n_r receive antennas. The input binary data streams are first fed into an outer TCM encoder to generate a sequence of complex-modulated symbols. The complex-modulated symbols $x_j (j = 1, 2, \ldots, n_t)$ are then fed into an inner space–time block encoder to generate the orthogonal transmitted code matrix (1). We define $x_{n_t}^{(n)}$ as the complex valued-modulated symbol transmitted from the n_tth transmit antenna in the nth signalling interval and $h_{lj}^{(n)}$ as the channel coefficient from the jth transmit antenna to the lth receive antenna at the same signalling interval, $j \in \{1, 2, \ldots, n_r, l \in \{1, 2, \ldots, n_r\}$.

Assuming that the signals of (1) are transmitted, the corresponding set of successive signal samples at the receiver at nth signalling interval is given by

$$\begin{aligned} r_l^{(n)} &= h_{l1}^{(n)} x_1^{(n)} \mathrm{e}^{j\theta^{(n)}} + h_{l2}^{(n)} x_2^{(n)} + \eta_l^{(n)} \\ r_{l+n_r}^{(n)} &= h_{l1}^{(n)} (-x_2^{(n)})^* \mathrm{e}^{j\theta^{(n)}} + h_{l2}^{(n)} (x_1^{(n)})^* + \eta_{l+n_r}^{(n)} \end{aligned} \tag{2}$$

where $l = 1, 2, \ldots, n_r$ and $x_l^{(n)}$ are independently identical distributed complex zero mean Gaussian noise samples, each sample with $\sigma^2/2$ per dimension. We assume that the channel undergoes Rayleigh fading, and the channel state information is known at the receiver.

3 Pairwise error probability

3.1 Derivations

To evaluate the PEP, that is, the probability of choosing the codeword sequence $\tilde{X} = (\tilde{x}_1^{(1)}, \tilde{x}_2^{(1)}, \ldots, \tilde{x}_{n_t}^{(1)} \tilde{x}_1^{(2)}, \tilde{x}_2^{(2)}, \ldots, \tilde{x}_{n_t}^{(2)} . \tilde{x}_1^{(N)}, \tilde{x}_2^{(N)}, \ldots, \tilde{x}_{n_t}^{(N)})$ when in fact the codeword sequence, $X = (x_1^{(1)}, x_2^{(1)}, \ldots, x_{n_t}^{(1)} x_1^{(2)}, x_2^{(2)}, \ldots, x_{n_t}^{(2)} \ldots x_1^{(N)}, x_2^{(N)}, \ldots, x_{n_t}^{(N)})$ was transmitted, we use the maximum likelihood metric corresponding to the correct path and the incorrect path. The maximum likelihood metric corresponding to the correct path is given by

$$\begin{aligned} m(r, X) = \sum_{n=1}^{N} \sum_{l=1}^{n_r} \Bigg[\left| r_l^{(n)} - \left(h_{l1}^{(n)} x_1^{(n)} \mathrm{e}^{j\theta^{(n)}} + h_{l2}^{(n)} x_2^{(n)} \right) \right|^2 \\ + \left| r_{l+n_r}^{(n)} - \left(h_{l1}^{(n)} \left(-x_2^{(n)} \right)^* \mathrm{e}^{j\theta^{(n)}} + h_{l2}^{(n)} \left(x_1^{(n)} \right)^* \right) \right|^2 \Bigg] \end{aligned} \tag{3}$$

The above is based on an observation of N blocks ($2N$ symbols), where each is described by (2).

For the incorrect path, the corresponding metric is given by (3) with $x_j^{(n)}, j = 1, 2$ and $\theta^{(n)}$ replaced by $\tilde{x}_j^{(n)}, j = 1, 2$ and $\tilde{\theta}_j^{(n)}$, respectively.

The realisation of the PEP over the entire frame length and for a given channel coefficient H is given by

$$\begin{aligned} P(X \rightarrow \tilde{X} | H) &= \Pr\{m(r, X) > m(r, \tilde{X})\} \\ &= \Pr\{m(r, X) - m(r, \tilde{X}) > 0\} \end{aligned} \tag{4}$$

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Substituting (3) and the corresponding expression for $m(r, \tilde{X})$ into (4) and simplifying gives

$$P(X \to \tilde{X}|H) = \Pr\left\{\sum_{n=1}^{N}\sum_{l=1}^{n_r}[|A|^2 + |B|^2] > 0\right\}$$
$$= \Pr\left\{\sum_{n=1}^{N}\sum_{l=1}^{n_r}\left\|H_l^{(n)}\boldsymbol{\Delta}_n\right\|^2 > 0\right\} \quad (5)$$

where

$$A = h_{l1}^{(n)}\left(\tilde{x}_1^{(n)}e^{j\tilde{\theta}^{(n)}} - x_1^{(n)}e^{j\theta^{(n)}}\right) + h_{l2}^{(n)}\left(\tilde{x}_2^{(n)} - x_2^{(n)}\right)$$
$$B = -h_{l1}^{(n)}\left(\tilde{x}_2^{(n)}e^{-j\tilde{\theta}^{(n)}} - x_2^{(n)}e^{-j\theta^{(n)}}\right)^* + h_{l2}^{(n)}\left(\tilde{x}_1^{(n)} - x_1^{(n)}\right)^*$$

and Δ_n is given as the codeword sequence matrix that characterises the HR-STTCM and its expression is given in (6) below and $H_l^{(n)} = \left[\begin{matrix} h_{l1}^{(n)} & h_{l2}^{(n)}\end{matrix}\right]$.

$$\boldsymbol{\Delta}_n = \begin{bmatrix} x_1^{(n)}e^{j\theta^{(n)}} - \tilde{x}_1^{(n)}e^{j\tilde{\theta}^{(n)}} & \left(-x_2^{(n)}\right)^* e^{j\theta^{(n)}} - \left(-\tilde{x}_2^{(n)}\right)^* e^{j\tilde{\theta}^{(n)}} \\ x_2^{(n)} - \tilde{x}_2^{(n)} & (x_1^{(n)})^* - (\tilde{x}_1^{(n)})^* \end{bmatrix} \quad (6)$$

The conditional PEP given in (5) can be expressed in terms of the complementary error function [13] as shown in (7).

$$P(X \to \tilde{X}|H) = \frac{1}{2}\text{erfc}\left(\sqrt{\frac{E_s}{4N_0}\sum_{l=1}^{n_r} H_l\boldsymbol{\Delta\Delta}^H H_l^H}\right) \quad (7)$$

The function $\boldsymbol{\Delta\Delta}^H$ is a diagonal matrix of the form shown in (8). This represents the codeword sequence matrix for the entire frame length. $(\,)^H$ represents the conjugate transpose of the matrix element and E_s/N_0 stands for the SNR per symbol.

$$\boldsymbol{\Delta\Delta}^H = \begin{bmatrix} \Delta_1\Delta_1^H & 0 & \cdots & \cdots & 0 \\ 0 & \Delta_2\Delta_2^H & 0 & \vdots & 0 \\ 0 & 0 & \ddots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \Delta_{N-1}\Delta_{N-1}^H & 0 \\ 0 & 0 & \cdots & 0 & \Delta_N\Delta_N^H \end{bmatrix} \quad (8)$$

The complementary error function, as defined integrally in [14, 7.4.11] is given by

$$\text{erfc}(b) = \frac{2}{\pi}\int_0^\infty \frac{e^{-b^2(t^2+1)}}{t^2+1}dt \quad (9)$$

Enumerating (7) using (9), we can then express the conditional PEP as an integral. Thus, with $E(x)$ denoting the average of x, one gets

$$P(X \to \tilde{X}) = \frac{1}{\pi}E\left[\int_0^\infty \frac{\exp\left[-(t^2+1)(E_s/4N_0)\sum_{l=1}^{n_r} H_l\boldsymbol{\Delta\Delta}^H H_l^H\right]}{t^2+1}dt\right] \quad (10)$$

We can simplify the above expression further using the results in [15]. For a complex circularly distributed Gaussian random row vector z with mean μ and covariance matrix $\boldsymbol{\sigma}_z^2 = E[zz^*] - \mu\mu^*$, and a Hermitian matrix $\boldsymbol{M}$, we have

$$E\left[\exp\left(-z\boldsymbol{M}(z^*)^T\right)\right] = \frac{\exp\left[-\mu\boldsymbol{M}\left(\boldsymbol{I} + \sigma_z^2\boldsymbol{M}\right)^{-1}(\mu^*)^T\right]}{\det\left(\boldsymbol{I} + \sigma_z^2\boldsymbol{M}\right)} \quad (11)$$

where $\boldsymbol{I}$ is an identity matrix. If we Apply (11) to solve (10) we get (12) since $\boldsymbol{z} = \boldsymbol{H}_l$, $\boldsymbol{M} = -(t^2+1)\cdot E_s/4N_0 \cdot \boldsymbol{\Delta\Delta}^H$ ($\boldsymbol{\Delta\Delta}^H$ is a diagonal matrix and $(t^2+1)\cdot E_s/4N_0$ is constant for a given SNR, thus making $\boldsymbol{\Delta\Delta}^H$ a Hermitian matrix), $\mu = 0$ (H_l has Rayleigh distribution) and $\sigma_z^2 = {\sigma_{H_l}}^2 = \boldsymbol{I}_{n_t}$.

$$P(X \to \tilde{X}) = \frac{1}{\pi}\int_0^\infty \frac{1}{t^2+1}\prod_{l=1}^{n_r}\frac{1}{\det\left[\boldsymbol{I}_{n_t} + (E_s/4N_0)\boldsymbol{\Delta\Delta}^H(t^2+1)\right]_l}dt \quad (12)$$

We can approximate the above expression (12) with the Gauss–Chebyshev Quadrature formula and details are given in the Appendix 1 leading to the following

$$P(X \to \tilde{X}) = \frac{1}{2k}\sum_{i=1}^{k}\prod_{l=1}^{n_r}\frac{1}{\det\left[\boldsymbol{I}_{n_t} + (E_s/4N_0)\boldsymbol{\Delta\Delta}^H \sec^2((2i-1)\pi/4k)\right]_l} + R_k \quad (13)$$

The term k is a small positive integer. As the term k increases, the remainder term R_k becomes negligible.

For slow fading (quasi-static) case, the channel coefficients are assumed to be constant for the entire frame duration, but varies from frame to frame, (13) therefore result in

$$P(X \to \tilde{X}) = \frac{1}{2k}\sum_{i=1}^{k}\prod_{l=1}^{n_r} \times \frac{1}{\det\left[\boldsymbol{I}_{n_t} + \frac{E_s}{4N_0}\sum_{n=1}^{N}\boldsymbol{\Delta}_n\boldsymbol{\Delta}_n^H \sec^2((2i-1)\pi/4k)\right]_l} + R_k \quad (14)$$

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The above method can be easily extended to a fast fading channel, but we will not discuss it here.

3.2 Numerical examples

As an example, we consider the rate $r = 1$ BPSK 2-state code [5, Fig. 1], whose trellis diagram is illustrated in Fig. 1, where two sets, each containing two pairs of BPSK symbols, are assigned to each state, that is, there is a pair of parallel path between each pair of states. The labelling $(s, l)/A(x_i, x_j, \theta)$ along each branch of the trellis refers to the pair of input BPSK symbols (s, l) and the corresponding output symbol function $A(x_i, x_j, \theta)$ using (1) to generate the orthogonal matrix.

First we consider the parallel paths, that is, $N = 1$, evaluating (6), the codeword matrix is given by

$$\boldsymbol{\Delta}_1 = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix}; \quad \boldsymbol{\Delta}_1\boldsymbol{\Delta}_1^{\mathrm{H}} = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} \tag{15}$$

Also we consider an error event path of length $N = 2$ with respect to the all zero path as the correct one. From the trellis diagram we have that $x_1^{(1)} = x_2^{(1)} = x_1^{(2)} = x_2^{(2)} = +1$, $\tilde{x}_2^{(1)} = \tilde{x}_1^{(2)} = -1$, $\theta^{(1)} = \tilde{\theta}^{(1)} = \theta^{(2)} = 0$ and $\tilde{\theta}^{(2)} = \pi$.

Evaluating the elements of the matrix in (6), $\boldsymbol{\Delta}_1$ and $\boldsymbol{\Delta}_2$, gives

$$\boldsymbol{\Delta}_1 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}; \quad \boldsymbol{\Delta}_1\boldsymbol{\Delta}_1^{\mathrm{H}} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$
$$\boldsymbol{\Delta}_2 = \begin{bmatrix} 0 & -2 \\ 0 & 2 \end{bmatrix}; \quad \boldsymbol{\Delta}_2\boldsymbol{\Delta}_2^{\mathrm{H}} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \tag{16}$$

We also consider an error event of length 3 with respect to the all zero path as the correct one. From the trellis diagram we have

$$x_1^{(1)} = x_2^{(1)} = x_1^{(2)} = x_2^{(2)} = x_1^{(3)} = x_2^{(3)} = \tilde{x}_1^{(1)} = \tilde{x}_2^{(3)} = +1,$$
$$\tilde{x}_2^{(1)} = \tilde{x}_1^{(2)} = \tilde{x}_2^{(2)} = \tilde{x}_1^{(3)} = -1$$
$$\theta^{(1)} = \tilde{\theta}^{(1)} = \theta^{(2)} = \theta^{(3)} = 0 \quad \text{and} \quad \tilde{\theta}^{(2)} = \tilde{\theta}^{(3)} = \pi$$

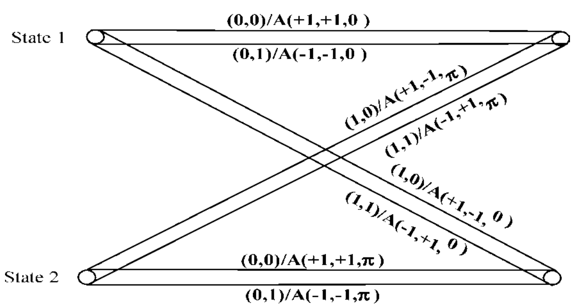

Figure 1 Trellis diagram for rate 1, two-state BPSK HR-STTCM

Evaluating the elements of the matrix (6), $\boldsymbol{\Delta}_1$, $\boldsymbol{\Delta}_2$ and $\boldsymbol{\Delta}_3$, gives

$$\boldsymbol{\Delta}_1 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}; \quad \boldsymbol{\Delta}_1\boldsymbol{\Delta}_1^{\mathrm{H}} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$
$$\boldsymbol{\Delta}_2 = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}; \quad \boldsymbol{\Delta}_2\boldsymbol{\Delta}_2^{\mathrm{H}} = \begin{bmatrix} 0 & 0 \\ 0 & 8 \end{bmatrix}$$
$$\boldsymbol{\Delta}_3 = \begin{bmatrix} 0 & -2 \\ 0 & 2 \end{bmatrix}; \quad \boldsymbol{\Delta}_3\boldsymbol{\Delta}_3^{\mathrm{H}} = \begin{bmatrix} 4 & -4 \\ -4 & 4 \end{bmatrix} \tag{17}$$

Substituting the codeword matrix obtained in (15)–(17) into (14), we can obtain the closed form expression for different error events in a slow fading channel.

3.3 Numerical result and discussion

In this section we provide numerical results for the closed form PEP enumerated in the previous sections. For our result, we assume that $k = 2$ and $n_r = 1$. Fig. 2 shows the PEP for slow fading (quasi-static) channel for $N = 1$, 2 and 3. The PEP at $N = 2$ is the worst case for slow fading channel.

4 Evaluation of average bit error probability

In this section, we use the PEPs previously derived to evaluate the average BEP that is, $P_b(E)$, based on accounting only for error events of length N.

Transfer function method [16] is a technique, which makes use of a code's state diagram to obtain error rate performance of trellis-based codes. This method takes into account error event of all lengths. In [11] as estimation of BEP was obtained through accounting for error event paths

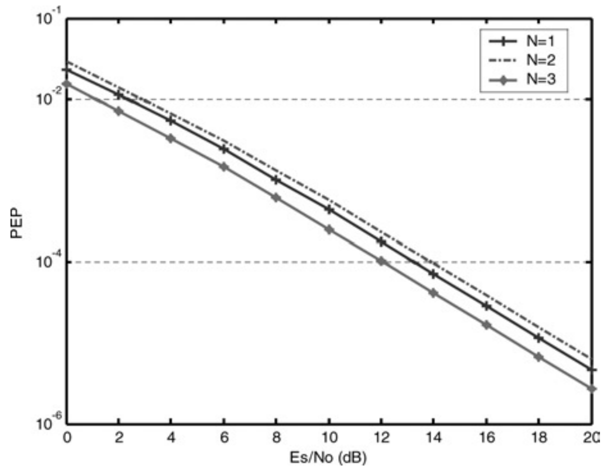

Figure 2 PEP performance of rate 1, two-state BPSK HR-STTCM over quasi-static fading Rayleigh Channel; one receive antenna

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of length to a pre-determined specific value using

$$P_b(E) \approx \frac{1}{g} \sum_{X \neq \tilde{X}} q(X \to \tilde{X}) P(X \to \tilde{X}) \qquad (18)$$

where g is the number of input bits per trellis transition and $q(X \to \tilde{X})$ the number of bit errors associated with each error event.

Assuming transmission of the all zeros sequence, then for the two-state HR-STTCM in Fig. 1, there is a single error event path of length 1, four error event paths of length 2 and eight error event paths of length 3. The single error event has PEP_1 obtain when $N = 1$ whereas the four error event paths of length 2 all have PEP_{II} and the eight error event paths of length 3 all have PEP_{III}. The PEP_I contributes one bit error, whereas the four paths of PEP_{II} contribute a total of 12 bit errors and finally the eight paths of PEP_{III} contribute a total of 28 bit errors.

The average BEP when considering error event paths of 1, 2 and 3 is given by P_{b1}, P_{b2} and P_{b3}, respectively.

$$P_{b1} \simeq \frac{1}{2}(PEP_I) \qquad (19)$$

$$P_{b2} \simeq \frac{1}{2}(PEP_I + 12 * PEP_{II}) \qquad (20)$$

$$P_{b3} \simeq \frac{1}{2}(PEP_I + 12 * PEP_{II} + 28 * PEP_{III}) \qquad (21)$$

Fig. 3 shows the average BEP of the code for slow fading channels. In the figure, the approximate average BEP performances are plotted accounting for error event lengths 1, 2, 3 which are based on (19)–(21). From the plot, we can observe that considering error events length up to 3 is sufficient for calculating the average BEP for slow fading channels as can be seen by a comparison with the simulated results provided for the true BEP. Each frame consists of 256 bits in the simulated BEP. This slower convergence of the PEP to the average BEP as a function of the length of paths considering for slow fading is consistent with a similar observation made [12] for orthogonal space–time trellis codes.

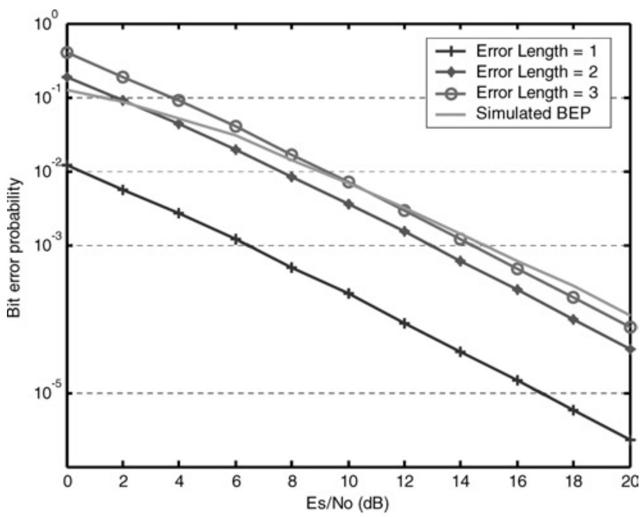

Figure 3 Average BEP of rate 1, two-state BPSK HR-STTCM over quasi-static fading Channel with one receive antenna

5 Conclusion

In this paper, we have derived the closed form expressions of the pairwise error probability of HT-STTCM using the orthogonal transmission matrix. We later used the PEP obtained to estimate the average BEP. The Gauss–Chebyshev Quadrature method used proved to be accurate and made the derivation of the closed form PEP easily obtainable. The method used here proved to be less complex than the others presented in the literature.

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7 Appendix

Here, the approximation in (12) is developed along with some error bounds. Consider the integral

$$I = \frac{1}{\pi}\int_0^\infty \frac{1}{t^2+1} f(t^2+1)\mathrm{d}t \qquad (22)$$

Substituting $y = 1/t^2 + 1$ into (22), (22) becomes

$$I = \frac{1}{2\pi}\int_0^1 \frac{1}{\sqrt{y(1-y)}} f(1/y)\mathrm{d}y \qquad (23)$$

Equation (23) is of the form of an orthogonal polynomial in [14, 25.4.38] and Gauss–Chebyshev quadrature formula of first kind can be applied to solve it.

$$\int_{-1}^1 \frac{f(u)}{\sqrt{1-u^2}}\mathrm{d}u = \sum_{i=1}^k w_i f(u_i) + R_k \qquad (24)$$

$$u_i = \cos\frac{(2i-1)\pi}{2k}$$
$$w_i = \frac{\pi}{k}$$
$$R_k \le \max_{-1>u>+1} \frac{\pi}{(2k)!2^{2k-1}} | f^{2k}(u)|$$

The expression in (23) can be reduced to (24), if we express $2y - 1 = u$

$$\begin{aligned} 2y - 1 &= \cos\frac{(2i-1)\pi}{2k} \\ 2y &= \cos\frac{(2i-1)\pi}{2k} + 1 \end{aligned} \qquad (25)$$

Using trigonometric function in (26), y can be expressed as (27).

$$\cos m = \cos\left(\frac{m}{2}+\frac{m}{2}\right) = \cos^2\frac{m}{2} - \sin^2\frac{m}{2} \qquad (26)$$

$$\begin{aligned} y &= \cos^2\frac{(2i-1)\pi}{4k} \\ 1/y &= \sec^2\frac{(2i-1)\pi}{4k} \end{aligned} \qquad (27)$$

Accordingly one has that

$$I = \sum_{i=1}^k w_i f(u_i) = \frac{1}{2k}\sum_{i=1}^k f\left(\sec^2\frac{(2i-1)\pi}{4k}\right) \qquad (28)$$