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Computational Modelling of Buckling of Woven Fabrics

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ABSTRACT

The fabric buckling model proposed by Grosberg and Swani has been modified by incorporating Huang's bilinear bending rule. The proposed model is an extension of the present model and also covers the special cases. The numerical results appear realistic and conform to the trend observed by other researchers. The effects of fabric bending parameters on the buckling behaviour of woven fabrics have been explored by numerical computations. The second part of this paper will include the recovery from large-scale buckling of woven fabric.

INTRODUCTION

The buckling, bending and drape behaviours of a woven fabric influence its performance during actual use and during the process of making-up into the end product. These properties are important, particularly when the fabric is limp, resulting in large-scale deformation even under small applied forces. The buckling properties of a woven fabric also influence the sewing of garments and their resultant quality. Moreover, the development of robotised sewing operations for reducing unit production cost is critically important if garment industries in developed countries are keen to improve their competitiveness vis-à-vis low labour cost countries.

The mechanics of the buckling behaviour of woven fabric started with the classical paper by Grosberg and Swani, and continued by the contribution from Clap and Peng and Kang et al. Huang modified Grosberg's idealized bending rule by incorporating bilinearity, which was further utilized by Leaf and Anandjiwala in proposing a generalized model of a plain woven fabric and subsequently for modifying Huang's extension analysis. Although, Kang et al have utilized Huang's bilinearity in their model, the obvious inconsistency of applying the classical beam theory to the textile problem remains. Frictional restraint to bending of the fabric arises from inter-fibre frictional forces resulting from the fact that the fibres act as individual units and do not form a solid beam. Nevertheless, it is interesting to note that Kang et al themselves have observed better results when incorporating Huang's bilinear bending behaviour into their model than when using their own exponential function! This reaffirms our belief in the bilinear bending rule, which is a more reasonable and realistic approximation of the real bending behaviour of yarn and fabrics, but relatively easy to handle in mathematical analysis compared to the exponential nonlinearity proposed by Kang et al and the quadratic nonlinearity by Abbott *et al.*

In this paper we have modified Grosberg and Swani's buckling theory for woven fabric by introducing Huang's bilinear bending rule. The assumption of infinitely high initial bending rigidity of the fabric in their model is thus removed. The proposed theory also eliminates obvious inconsistencies in several earlier approaches based on Grosberg's idealized bending rule. The methodology of the theoretical treatment employed here is similar to that proposed in an earlier paper by Leaf and Anandjiwala but is further suitably modified to accommodate both Grosberg and Swani's model and elastic buckling theory of classical engineering mechanics as special cases. A numerical solution of the model is developed and numerical results are presented to study the effect of fabric bending parameters, such as initial and final bending rigidities, B^* and B, respectively, and transition couple, M_a on the buckling of woven fabric.

THEORETICAL (GOVERNING EQUATIONS)

$$K = \frac{M}{B^*} \qquad M \le M_a \tag{1}$$

$$K = \frac{M_a}{B^*} + \frac{M - M_a}{B} \qquad M > M_a \tag{2}$$

$$M_a = \frac{M_o}{(1 - \frac{B}{B^*})} \tag{3}$$

$$l_1 = \sqrt{\frac{B^*}{P}}\left\{F\left(k^*, \frac{\pi}{2}\right) - F\left(k^*, \phi_A^*\right)\right\} \tag{4}$$

$$x_A = 2k^*\sqrt{\frac{B^*}{P}}\cos\phi_A^* \tag{5}$$

$$y_A = \sqrt{\frac{B^*}{P}}\left[2\left\{E\left(k^*, \frac{\pi}{2}\right) - E\left(k^*, \phi_A^*\right)\right\} - \left\{F\left(k^*, \frac{\pi}{2}\right) - F\left(k^*, \phi_A^*\right)\right\}\right] \tag{6}$$

where,

$$k^* = \sin\frac{\theta}{2} \tag{7}$$

$$\psi_A = \cos^{-1}\left(\cos\theta + \frac{M_a^2}{2PB^*}\right) \tag{8}$$

$$\phi_A^* = \sin^{-1}\left(\frac{\sin\frac{\psi_A}{2}}{k^*}\right) \tag{9}$$

$$\frac{l}{2} = \sqrt{\frac{B}{P}}\{F(k, \phi_A)\} + l_1 \tag{10}$$

$$D = 2k\sqrt{\frac{B}{P}}\left(1 - \cos\phi_A\right) + x_A \tag{11}$$

$$\frac{y_2}{2} = \sqrt{\frac{B}{P}}\left\{2E\left(k, \phi_A\right) - F\left(k, \phi_A\right)\right\} \tag{12}$$

Where,

$$k = \sqrt{k^{*2} - \frac{M_a^2}{4PB^*}\left(1 - \frac{B}{B^*}\right)} \tag{13}$$

$$\phi_A = \sin^{-1}\left(\frac{k^* \sin\phi_A^*}{k}\right) \tag{14}$$

$$\psi_z = 0 \tag{15}$$

The fractional compression δ is then given by:

$$\delta = 1 - \frac{2y_A + y_2}{2l_1 + l_2} \tag{16}$$

$F(k, \phi_A)$ and $E(k, \phi_A)$ are the incomplete elliptic integrals of the first and the second kind, respectively.

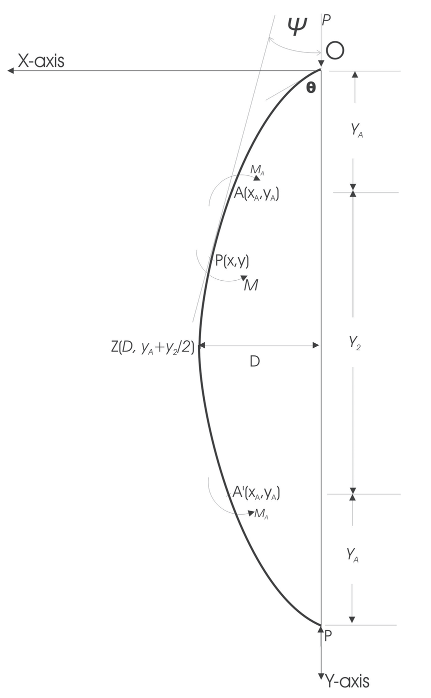

Figure 1: Force – Compression Curve of a Buckled Fabric

GROSBERG'S BUCKLING MODEL

Grosberg and Swani assumed a bending law in which the initial bending rigidity B^* tends to infinity and therefore the initial region OA was assumed to be a straight line with curvature $K = 0$, which implies that $(B/B^*) = 0$, therefore, from Equation (3) $M_a = M_o$. By substituting this in Equations (4) to (15) and deriving the simple trigonometric relationships for the initial straight line region OA, the following is obtained:

Equations (17) to (20) are identical to the buckling model reported by Grosberg and Swani.

$$\frac{l}{2} = \sqrt{\frac{B}{P}}\left\{F(k, \frac{\pi}{2})\right\} + \frac{M_o}{P}\operatorname{cosec}\theta \tag{17}$$

$$D = 2k\sqrt{\frac{B}{P}} + \frac{M_o}{P} \tag{18}$$

$$\frac{y_2}{2} = \sqrt{\frac{B}{P}}\left\{2E\left(k, \frac{\pi}{2}\right) - F\left(k, \frac{\pi}{2}\right)\right\} \tag{19}$$

$$y_A = \frac{M_o}{P}\cot\theta \tag{20}$$

ELASTIC BUCKLING MODEL

The elastic buckling theory in classical mechanics assumed a linear relationship between moment and curvature, ignoring the effect of the frictional couple which influences the behaviour of textile materials, such as yarns and fabrics. This implies that $M_a = 0$ and $B = B^*$. When substituting these values in Equations (4) to (16) equations are obtained that are similar to the buckling of a strut reported in standard textbooks of mechanics.

NUMERICAL RESULTS

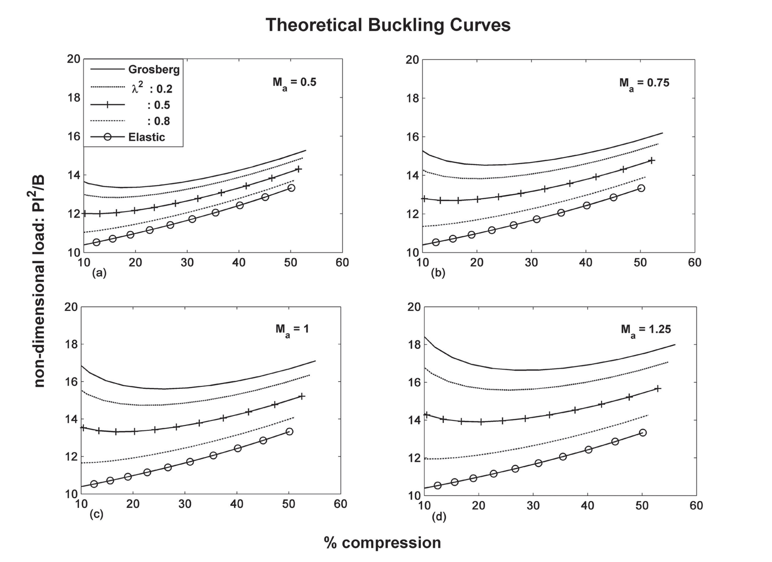

Figure 2: Effect of λ^2 on Fabric Buckling for (a)$\overline{M}_a$ = 0.5, (b)$\overline{M}_a$ = 0.75, (c)$\overline{M}_a$ = 1.0 and (d)$\overline{M}_a$ = 1.25.

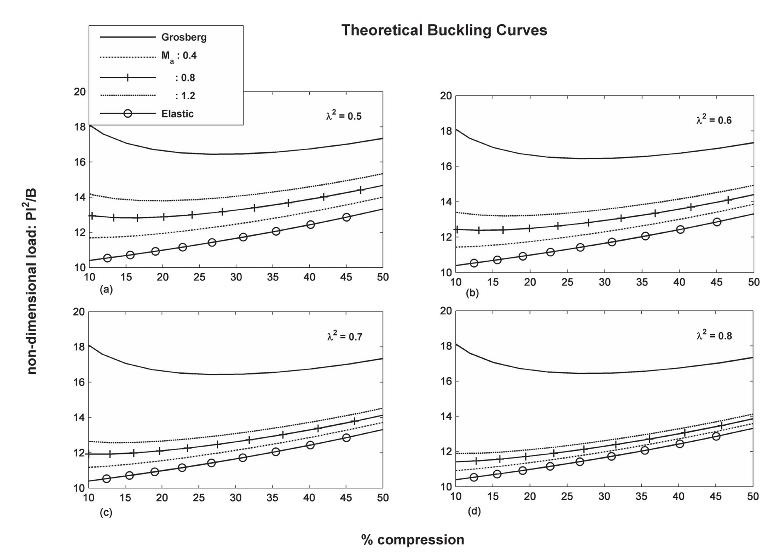

Figure 3: Effect of $\overline{M}_a$ on Fabric Buckling for (a) λ^2 = 0.5, (b) λ^2 = 0.6, (c) λ^2 = 0.7 and (d) λ^2 = 0.8.

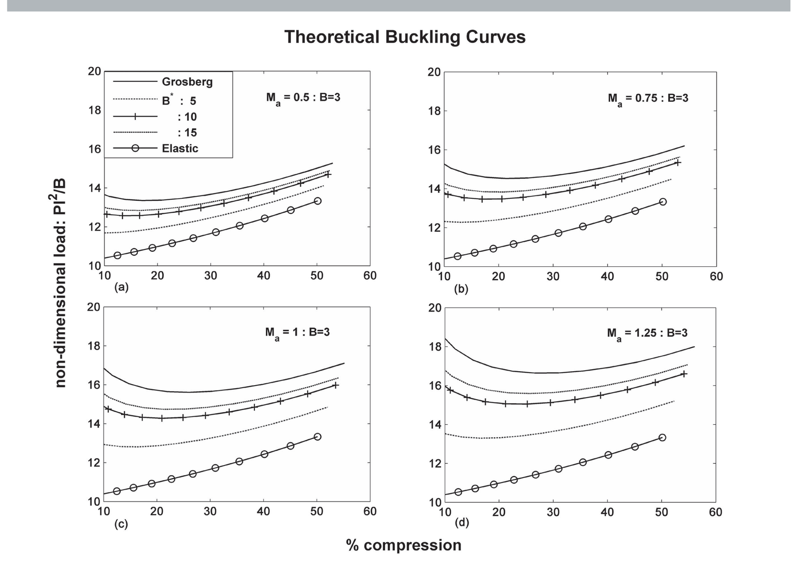

Figure 4: Effect of the Initial Bending Rigidity, B^* at a Constant Value of the Final Bending Rigidity, B, for (a) $\overline{M}_a$ = 0.5, (b) $\overline{M}_a$ = 0.75, (c) $\overline{M}_a$ = 1.0 and (d) $\overline{M}_a$ = 1.25.

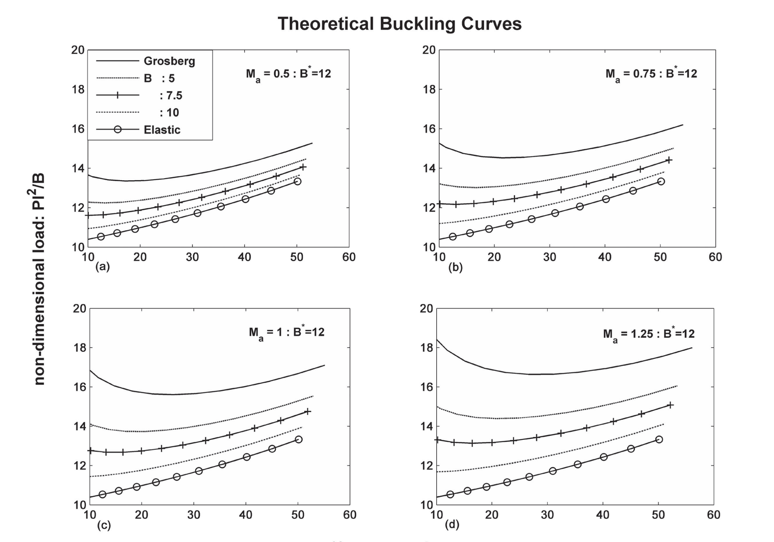

Figure 5: Effect of the Final Bending Rigidity, B at a Constant Value of the Initial Bending Rigidity, B^*, for (a) $\overline{M}_a$ = 0.5, (b) $\overline{M}_a$ = 0.75, (c) $\overline{M}_a$ = 1.0 and (d) $\overline{M}_a$ = 1.25.

CONCLUSIONS

The theoretical buckling model of fabric presented here, which is based on the bilinear bending rule, modifies the Grosberg and Swani's buckling model. The proposed model also includes, as subsets, other models, such as that of Grosberg and Swani's and the elastic buckling models. The proposed model behaves well numerically and its solution can be easily obtained by utilizing established and commercially available numerical analyses programs. The numerical results appear realistic and are consistent with theoretical assumptions. The second part of this paper will deal with the recovery behaviour of the fabric buckled to a finite deformation and the effect of bending hysteresis.