

Performance of Super-Orthogonal Space-time Trellis Code in a Multipath Environment

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Abstract—This paper investigates the performance of Super-Orthogonal Space-time Trellis Code (SOSTTC) designed primarily for non-frequency selective (i.e. flat) fading channel but now applied to a frequency selective fading channel. A new decoding trellis is proposed for the SOSTTC in frequency selective fading environment. It was apparent that the number of state of the decoding trellis of the SOSTTC in a frequency selective channel is a function of the number of divergent paths per state, the multipath ray and the original state number of the SOSTTC. Simulation result (i.e. frame error rate) shows that the SOSTTC in such environment (i.e. frequency selective) could not take full advantage of the multipath environment to increase coding advantage (i.e. shift of error rate curve) due to increase in parallel path of the decoding trellis. However the diversity order (i.e. slope of the error rate curve) performance is equivalent to that obtained in flat fading channel.

Index Terms—super-orthogonal codes, space-time codes frequency selective channel, multiple-input multiple-output

I. INTRODUCTION

THE use of channel code in combination with multiple transmit antenna achieves diversity, but the drawback is loss in bandwidth efficiency. Diversity can be achieved without any sacrifices in bandwidth efficiency if the channel codes are specifically designed for multiple transmit antennas. Space-time coding is a bandwidth and power efficient method of communication over fading channels. It combines the design of channel coding, modulation, transmit diversity and, receive diversity. Space-time codes provide better performance compared to an uncoded system. Some of the basic technique of space-time codes includes layered space-time code [1], space-time trellis code [2], [3], space-time block code [4], [5] and super-orthogonal space-time trellis code [6]. SOSTTC is a new class of space-time code that combines the set partitioning and a super set of orthogonal space-time block codes in a systematic way to provide full diversity and improved coding gain when compared with earlier space-time trellis construction [2], [3], [4] and [5]. SOSTTC does not only

provides a scheme that has an improvement in coding gain when compared with earlier construction, but it also answers the question of a systematic design for any rate, number of states and the maximization of coding gain. The orthogonal transmission matrix used in the design of SOSTTC is given by (1)

$$A(x_1, x_2, \theta) = \begin{pmatrix} x_1 e^{j\theta} & x_2 \\ -x_2^* e^{j\theta} & x_1^* \end{pmatrix} \quad (1)$$

For an M-Phase Shift Keying (PSK) modulation with constellation signal represented by $x_i \in e^{j\frac{2\pi a}{M}}$, $i = 1, 2, \dots, a = 0, 1, \dots, M-1$, one can pick $\theta = 2\pi a/M$, where $a = 0, 1, \dots, M-1$. In this case, the resulting transmitted signals of (1) are also member of the M-PSK constellation and, therefore do not expand the constellation signals. Since the transmitted signals are from a PSK constellation, the peak-to-average power ratio of transmitted signals is equal to one. The choice of θ that can be used in equation 1 for both Binary PSK (BPSK) and Quaternary PSK (QPSK) is given as $0, \pi$ and $0, \pi/2, \pi, 3\pi/2$ respectively.

It should be noted that when $\theta = 0$, equation 1 becomes the code presented in [4] (i.e. Alamouti code). The construction of the SOSTTC is based on the expansion of the orthogonal transmission matrices and standard set partitioning method [7]. In [6], the set partitioning for the SOSTTC's are shown and the way the code maximizes coding gain, without sacrificing rate.

However all the above mentioned techniques for space-time codes (i.e. STBC, STTC and SOSTTC) performances in [2], [4] and [6] is based on two fundamental assumptions on the fading channel, viz. a) Non-frequency selective channel – the channel does not have temporal interference, b) The fading from each transmit antenna to any receive antenna are independently identically distributed – this assumption is valid if the antennas are located far apart from each other (to be precise $\lambda/2$ separation between antennas).

The first assumption may not be guaranteed to be possible in an outdoor setting where delay spreads are significantly large making the channel frequency selective and thereby

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causing temporal interference of signals.

Temporal signal interference can severely degrade the performance of space-time codes. Space-time codes typically suffer from irreducible floor of error probability both in terms of the frame error rate and in term of the bit error rate [8]. Two main approaches are found in literature that are used to enhance the performance of space-time codes in a multipath frequency selective fading channel are

- Orthogonal frequency-division multiplexing [9] i.e. temporal signal interferences are reduced by converting the frequency selective fading channel into parallel flat fading channels
- Employing maximum likelihood sequence estimation can be employed with equalization [10].

In this paper, the later is used to mitigate the effect of the temporal signal interference. The paper proposes a new decoding trellis for the SOSTTC to mitigate the effect of temporal signal interference. The performance of the SOSTTC when equipped with the proposed trellis is shown with error rate simulation results for a 2-state and a 4-state QPSK SOSTTC system.

The paper is organized as follows. In Section II, the system model of the SOSTTC is described in frequency selective fading channel. The decoding scheme of our SOSTTC in frequency selective channel with the new decoding trellis is described for a 2-state and 4-state SOSTTC scheme in section III. The Simulation results are presented in Section IV and the conclusion are drawn in Section V.

II. SYSTEM MODEL OF SUPER-ORTHOGONAL SPACE-TIME TRELLIS CODE FREQUENCY SELECTIVE FADING CHANNEL

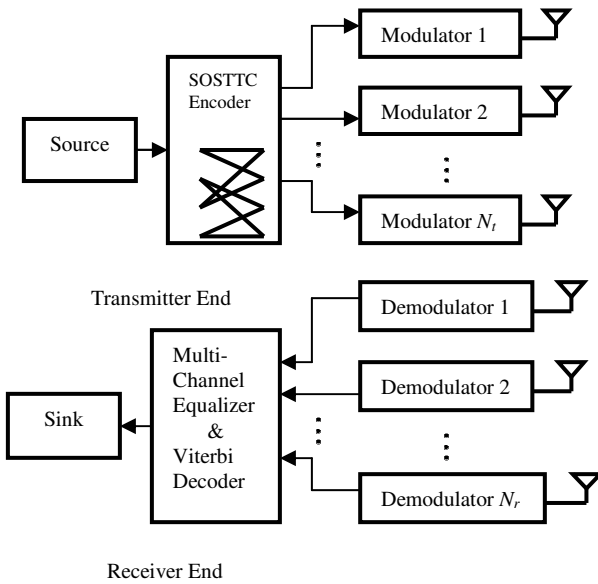


Figure 1 Block diagram of SOSTTC transmission in a frequency selective channel.

The above figure shows the proposed SOSTTC system equipped with N_t transmit antennas and N_r receive antennas. The transmitter employs a concatenated scheme where a Multiple-Trellis Coded Modulation (M-TCM) encoder with multiplicity of N_c is used as an outer code and an $N_c \times N_t$ space-time block code is used as the inner code. The transmitter encodes k_c information bits into $N_c N_t$ symbols (i.e. $N_c \times N_t$ matrix dimension) corresponding to the edge in the trellis of the space-time block with 2^v states, where v is the memory of the space-time encoder. The encoded symbols x_i are divided into N_t streams, each of which is linearly modulated and simultaneously transmitted via each antenna using the orthogonal transmission matrix in (1). We define $x_i^{(n)}$ as the modulated symbol transmitted from the i^{th} transmit antenna in the n^{th} signaling interval.

The transmission trellis for a 2-state and 4-state SOSTTC schemes are given below:

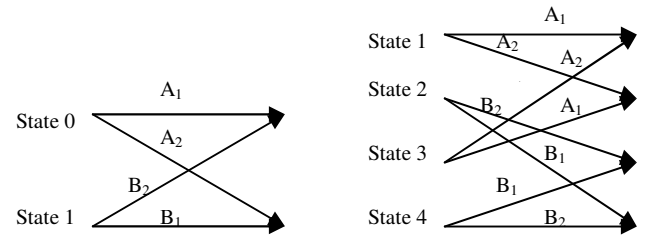


Figure 2 2-State and 4-State 2-bits/symbol SOSTTC

For QPSK symbols, the orthogonal transmission matrices partitioning is given below as

$$\begin{aligned}
 A_1 &\equiv \{(\pm 1, \pm 1, 0), (\pm j, \pm j, 0)\} \\
 A_2 &\equiv \{(\pm 1, \pm j, 0), (\pm j, \pm 1, 0)\} \\
 B_1 &\equiv \{(\pm 1, \pm 1, \pi), (\pm j, \pm j, \pi)\} \\
 B_2 &\equiv \{(\pm 1, \pm j, \pi), (\pm j, \pm 1, \pi)\}
 \end{aligned} \tag{2}$$

The solution for temporal signal interference for space-time block codes as stated in [11] assume that the space-time coding is done over two large blocks of data symbols instead of just two symbols as in the original proposed scheme (i.e. [4]). Further in the receiver of the scheme proposed in [11], there is an increase in complexity due to the doubled front-end convolution of the overall system. Asokan *et.al.* [12] proposed a scheme that uses a transmitter with original space-time coding that is over every two-symbol block.

In this paper, we assume that the temporal interference is over every two-symbol as used in [12] for orthogonal space-time block code transmission.

Based on the above assumptions and if the number of receiver i.e. $N_r=1$ and the channel state information is know at the receiver we can write equation 3 as:

$$\begin{aligned}
\begin{bmatrix} r_1^{(n)} \\ r_2^{(n)} \end{bmatrix} &= \begin{bmatrix} x_1^{(n)} e^{j\theta^{(n)}} & x_2^{(n)} \\ (-x_2^{(n)})^* e^{j\theta^{(n)}} & (x_1^{(n)})^* \end{bmatrix} \begin{bmatrix} h_{0,1}^{(n)} \\ h_{0,2}^{(n)} \end{bmatrix} + \\
&= \begin{bmatrix} (-x_2^{(n-1)})^* e^{j\theta^{(n-1)}} & (x_1^{(n-1)})^* \\ x_1^{(n)} e^{j\theta^{(n)}} & x_2^{(n)} \end{bmatrix} \begin{bmatrix} h_{1,1}^{(n)} \\ h_{1,2}^{(n)} \end{bmatrix} + \quad (3) \\
&= \dots + \underline{\mathbf{K}} \begin{bmatrix} h_{k-1,1}^{(n)} \\ h_{k-1,2}^{(n)} \end{bmatrix} + \begin{bmatrix} \eta_1^{(n)} \\ \eta_2^{(n)} \end{bmatrix}
\end{aligned}$$

Where k is the channel tap, the suffix in $h_{u,w}^{(n)}$ stands for the u^{th} channel tap from transmitter w and

$$\underline{\mathbf{K}} = \begin{cases} \begin{bmatrix} x_1^{(2n-k+1)/2} e^{j\theta^{(2n-k+1)/2}} & x_2^{(2n-k+1)/2} \\ (-x_2^{(2n-k+1)/2})^* e^{j\theta^{(2n-k+1)/2}} & (x_1^{(2n-k+1)/2})^* \end{bmatrix} & \text{If } k \text{ is odd} \\ \begin{bmatrix} (-x_2^{(2n-k)/2})^* e^{j\theta^{(2n-k)/2}} & (x_1^{(2n-k)/2})^* \\ x_1^{(2n-k+2)/2} e^{j\theta^{(2n-k+2)/2}} & x_2^{(2n-k+2)/2} \end{bmatrix} & \text{If } k \text{ is even} \end{cases}$$

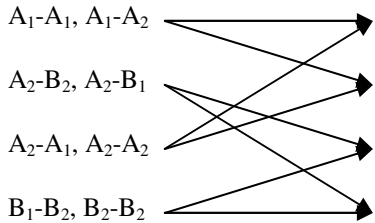
Equation 3 represents the corresponding set of successive sample at two output times. $\eta_i^{(n)}$ are independent identically distributed complex zero mean Gaussian noise samples, each sample with $\sigma^2/2$ per dimension. It is assumed that the channel elements undergo Rayleigh Fading.

III. RECEIVER STRUCTURE OF THE SUPER-ORTHOGONAL SPACE-TIME TRELLIS CODE IN MULTIPATH FADING CHANNEL

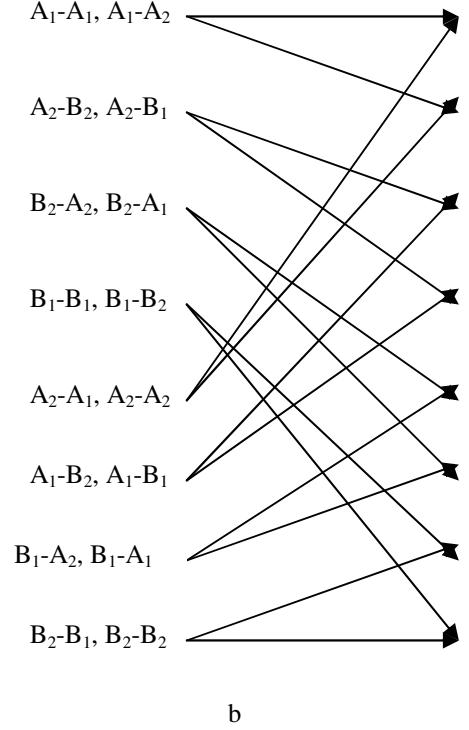
In this section, the receiver structures with maximum likelihood criterion for the SOSTTC are enumerated to mitigate the effect of temporal signal interference.

In this paper, we assume there are two rays in each subchannel and that the temporal interference spans two-symbol blocks. If there are more than two rays in each subchannel, the method enumerated here can be straightforwardly extended.

The number of states of the receiver structure for the 2-state and 4-state QPSK SOSTTC coding system increase to 4 and 8 respectively. The resultant code trellis for the receiver structure is given below.



a



b

Figure 3 a) Decoding trellis of the receiver for the 2-state SOSTTC b) Decoding trellis of the receiver for the 4-state SOSTTC

The above trellises represent the equivalent decoding trellis for the 2-state and the 4-state SOSTTC in a frequency selective fading channel with temporal interference that spans two-symbol block. The transition per state contains 64 parallel paths of signal sets. In the trellises above, A_i (or B_j) - A_j (or B_i) represent the delayed version of two-symbol block A_i (or B_i) affected by the second tap and A_j (or B_j) represent the two-symbol block affected by the first tap (our analysis assume $k=2$).

It is apparent that the number of states of the receiver trellis for the SOSTTC when the temporal interference spans two-symbol block with k rays is given by $2^{*(k-1)*l}$, where l is the original state number of the super-orthogonal space-time trellis code.

IV. SIMULATION RESULTS

In this section, the simulation results of the receiver for the 2-state and 4-state super-orthogonal space-time trellis code are stated under the assumption that the receiver has perfect knowledge of the channel state information. The channel is modeled as a two-ray quasi-static frequency selective fading Rayleigh channel with uncorrelated channel rays. There are 130 symbols transmitted from the two transmit antennas (trellis in Figure 2 is used at the transmitter for both 2-state and 4-state SOSTTC). At the receiver, the receiver structure enumerated in Figure 3 is used with Viterbi algorithm [13] to

search for path with the lowest accumulated metric.

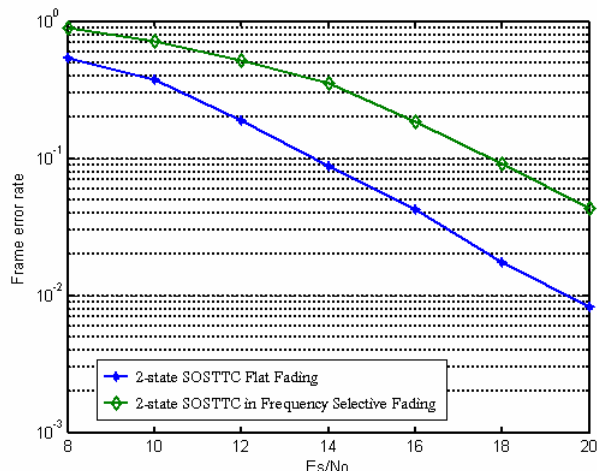


Figure 4 FER of a 2-state SOSTTC with $N_t=2$, $N_r=1$ in fading channels

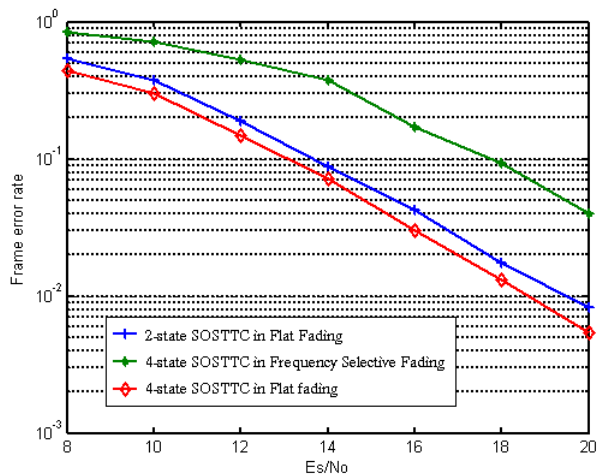


Figure 5 FER of SOSTTC schemes with $N_t=2$, $N_r=1$ in fading channel.

Figure 4 shows the frame error rate of 2-state SOSTTC in both frequency selective and non-frequency selective fading channel. The simulation parameters were chosen specifically to be able to compare the diversity advantage of the scheme with similar scheme in [6]. From Figure 4 we can see that the 2-state SOSTTC scheme in frequency selective channel achieve the same diversity order (i.e. slope of the error rate curve) as compared with the scheme in flat fading scenario, although the scheme suffer some coding gain loss. The simulation in Fig. 4 shows about a 4dB performance degradation of the 2-state SOSTTC scheme in frequency selective channel. This performance degradation in Figure 4 and 5 can be attributed to the increase in number of parallel path transitions per state. If we assume that all the 64 transitions per branch in the decoding trellis (i.e. Fig. 3 a & b) are equally likely to be decoded, the probability of decoding correctly a transmitter codeword per state is equal to $1/64 \times$

$1/64 = 1/4096$. The probability of decoding a transmitted codeword in the scheme under non-frequency selective fading assumption is $1/8 \times 1/8 = 1/64$.

For a 2-state decoder (i.e. SOSTTC code in a non-frequency selective fading channel) the probability per decoding interval becomes $1/64 \times 2 = 1/32$ while for a 4-state decoder (i.e. SOSTTC in a frequency selective fading channel) the probability per decoding interval becomes $1/4096 \times 4 = 1/1024$.

Although there is an increase in state in the SOSTTC code decoder in frequency selective channel, the probability of decoding correctly is still lower as compared with the case of the code in a non-frequency selective channel. This accounts for the performance loss obtained in terms of the coding advantage (i.e. shift in the error curve upward). This above stated explanation is also applicable to the performance degradation of the code in Figure 5.

V. CONCLUSION

In this paper, the receiver structure of a super-orthogonal space-time trellis code in a frequency selective channel is designed. The decoding trellises for a 2-state and 4-state code schemes are given. The formula for deriving the number of states of the SOSTTC in frequency selective was derived as a function of the number of divergent path per state, the multipath ray and the original number of state of the SOSTTC. The simulations results proved that although the code was designed for flat fading channel, it provide at least the same diversity advantage when applied to a frequency selective Rayleigh fading channel.

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