

# An alternative method for the measurement of the mechanical impulse of a vertically directed blast

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An alternative method for the measurement of the total mechanical impulse of a vertically directed blast due to an explosive charge is presented. The method differs from apparatus that employ a vertically displaced mass (similar in principle to the ballistic pendulum) in that a relatively compact spring-damper system is employed to constrain the movement of the mass. The mechanical impulse is determined by integrating, with respect to time, the net force applied to the spring-damper system. The details of an explosive impulse measuring instrument rated to 12 kN s are presented. © 2008 American Institute of Physics. [DOI: 10.1063/1.2949391]

## INTRODUCTION

Taylor *et al.*<sup>1</sup> have described the use of the vertical impulse measurement fixture at the Army Research Laboratory, Aberdeen, MD, for large scale vertical impulse measurements of explosive charges placed beneath a target of known mass. The movement of the target is confined to the vertical by a guide rail which passes through a guide housing. Impulse calculations are performed by measuring the height to which the rail rises, subsequent to the detonation of the explosive charge beneath the target. A distinct disadvantage of this method is the size of the apparatus due to the unconstrained height to which the target may rise. An alternative to a free-moving target is a target constrained by a system of springs and dampers. This has the potential to reduce the overall size of the apparatus, but greatly increases the force upon the frame which supports the target during an explosive event. If the force exerted by the target against the frame exceeds the weight of the frame (including any structure to which the frame is anchored), the frame will hop, potentially shifting its position and destroying its alignment.

## THEORY

A simple mechanical analog of an explosive impulse measuring instrument (EIMI) is shown in Fig. 1. The EIMI comprises a target attached to a static frame by a system of springs and dampers. The target is confined to move in the vertical direction only. For reasons that will become apparent, the spring-damper pairs cannot be operated in tension, and thus, the distance between the two halves of the target is adjusted such that the upper and lower spring-damper pairs are maintained in compression throughout the range of motion of the target.

From first principles,

$$m \frac{d^2x}{dt^2} = F_1 - F_2 - mg, \quad (1)$$

where  $m$  and  $x$  are the mass and vertical displacement of the target,  $F_1$  and  $F_2$  are the upper and lower spring-damper

forces, respectively, and  $g$  is the acceleration due to gravity. Here,  $g = 9.8 \text{ m/s}^2$ .

Integrating Eq. (1) with respect to time,

$$m \frac{dx}{dt} = \int (F_1 - F_2 - mg) dt + p_0, \quad (2)$$

where  $p_0$  is the momentum imparted to the target by the explosive event at time  $t=0$ . Component values are selected such that the duration of the explosive event is negligible compared to the response time of the mechanical system. Since the final velocity of the target is zero, then

$$p_0 = - \int_0^T (F_1 - F_2 - mg) dt. \quad (3)$$

Thus, the momentum imparted to the target may be determined by integrating with respect to time the *net* force exerted *upon* the target by the system of *springs and dampers* and *gravity*. The former may be gauged by instrumenting the spring-damper assembly with force gauges or *load cells*, the latter is a known constant. The spring-damper forces are recorded until such time  $T$  as the target has returned to rest. Equation (3) is valid *regardless* of whether the springs and dampers behave as *linear* or *nonlinear* elements. The kinetic energy acquired by the target is ultimately dissipated as heat into the dampers.

Assuming the spring-damper pairs are linear and identical, then

$$-F_1 = k(x - d_1) + c \frac{dx}{dt}, \quad (4)$$

and

$$F_2 = k(x + d_2) + c \frac{dx}{dt}, \quad (5)$$

where  $k$  is the spring coefficient of stiffness,  $c$  is the damper coefficient of friction, and  $d_1$  and  $d_2$  are the dimensions by which the upper and lower springs are compressed, at rest, relative to their uncompressed dimensions, in order to prevent the springs from operating in tension.

From Eqs. (4) and (5), the force against the frame supporting the target is

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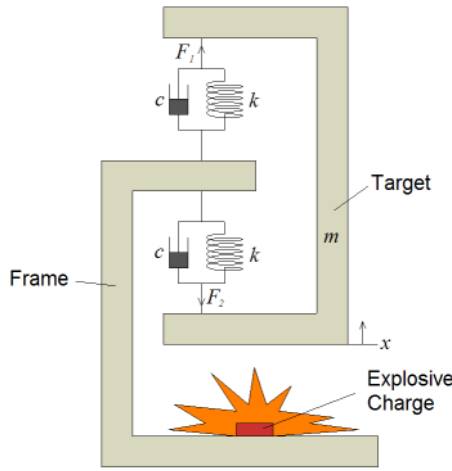


FIG. 1. A simple mechanical analog of an explosive impulse measuring apparatus.

$$F_2 - F_1 = 2kx + 2c \frac{dx}{dt} - k(d_1 - d_2). \quad (6)$$

Substituting Eqs. (4) and (5) into Eq. (1) and solving yields the result

$$x(t) = e^{-ct/m} \left[ \left( \frac{v_0}{\omega} - x_0 \frac{c}{m\omega} \right) \sin \omega t - x_0 \cos \omega t \right] + x_0, \quad (7)$$

where

$$v_0 = \frac{dx}{dt} \Big|_{t=0} = p_0/m \quad (8)$$

is the initial velocity of the target,

$$\omega = \sqrt{2k/m - (c/m)^2} \quad (9)$$

is the characteristic frequency of oscillation, and

$$x_0 = \frac{1}{2} \left[ (d_1 - d_2) - \frac{mg}{k} \right] = 0, \quad (10)$$

since  $k(d_1 - d_2) = mg$  when the target is at rest. Substituting Eq. (10) into Eq. (7),

$$x(t) = \frac{v_0}{\omega} e^{-ct/m} \sin \omega t. \quad (11)$$

Thus,

$$\frac{dx}{dt}(t) = v_0 e^{-ct/m} \left( \cos \omega t - \frac{c}{\omega m} \sin \omega t \right). \quad (12)$$

Solving Eq. (3),

$$p(t) = mv_0 \left[ 1 - e^{-ct/m} \left( \cos \omega t - \frac{c}{\omega m} \sin \omega t \right) \right]. \quad (13)$$

Substituting Eqs. (11) and (12) into Eq. (6),

$$\begin{aligned} F_2 - F_1 &= 2k \frac{v_0}{\omega} e^{-ct/m} \sin \omega t \\ &+ 2cv_0 e^{-ct/m} \left( \cos \omega t - \frac{c}{\omega m} \sin \omega t \right) \\ &- mg. \end{aligned} \quad (14)$$

Plots of Eqs. (11)–(14) for a 50 kN s impulse are shown in

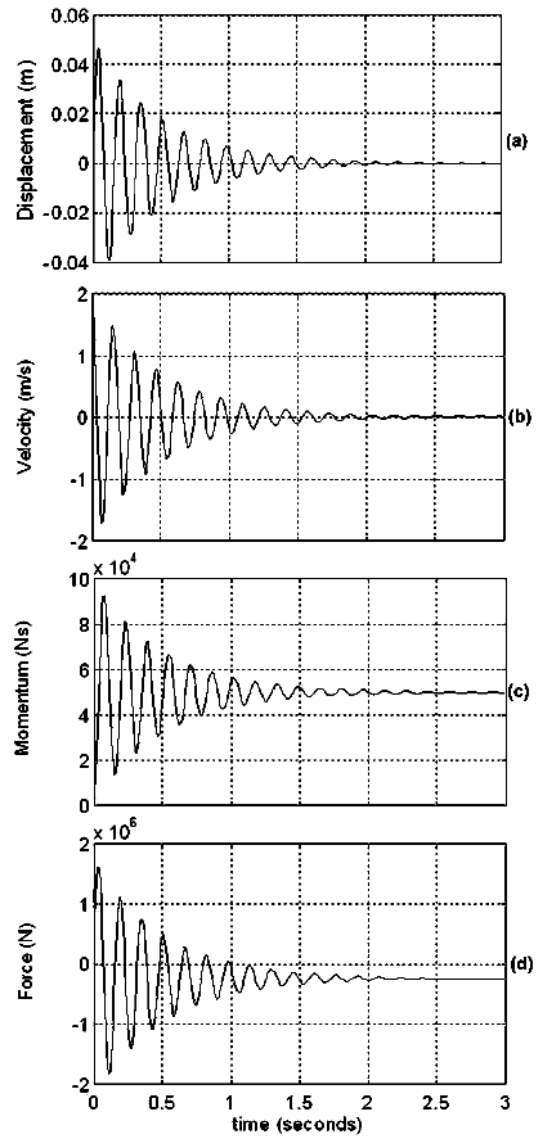


FIG. 2. Calculated target displacement (a), target velocity (b), dissipated momentum (c), and frame force (d), with time.

Figs. 2(a)–2(d), respectively. The parameters of the mechanical system are presented in Table I.

Results of the calculations indicate that the  $2.5 \times 10^4$  kg target is displaced a maximum of 46.3 mm upward and 39.6 mm downward due to the 50 kN s impulse imparted to the target. Equation (8) determines an initial target velocity of 2 m/s and Eq. (9) determines a resonant frequency of 6.36 Hz. The target takes approximately 3 s to come to rest after the detonation. The maximum lifting force exerted upon the frame (as the target rises) is  $1.62 \times 10^6$  N.

TABLE I. Parameters of the simulated mechanical system.

$k$	$= 2 \times 10^7$ N/m
$c$	$= 5 \times 10^4$ Ns/m
$m$	$= 2.5 \times 10^4$ kg
$g$	$= 9.8$ m/s <sup>2</sup>
$p_0$	$= 5 \times 10^4$ Ns



FIG. 3. The SIIMA.

Thus, if the mass of the frame does not exceed  $1.65 \times 10^5$  kg, the frame will lift during an impulse measurement. Figure 2(d) indicates that Eq. (3) settles to a value of  $5 \times 10^4$  N s, as required.

### SCIENTIFICALLY INSTRUMENTED IMPULSE MEASURING APPARATUS

An existing EIMI, the scientifically instrumented impulse measuring apparatus (SIIMA), shown in Fig. 3, was modified to test the presented theory. Due to space constraints within the frame, the maximum target mass that could be accommodated was approximately  $8.5 \times 10^3$  kg, manufactured from WA300 mild steel. The existing frame weighs approximately  $3 \times 10^4$  kg and is anchored to a  $1.4 \times 10^5$  kg steel reinforced concrete slab with a circular sandpit at the center. A cross section of the modified SIIMA is shown in Fig. 4. The spring-damper system was assembled from 32 polyurethane cylinders arranged in 16 columns of two cylinders each, compressed between the frame and the target. Polyurethane behaves as a viscoelastic material, and thus demonstrates the properties of both a spring and a damper. Each of the 16 polyurethane spring dampers was

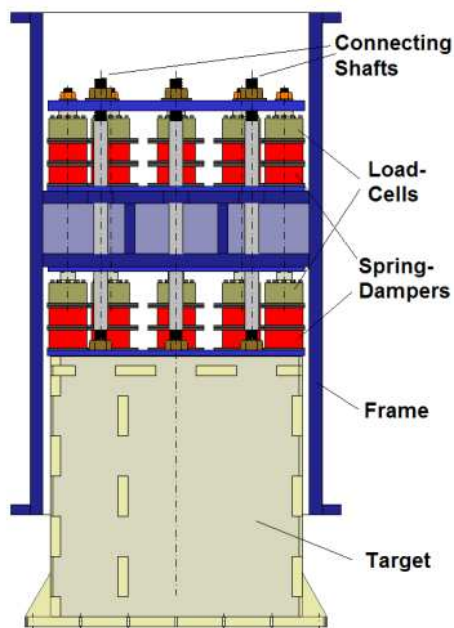


FIG. 4. A cross section of the SIIMA.

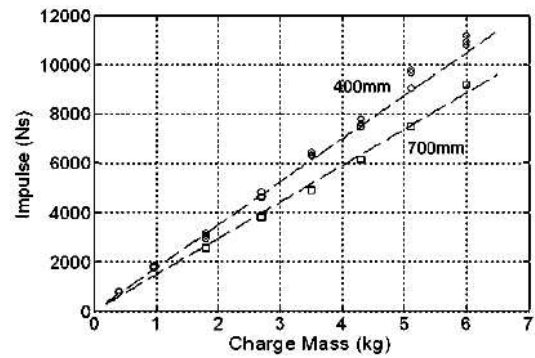


FIG. 5. Impulse data as a function of charge mass.

instrumented with a single universal low profile load cell rated to  $2 \times 10^4$  kg or  $19.6 \times 10^4$  N. Thus, the SIIMA was capable of measuring forces of up to  $15.7 \times 10^5$  N in both the upward and downward directions. To prevent the polyurethane spring dampers from operating in tension and tearing from their mountings, the nuts on 12 shafts, connecting the upper and lower sections of the target, were progressively tightened until a static load of approximately half the dynamic range of the load cells was registered. The difference between the upper and lower load-cell values is conveniently the weight of the target.

The presented model is useful for demonstrating the principle of operation of an EIMI, but is insufficiently detailed to describe the behavior of the SIIMA due to the nonlinear behavior of the polyurethane spring dampers. To this end a nonlinear mathematical model was constructed in which the equivalent spring stiffness and damper friction of the polyurethane cylinders were varied dynamically with the target displacement and velocity. The resultant nonlinear differential equations of motion were solved numerically. The results of the model served to indicate that the range of motion of the target was within the limitations of the polyurethane spring dampers for an impulse of up to 12 kN s. It should be kept in mind that Eq. (1) is universally true for mechanical systems of this type and Eq. (3) therefore describes a valid method for determining the impulse of an explosive charge detonated below the target, irrespective of the behavior of the springs and dampers.

### EXPERIMENTAL RESULTS

A series of experiments were undertaken with PE4 plastic explosive charges. Each charge was hand pressed into a right-regular cylindrical plastic mould with diameter to height ratio of 2:1; weighed to an accuracy of 10 mg; and buried flush with the surface of the sandpit, 400 and 700 mm, from the underside of the target. The results of tests conducted with charges ranging from 0.4 to 6 kg mass are shown in Fig. 5. A basic prediction of the expected impulse as a function of the charge mass was performed using the modified open-face sandwich equation of Gurney<sup>2</sup>,

$$I = m_m \Lambda \sqrt{2E} \left[ \frac{1 + \left(1 + 2 \frac{m_m}{m_c}\right)^3}{6 \left(1 + \frac{m_m}{m_c}\right)} + \frac{m_m}{m_c} \right]^{1/2}, \quad (15)$$

where  $I$  is the predicted impulse,  $\sqrt{2E}=2680$  m/s is the characteristic Gurney velocity of PE4,  $m_m$  and  $m_c$  are the masses of the target and explosive charge, respectively, and

$$\Lambda = \frac{b^2}{ab + b^2} \quad (16)$$

is a geometric correction factor introduced to account for the separation between the charge and the target. Here,  $a$  is the distance measured between the underside of the target and the top of the explosive charge, and  $b^2$  is the surface area of the target face. If the mass of the target is large in comparison to the mass of the explosive charge then Eq. (15) reduces to

$$I \approx \Lambda m_c \sqrt{2E} \sqrt{\frac{3}{4}}, \quad (17)$$

which is independent of the mass of the target and indicates a linear relationship between the explosive impulse and the mass of the explosive charge, as shown in Fig. 5.

## CONCLUSIONS

The theory of operation of an EIMI employing a target constrained by a system of springs and dampers, has been presented. The theory was developed into a simple analytical

model in order to predict the displacement, velocity, and momentum of the target with time. The model has been expanded to describe the operation of the SIIMA. The SIIMA, an existing EIMI, was modified to test the presented theory and employs nonlinear viscoelastic polyurethane cylinders in place of conventional linear springs and dampers.

A series of experimental tests were undertaken with the modified SIIMA in order to determine the explosive impulse as a function of charge mass for charges of PE4 ranging from 0.4 to 6 kg in mass. Recorded waveforms were similar to those shown in Fig. 2 and indicated a characteristic frequency of oscillation of approximately 18 Hz. A basic prediction of the impulse was performed using the modified open-face sandwich equation of Gurney. Results compared well with predictions. The method of impulse calculation described by Eq. (3) is thus considered theoretically and practically sound.

## ACKNOWLEDGMENTS

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<sup>1</sup>L. C. Taylor, R. R. Skaggs, W. Gault, *Fragblast* 9, 19 (2005).

<sup>2</sup>Paul W. Cooper, *Explosives Engineering* (Wiley-VCH, 1996), Chap. 27, p. 390.