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Use of Eigenvectors in the Solution of the Flutter Equation

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Nomenclature

$[A]$ = aerodynamic matrix
c = reference chord
g = structural damping
$[K]$ = stiffness matrix
k = $\omega c/V$, reduced frequency
$[M]$ = inertia matrix
p = $\gamma k \pm ik$, eigenvalue of the flutter equation
q = vector of degrees of freedom
V = true air speed
X, Y = complex vectors of length n
γ = damping coefficient
ρ = air density
ω = angular frequency

Introduction

THE flutter equation and its solution is central to almost all aeroelasticity problems. Various formulations of the flutter equation exist, each involving some approximations. The *p-k* formulation of Hassig[1] is generally accepted as the one giving the most realistic damping values. However, the determinant iteration solution procedure proposed by him may fail under certain conditions, and some trial and error is usually required to obtain a satisfactory solution. An alternative solution procedure using eigenvectors to assign eigenvalues to modes is suggested which will in most cases substantially reduce the chance of failure.

Although the use of eigenvectors is presented for the *p-k* formulation only, the method can be applied equally well to the *V-g* formulation. The method was originally developed to assign eigenvalues to modes in the solution of the latter formulation, "the fundamental and troublesome problem," according the Desmarais and Bennett.[2] The description of the method given here is directly applicable to the *V-g* formulation.

p-k Method

A simplified form of the *p-k* formulation of Hassig is

$$\left[\frac{V^2}{c^2}[M]p^2 + (1 + ig)[K] - \frac{1}{2}\rho V^2[A(k)]\right] q = 0 \qquad (1)$$

Aerodynamic matrices are calculated for a relatively small number of reduced frequencies and are interpolated to other reduced frequencies as required during the solution process. The solution consists of a frequency and damping coefficient, calculated from the eigenvalues of the flutter equation, for each mode at a given set of true air speeds. The eigenvalues determined at previous speeds are used to calculate initial values for a determinant iteration procedure to determine the eigenvalues at the next speed. The solution is started at very low speed where the eigenvalues are close to the structural values determined by either ground vibration testing or finite element analysis.

Two commonly encountered conditions may cause the procedure to fail. If an eigenvalue changes rapidly with increasing speed, the determinant iteration procedure may not converge or converge to the wrong eigenvalue due to too large an error in the initial value of p. Decreasing the speed increment usually solves this problem at the cost of increased computer time. When two eigenvalues are close to each other or even identical, it is sometimes impossible to prevent the procedure from converging to the wrong eigenvalue, even with very small speed increments.

Alternative Procedure

One possible way to avoid the problems of failure to converge or converging to the wrong eigenvalue, is to use an eigenvalue routine which can handle repeated eigenvalues. Such a routine will return a number of eigenvalues equal to the number of modes, and it must still be determined which is the desired eigenvalue. Eigenvectors can be used effectively to select the eigenvalue even if the eigenvalues have changed substantially from the previous speed, justifying the use of a more expensive (in computer time) eigenvalue routine by the possibility of using larger speed increments and having fewer failures.

Implementation

In the present implementation, the solution for one mode at all speeds is found before the solution for the next mode is started. The explanation which follows is for the solution of mode i only. Although only the solution for mode i is valid (because the aerodynamic matrix is a function of reduced frequency), the eigenvalues and eigenvectors corresponding to all the other modes are calculated.

The natural frequency of mode i is used in the first calculation of the reduced frequency at the first speed. The aerodynamic matrix is interpolated and the eigenvalue routine is called to solve the eigenvalues and eigenvectors. The eigenvectors are compared to unit vectors, corresponding to the natural modes, in order to determine which eigenvalue corresponds to mode i. This eigenvalue is used in the next calculation of the reduced frequency and the eigenvalue routine is called again. The process is repeated until the reduced frequency converges. At each successive speed, the converged eigenvalue of the previous speed is used in the first calculation of the reduced frequency. The aerodynamic matrix is interpolated and the eigenvalue routine is called to solve the eigenvalues and eigenvectors. The eigenvectors are compared to the converged eigenvectors of the previous speed to determine which eigenvalue corresponds to mode i. This eigenvalue is used in the next calculation of the reduced frequency.

Eigenvectors are compared as follows: A matrix of scalar products of the converged eigenvectors of the previous speed and the new eigenvectors is calculated. Each column of the matrix corresponds to a new eigenvector and each row to an old eigenvector. The element in row i and column j of the matrix is the scalar product of old eigenvector i and new eigenvector j. The matrix is then searched for the largest element. The corresponding old and new eigenvectors are taken to belong to the same mode. The corresponding row and column is zeroed and the process is repeated until the whole matrix is zero.

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The scalar product of two complex vectors must be defined to be independent of scaling and phase. A definition which satisfies these conditions is

$$X \cdot Y = \frac{\sqrt{S_1^2 + S_2^2}}{\sqrt{S_3 S_4}} \tag{2}$$

where

$$S_1 = \sum_{i=1}^{n} \mathrm{Re}(X_i)\mathrm{Re}(Y_i) + \mathrm{Im}(X_i)\mathrm{Im}(Y_i) \tag{3}$$

$$S_2 = \sum_{i=1}^{n} \mathrm{Re}(X_i)\mathrm{Im}(Y_i) - \mathrm{Im}(X_i)\mathrm{Re}(Y_i) \tag{4}$$

$$S_3 = \sum_{i=1}^{n} \|X_i\|^2 \tag{5}$$

$$S_4 = \sum_{i=1}^{n} \|Y_i\|^2 \tag{6}$$

In the present implementation, the LZ eigenvalue routine of Kaufman[3,4] is used.

The present method has proved to be reliable and faster than the determinant iteration procedure. One problem that has been encountered is that the eigenvalue routine sometimes failed to find all the eigenvalues. This occurred when generalized masses varied by a factor of more than approximately 30, and the problem could be solved by rescaling the modes to unit generalized mass.

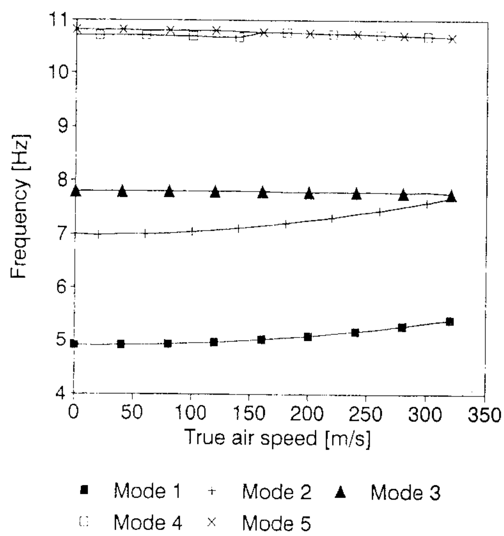

Fig. 1 Frequencies calculated with the determinant iteration procedure.

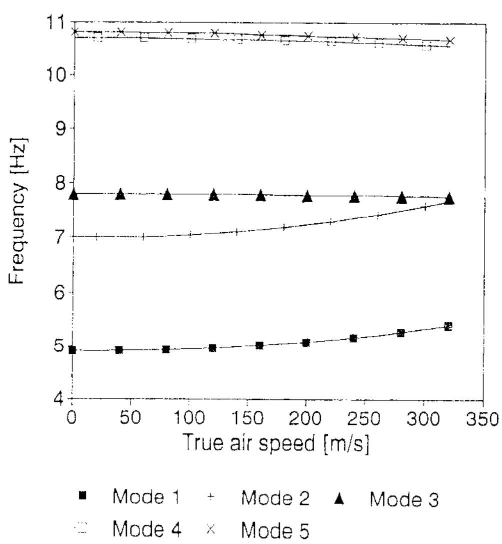

Fig. 2 Frequencies calculated with the present method.

Results

It is impossible to make an absolute comparison between the two methods of solving the flutter equation because of the many different implementations and possible refinements. The results presented here are only used to illustrate a potential problem of the determinant iteration procedure and to show that it is overcome by the present method. The test case is taken from an actual flutter clearance in which 16 modes were identified by ground vibration testing. Modes 4 and 5 are of particular interest because they were almost identical modes of the left and right wings, respectively. Due to slight structural asymmetry of the aircraft, their frequencies differed by 0.1 Hz and they were treated as separate modes. The measured structural dampings of the two modes were almost equal. For the sake of clarity, only the first five modes were used to generate these results.

In Fig. 1, calculated using the determinant iteration procedure and a speed increment of 20 m/s, a typical failure of the procedure is illustrated. At a speed of 160 m/s, the eigenvalue of mode 4 converges to the eigenvalue of mode 5. The speed increment was reduced to 5 m/s, with the same result. It seems likely that it was not the large error in the initial value of p that caused the failure, but the large perturbation in p used to calculate the derivatives of the determinant. Decreasing the perturbation in p may cause numerical instabilities and is, therefore, not always an acceptable remedy.

Figure 2, calculated using the present method and a speed increment of 20 m/s, shows that the method is successful in distinguishing between modes 4 and 5 up to the maximum speed of 320 m/s. To further test the robustness of the method, the speed increment was progressively increased until a single calculation at 320 m/s was performed. The results for the five modes were within the specified tolerances of the results calculated with a speed increment of 20 m/s. A similar calculation with all 16 modes was also successful.

Conclusions

The present method has the potential to overcome some of the problems of the determinant iteration procedure of solving the flutter equation. The use of eigenvectors to assign eigenvalues to modes has the following advantages: 1) it is possible to use a general eigenvalue routine capable of solving repeated eigenvalues; 2) eigenvalues can be distinguished effectively; 3) initial frequency values are only required for the first calculation of reduced frequency at each speed; 4) control laws can be accommodated as easily as with the determinant iteration procedure; and 5) computation time is reduced in most cases.

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