IDENTIFICATION OF ROCK MASS DISCONTINUITIES IN A CLUSTER OF SEISMIC EVENT HYPOCENTERS

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CONTENTS

Abstract

A technique to interpret a cluster of seismic events in terms of causative structures is described. The process consists of moving the located hypocenters of events within their confidence ellipsoids until a simplified pattern of the seismic event cluster is obtained. This simplified pattern might then be interpreted in terms of a fault, a system of faults or in general, as a rock mass discontinuity. The process is tested and demonstrated for three synthetic sets of data: a single fault, two joined faults and three separate faults.

3. Introduction

A technique to interpret a cluster of seismic events in terms of causative structures is presented. In the application of the procedure, a successful interpretation crucially depends on information on the confidence ellipsoid of the hypocenter co-ordinates. The next section describes in detail the technique for estimating such ellipsoids for local events generated by mining. The following section demonstrates this technique, giving an example of expected errors for a certain network configuration. The remaining section illustrates the procedure of cluster interpretation.

2. Approaches to Confidence Ellipsoid Determination

The error in seismic event co-ordinates might be considered to consist of two components. The first component is caused by errors in arrival time measurements. The second is caused by the velocity model used in the location procedure, which is only an approximation of the real (unknown) model. The proper procedure to evaluate the error ellipsoid of hypocenter co-ordinates, should take these two components into account.

2.1. Classical Approach : Confidence Ellipsoids from Data Uncertainty.

Let vector $\boldsymbol{\theta} = (t_0, x_0, y_0, z_0)^T$ denotes the focal parameters of a seismic event, where t_0 is the origin time, and (x_0, y_0, z_0) are hypocenter co-ordinates in the Cartesian system (x, y, z) and matrix operator T stands for matrix transposition. The classical theory of inversion says, that if the parameters $\boldsymbol{\theta}$ are estimated by the least-squares procedure, an approximate confidence ellipsoid for parameters $\boldsymbol{\theta}$ is of the following form:

$$(\theta - \hat{\theta})^T \boldsymbol{C}_\theta^{-1} (\theta - \hat{\theta}) \le constant, \tag{1}$$

where $\hat{\theta}$ is the estimate of $\boldsymbol{\theta}$, *constant* is an appropriate quantity from the χ_4^2 distribution, and $\boldsymbol{C}_\theta$ is the covariance matrix of the focal parameters $\boldsymbol{\theta}$. By definition of the covariance matrix, the variances of the parameters $\boldsymbol{\theta}$ are equal to the respective diagonal elements of the matrix $\boldsymbol{C}_\theta$, where

$$\boldsymbol{C}_\theta = (\boldsymbol{A}^T \boldsymbol{C}_t^{-1} \boldsymbol{A})^{-1}. \tag{2}$$

$\boldsymbol{C}_t$ is the covariance matrix of arrival times $\boldsymbol{t} = (t_1,\dots, t_n)^T$, $t_i = t_0 + T_i(x_0, y_0, z_0,\boldsymbol{V}) + \varepsilon_i$, $T_i(x_0, y_0, z_0,\boldsymbol{V})$ is the travel time from the hypocenter (x_0, y_0, z_0) to the i-th station, $\boldsymbol{V}$ is velocity model, ε_i is the error in the arrival time determination at the i-th station, $i = 1,\dots, n$, and n denotes number of seismic stations. $\boldsymbol{A}$ is the matrix of partial derivatives at the solution

$$\boldsymbol{A} = \begin{pmatrix} 1, & \frac{\partial T_1}{\partial x_0}, & \frac{\partial T_1}{\partial y_0}, & \frac{\partial T_1}{\partial z_0}, \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 1, & \frac{\partial T_n}{\partial x_0}, & \frac{\partial T_n}{\partial y_0}, & \frac{\partial T_n}{\partial z_0}, \end{pmatrix}. \tag{3}$$

Assuming statistical independence between arrival times errors determination (i.e. $Cov[\varepsilon_i,\varepsilon_j] = 0$, for $i \neq j$, and $Cov[\varepsilon_i,\varepsilon_j] = \sigma_{t_i}^2$ for $i = j$, where $\sigma_{t_i}^2$ denotes the variance of the arrival time t_i determination at station i, and $i, j = 1,\dots, n$), the covariance matrix of the data $\boldsymbol{t}$ is

$$\boldsymbol{C}_t = diag[\sigma_{t_1}^2, \sigma_{t_2}^2, \dots, \sigma_{t_n}^2]. \tag{4}$$

The above formulas follows from the classical least-squares inversion procedure developed for the case where the error in the velocity model is not explicitly taken into account.

2.2. Generalised Approach : Confidence Ellipsoids from Data Uncertainty and Velocity Model Uncertainty.

The velocity model parameters are never known exactly. Let us assume that the real (unknown) velocity model $\boldsymbol{V}$ can be expressed as

$$\boldsymbol{V} = \overline{\boldsymbol{V}} + \delta\boldsymbol{V}, \tag{5}$$

where $\overline{\boldsymbol{V}}$ is the **mean** (known) velocity model and $\boldsymbol{\delta V}$ are the unknown errors reflecting the deviation of the velocity model from the average one. Let us assume that these errors are of random Gaussian character with the mean value equal to zero and known covariance matrix $\boldsymbol{C}_V$. Assuming further that the errors of the velocity model parameters are mutually independent, the matrix $\boldsymbol{C}_V$ becomes diagonal with the elements

$$\boldsymbol{C}_V = diag[\sigma^2_{V_1}, \sigma^2_{V_2}, \ldots, \sigma^2_{V_k}], \tag{6}$$

where $\sigma^2_{V_i}$ are the known variances of the velocity model parameters V_i, $i = 1,\ldots, k$, and k is the number of parameters describing the velocity model $\boldsymbol{V}$.

It can be shown that the introduction of the errors of velocity model leads to characteristic disturbance of travel times $\boldsymbol{T}$, $\boldsymbol{\delta T}$, with the mean value

$$E(\delta T) \equiv \Delta T = \frac{1}{2}\mathrm{Sp}\left(\mathbf{C}_V \, \frac{\partial^2 T}{\partial \mathbf{V} \partial \mathbf{V}^T}\right)\Big|_{\mathbf{V}=\overline{V}}, \tag{7}$$

and variance

$$Var(\delta T) \equiv \sigma^2_T = \mathrm{Sp}\left(\mathbf{C}_V \, \frac{\partial T}{\partial \mathbf{V}} \frac{\partial T}{\partial \mathbf{V}^T}\right)\Big|_{\mathbf{V}=\overline{V}}. \tag{8}$$

From above relations it follows that if **both**, the **errors of arrival times determination** and the **uncertainty in velocity model parameters**, are taken into account, the covariance matrix of the focal parameters $\boldsymbol{\theta}$ is

$$\boldsymbol{C}_\theta = (\tilde{\boldsymbol{A}}^T \tilde{\boldsymbol{C}}_t^{-1} \tilde{\boldsymbol{A}})^{-1}, \tag{9}$$

where

$$\tilde{\boldsymbol{A}} = \begin{pmatrix} 1, & \frac{\partial T_1}{\partial x_0}, & \frac{\partial T_1}{\partial y_0}, & \frac{\partial T_1}{\partial z_0}, \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 1, & \frac{\partial T_n}{\partial x_0}, & \frac{\partial T_n}{\partial y_0}, & \frac{\partial T_n}{\partial z_0}, \end{pmatrix}, \tag{10}$$

matrix $\tilde{\boldsymbol{C}}_t = \boldsymbol{C}_t + \boldsymbol{C}_T$, and $\boldsymbol{C}_T$ is the diagonal matrix with diagonal elements equal to the travel time variances $\sigma_{T_i}^2$, caused by the uncertainty in velocity model $\boldsymbol{V}$.

The meaning of these relations is simple. As a result of error in arrival times determination and error in velocity model, the observed and calculated travel times differ by some random values. Knowing the mean and variances of these uncertainties and by using (7)-(10), we are able to estimate the mean value and the variance of the random component of travel times. In addition, according to the formula (9), we are able to calculate the covariance matrix of the focal parameters θ, where both sources of errors are taken into consideration.

4.3. Special Case : Half-Space Velocity Model.

All above relations are of a general nature. Let us consider one of the most important special cases of the above formalism, viz. when velocity model V is characterised by only one parameter: *P*-wave velocity in half-space (the more complicated case when both *P*- and *S*-wave velocity parameters are considered is treated in the Appendix). Clearly, in such a case the *P*-arrival time is described by the formula

$$t_i = t_0 + T_i + \varepsilon_i, \tag{11}$$

where the travel time from the hypocenter (x_0, y_0, z_0) to the ith seismic station with co-ordinates (x_i, y_i, z_i) is

$$T_i = \frac{d_i}{V_i} = \frac{\sqrt{(x_i - x_0)^2 + (y_i - y_0)^2 + (z_i - z_0)^2}}{V_i}, \tag{12}$$

where d_i is the hypocentral distance, V_i denotes an unknown *P*-wave velocity, and $i = 1,\ldots,n$. Assuming that the *P*-wave velocities between the hypocenter and seismic stations are different but oscillates around the same, known value of $\overline{V}$, relations (7) and (8) take the simple form

$$\begin{aligned} \Delta T &= \frac{1}{2}\sigma_V^2 \frac{\partial^2}{\partial V^2}\left(\frac{d}{V}\right)\Big|_{V=\overline{V}} \\ &= \frac{d}{\overline{V}} q^2 \\ &= Tq^2 \end{aligned} \tag{13}$$

and

$$\begin{aligned} \sigma_T^2 &= \sigma_V^2 \left[\frac{\partial}{\partial V}\left(\frac{d}{V}\right)\right]^2 \Big|_{V=\overline{V}} \\ &= \left(\frac{d}{\overline{V}}\right)^2 q^2 \\ &= T^2 q^2. \end{aligned} \tag{14}$$

Since the mean value of the travel time $T + \Delta T$ takes the simple form

$$\begin{aligned} T + \Delta T &= \frac{d}{\overline{V}} + q^2 \frac{d}{\overline{V}} \\ &= \frac{d}{V^*}, \end{aligned} \tag{15}$$

where $V^* = \overline{V}/(1+q^2)$, the matrix of the partial derivatives

$$\tilde{\boldsymbol{A}} = \begin{pmatrix} 1, & \frac{(x_1 - x_0)}{V^*}, & \frac{(y_1 - y_0)}{V^*}, & \frac{(z_1 - z_0)}{V^*}, \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ 1, & \frac{(x_n - x_0)}{V^*}, & \frac{(y_n - y_0)}{V^*}, & \frac{(z_n - z_0)}{V^*}, \end{pmatrix}. \tag{14}$$

and the covariance matrix of the data is a diagonal matrix with diagonal elements equal to

$$\tilde{\boldsymbol{C}}_t \equiv \{\tilde{c}_t\}_{ii} = \sigma_{t_i}^2 + \frac{d_i^2}{\overline{V}^2} q^2 , \tag{15}$$

where i = 1,…, n. Relation (9), together with (13)-(15), form the basis for the determination of the covariance matrix of the focal parameters $\boldsymbol{\theta}$, where both sources of errors are taken into consideration. The diagonal elements of the covariance matrix (9) are the variances of the estimated focal parameters, correspondingly. Thus,

$$\{\boldsymbol{C}_\theta\}_{ii} = \begin{cases} \sigma_{t_0}^2, & \text{for } i=1, \\ \sigma_{x_0}^2, & \text{for } i=2, \\ \sigma_{y_0}^2, & \text{for } i=3, \\ \sigma_{z_0}^2, & \text{for } i=4. \end{cases} \tag{16}$$

where $\sigma_{t_0}, \sigma_{x_0}, \sigma_{y_0}$ and σ_{z_0} denotes respectively expected errors of origin time t_0, and hypocenter co-ordinates x_0, y_0, z_0 .

3. Numerical Example of Expected Location Errors

Figures 1-3 show the expected errors of hypocenter co-ordinates x_0, y_0 and z_0 determination by hypothetical seismic network of 12 seismic sensors similar to that at the one of deep gold mines in South Africa. In all subsequent numerical examples we assume standard deviation in arrival time determination equal to σ_t = 0.05 sec, the half-space velocity model with the mean value of the P-wave velocity $\overline{V}$ = 5.925 km/sec, and its standard deviation σ_V = 0.15 km/sec. The assumption is made that every seismic event is recorded at all stations. Of course this

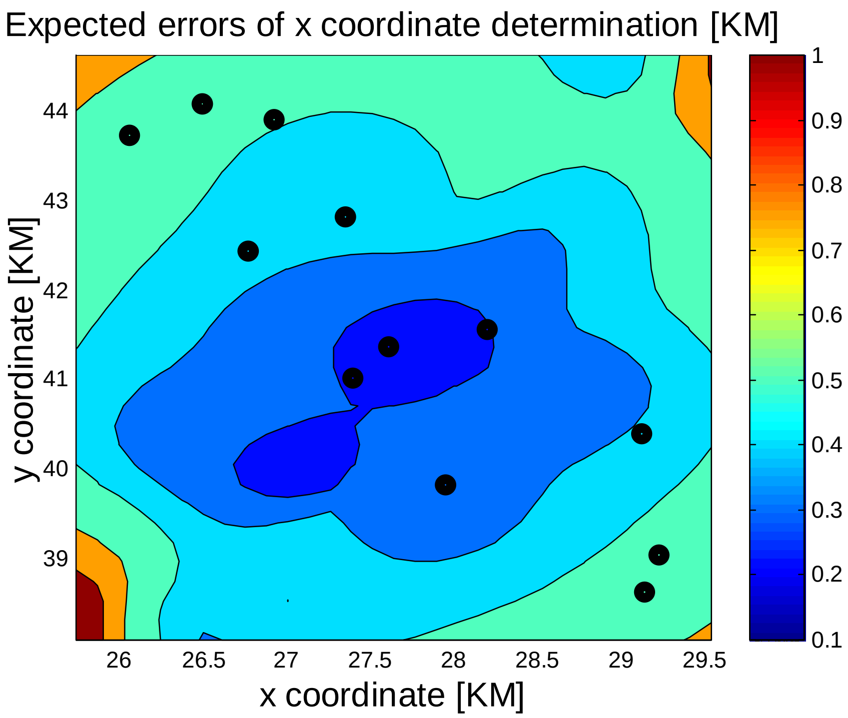


Figure 1. *Expected error of co-ordinate x_0 determination by a network of 12 sensors depending on the seismic event epicenter position. Seismic sensors are marked by dots.*

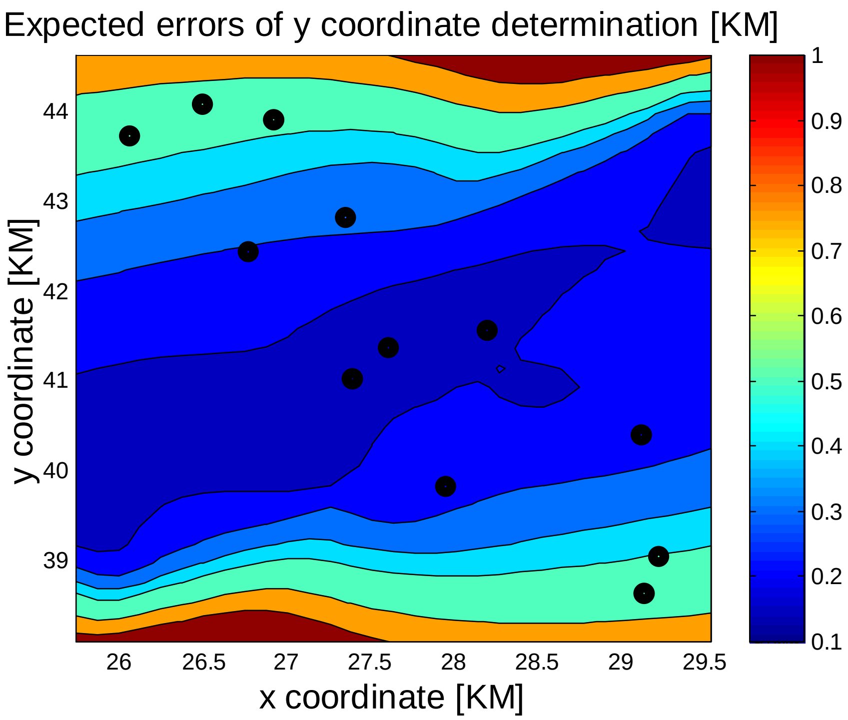


Figure 2. *Expected error of co-ordinate y_0 determination by a network of 12 sensors depending on the seismic event epicenter position. Seismic sensors are marked by dots.*

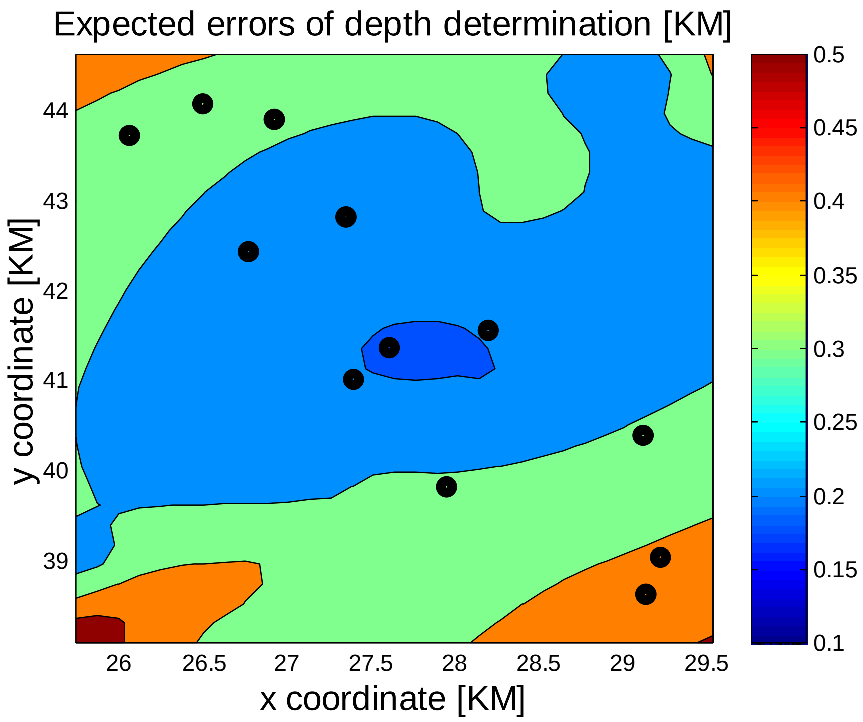


Figure 3*. Expected error of seismic event depth determination by a network of 12 sensors depending on the seismic event epicenter*

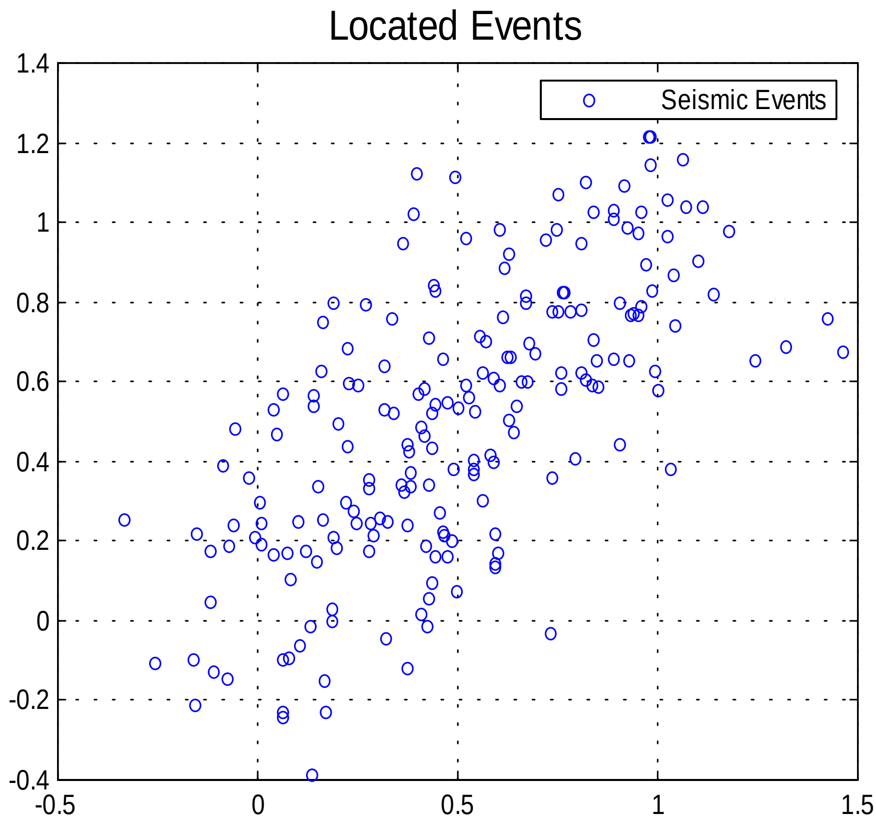
assumptions is not required and even not true, since we know that the farther stations, which are usually not recording the event, do not influence the solution strongly.

4. The Technique for Rock Mass Discontinuities Identification.

Conceptionaly, the technique is simple. The core of the technique is based on the knowledge of the confidence ellipsoid of the hypocenter co-ordinates determination. For each seismic event, one has to find all other events whose hypocenters lie within this hypocenter ellipsoid. The site with the co-ordinates equal to the mean co-ordinates of all of these events is then calculated, and the event is shifted in the direction of this mean-site. The procedure is applied for each event and repeated several times, until the minimum in the misfit between the distribution of shifted hypocenters and χ_3^2 is obtained. The number of degrees of freedom in the χ^2 distribution was reduced from 4 (eq. 1) to 3, because in our procedure we are using only 3 spatial components of the 4-dimensional ellipsoid.

Figures 4 to 6 demonstrates the procedure for different sets of synthetic data.

(a)



(b)

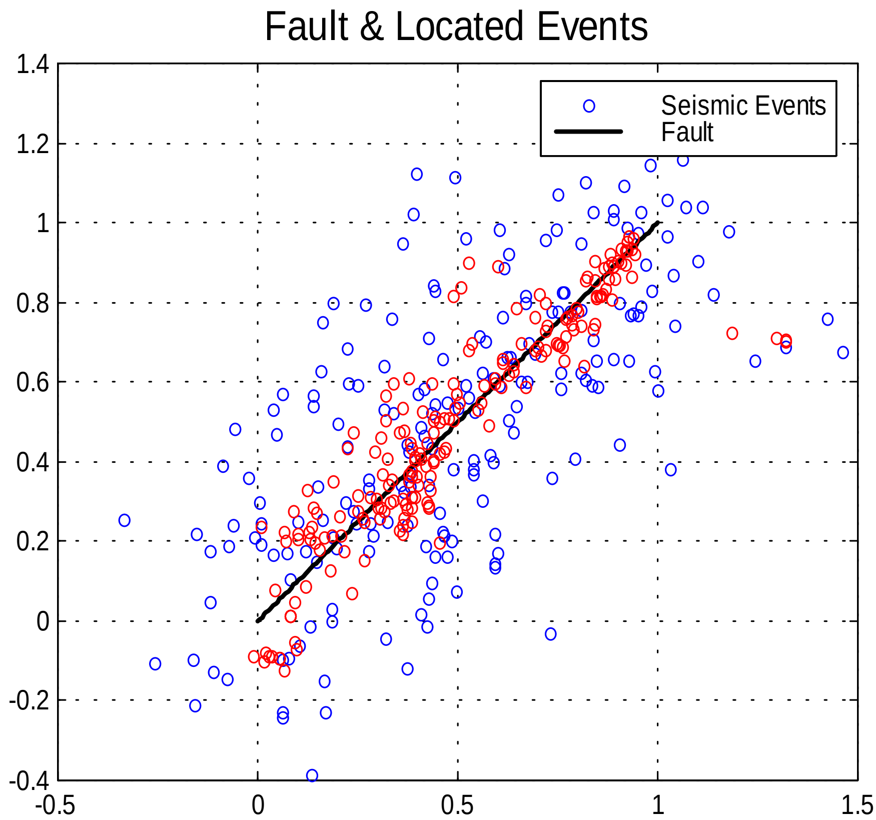
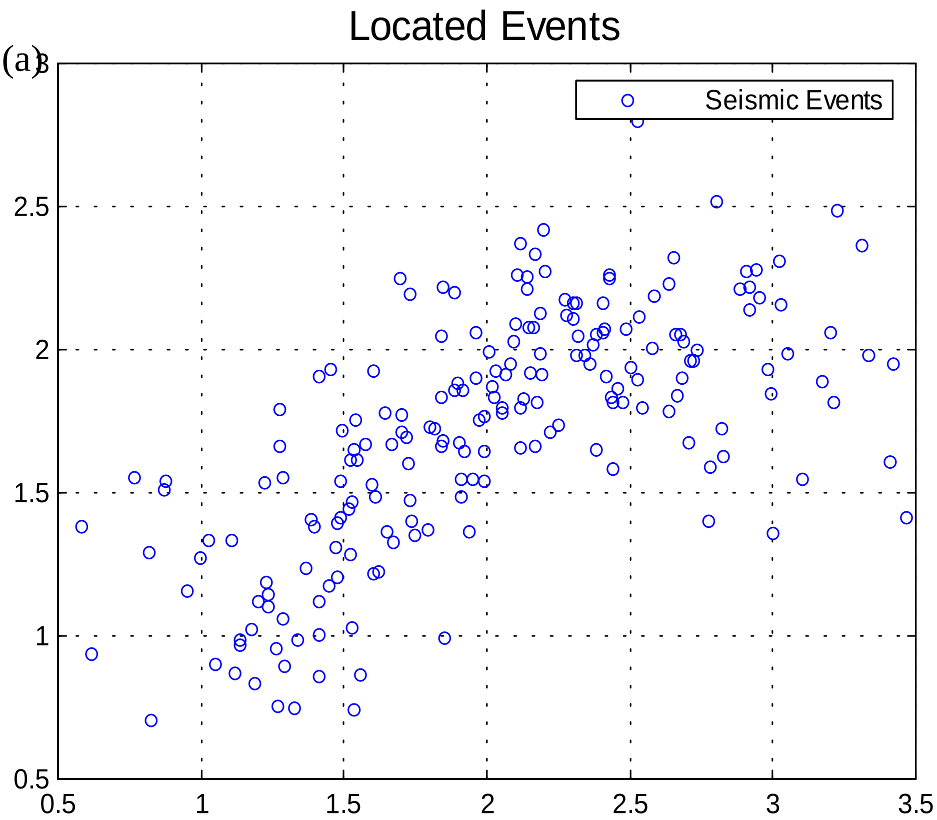


Figure 4*. (a) The location of seismic events synthetically generated by a fault. (b) The location of the fault that generated the synthetic events as well as the relocated positions of these events (red).*

(a)



(b)

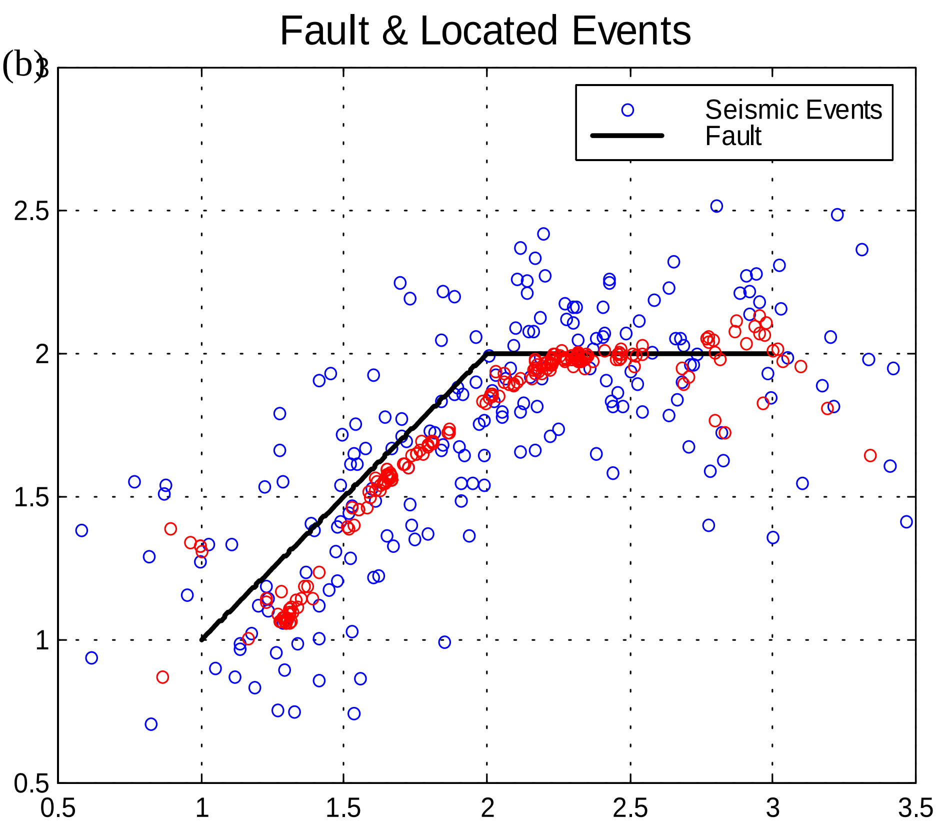
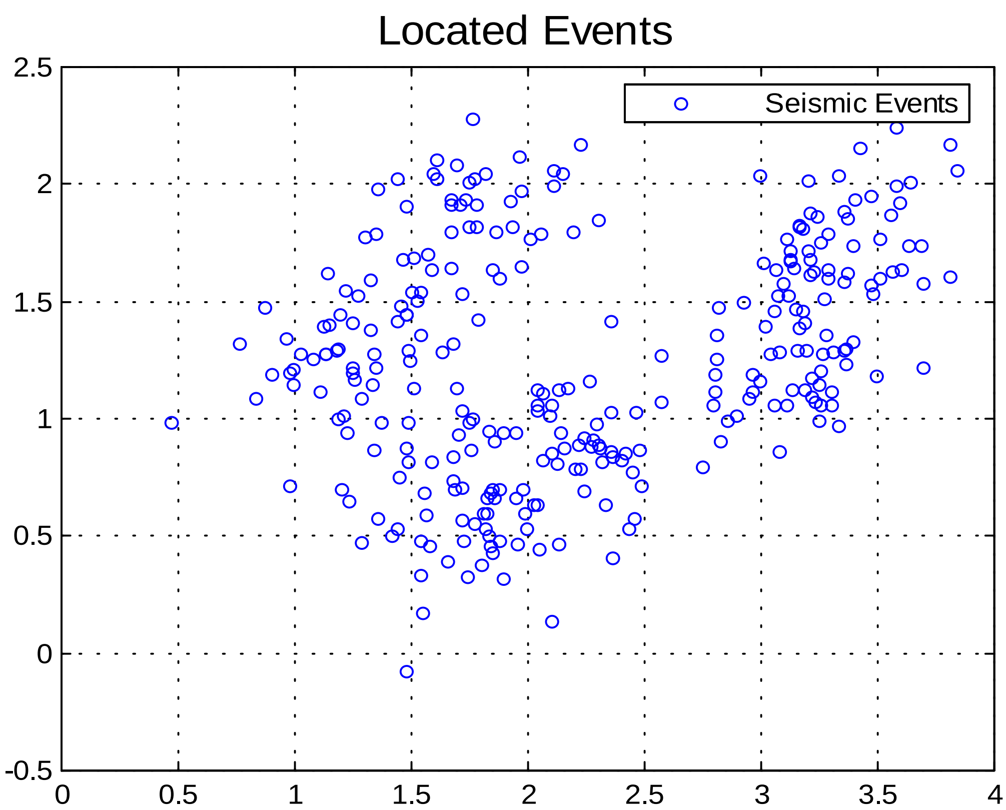


Figure 5. *(a) The location of seismic events synthetically generated by two joined faults. (b) The location of the joined faults that generated the synthetic events as well as the relocated positions of these events (red).*

(a) 

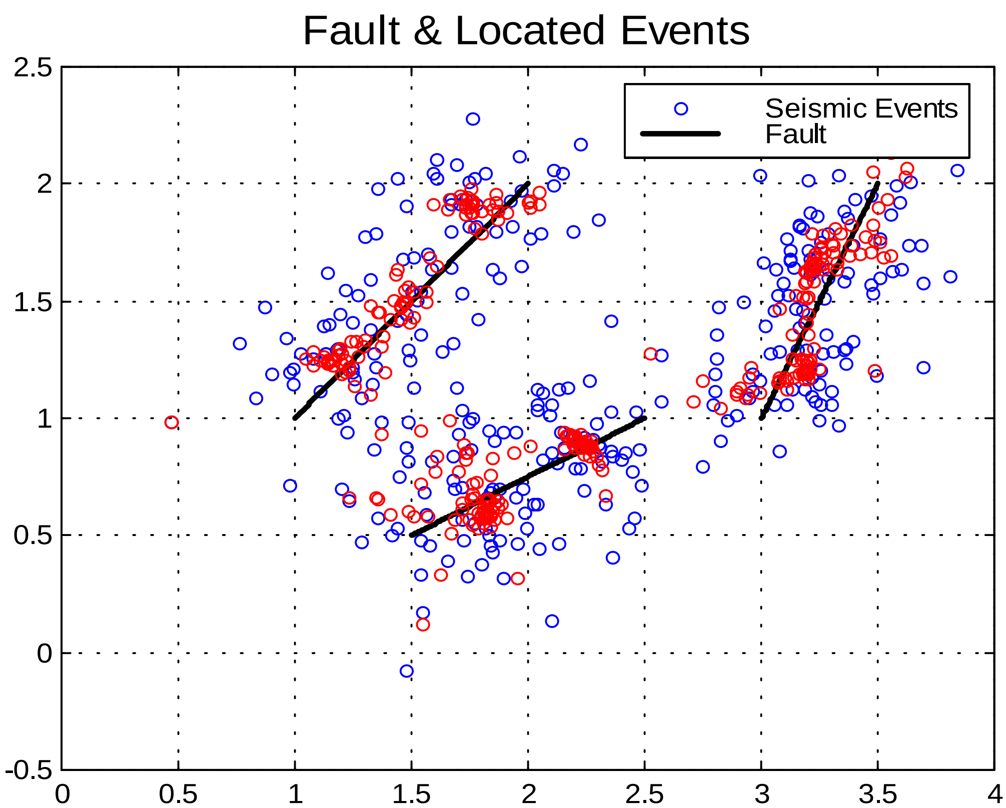
(b) 

Figure 6. *(a) The location of seismic events synthetically generated by three faults. (b) The location of the three faults that generated the seismic events as well as the relocated positions of these events (red).*

Figure 5(a) and (b) represents the case of two joined faults. From Figure 5(b) it can be seen that the event locations converge towards both faults, thus delineating the causative structures.

Figures 6(a) and (b) demonstrates the case of three sources of seismic events, with the procedure capable of delineating all three causative structures.

It should be strongly emphasised that the above procedure is far from unique, and that the problem of identification of rock mass discontinuities in a cluster of seismic event hypocenters can be solved by the application of several alternative techniques. For example, to infer the orientation of active fault planes, the procedure of principal parameters can be used. Essentially, the method involves the sliding of a temporal window of a fixed number of events and the estimation of the eigenvalues and eigenvectors of a spatial matrix for each window-set. Such analysis of the sequence of seismic events makes it possible to isolate and select different trends of seismicity patterns. Usually, the trends are consistent with one of the focal planes of the fault plane solutions and the method offers a simple way to infer the average active fault geometry from the hypocentral location only.

Another procedure for finding statistically significant planes of any orientation in a cluster of seismic events is based on the analysis of the distribution of observed directions with respect to some fixed direction, axis, or plane. Several statistical tests are based on this concept and they require computation of the cumulative distribution function for isotropically distributed directions. Based on this concept it is possible to design a simple graphical procedure which makes it possible to answer the question, whether a set of direction vectors are clustered with respect to particular directions, axes or planes.

Another procedure, conceptionaly close to the last one, is the technique where the dominant directions of the distribution of hypocenters are described by the deflections of straight lines connecting the hypocenters of every two consecutive events, where directions are measured from a certain fixed axe. In case of the lack of any a priori information about the process, the series of deflections can be analysed and quantified by a non-parametric probability distribution function.

6. Conclusion

A technique involving the confidence ellipsoids of seismic event locations was illustrated to be useful in the interpretation of a cluster of seismic events in terms of causative structures.

Determination of the confidence ellipsoids by means of the generalised approach takes errors in arrival time measurements as well as velocity model uncertainties into account. The half-space velocity model forms an important special case of the generalised approach.

Using the calculated dimensions of the confidence ellipsoids, a relocation algorithm was used to alter the locations of the seismic events. The relocation algorithm has the effect of delineating the source of generation when the scatter of events about the source is assumed to be random.

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Appendix

Modelling of the expected errors of epicenter location in the presence of S-waves

The time residuals of location errors consist of two components: **reading errors** of arrival time (say δt_R), and **travel time anomalies** due to the incorrectness of the crustal **model** (say δt_M).

The P-arrival time residual at each station is

$$\delta t_P = \delta t_{R(P)} + \delta t_{M(P)}, \tag{1}$$

and the respective S-arrival time residual is

$$\delta t_S = \delta t_{R(S)} + \delta t_{M(S)}. \tag{2}$$

Let us consider the simplest case of a velocity model, viz. the half-space with P-wave velocity equal to V_P and S-wave velocity equal to V_S. Clearly, the respective travel time anomalies for P and S-waves are equal to

$$\delta t_{M(P)} = \frac{-d}{V_P^2}\delta V_P \quad \text{and} \quad \delta t_{M(S)} = \frac{-d}{V_S^2}\delta V_S\,, \tag{3}$$

where d is the hypocentral distance and δV_P and δV_S denote the respective anomalies of the P and S-waves velocities.

Let us assume that the expected travel time anomaly values of the P and S-waves are equal 0, viz. $E(\delta t_{M(P)}) = 0$, $E(\delta t_{M(S)}) = 0$, and that the parameter ρ denotes the ratio between the V_P and V_S, so that $\delta V_S = \delta V_P/\rho$. With the variances of $\delta t_{M(P)}$ and $\delta t_{M(S)}$ being equal to

$$VAR[\delta t_{M(P)}] = \frac{d^2}{V_P^2} q^2 \tag{4}$$

and

$$VAR[\delta t_{M(S)}] = \frac{d^2}{V_S^2} q^2, \tag{5}$$

where $q = \sigma_{Vp}/V_P = \sigma_{Vs}/V_S$ and σ_{Vp} and σ_{Vs} denote respectively the standard deviation of the *P* and *S*-velocity, it is easy to show that the covariance $Cov[\delta t_{M(P),}\ \delta t_{M(S)}]$ is equal to

$$\begin{aligned} Cov\left[\delta t_{M(P)}, \delta t_{M(S)}\right] &= E\left[\frac{d^2 \delta V_P \delta V_S}{V_P^2 V_S^2}\right] \\ &= \frac{d^2}{\rho V_S^2} q^2 \\ &= \frac{d^2 q^2}{V_P V_S}. \end{aligned} \tag{6}$$

If the *P* and *S*-arrival time **reading errors** at the same station are not correlated, one obtains

$$Cov[\delta t_{R(P)}, \delta t_{R(S)}] = 0, \tag{7}$$

and

$$E[\delta t_{R(P)}{}^2] = E[\delta t_{R(S)}{}^2] = \sigma_t^2\,, \tag{8}$$

where σ_t is the standard deviation of arrival time determination.

The least squares procedure for the location of a seismic event is equivalent to the minimization of functional

$$(\boldsymbol{t}^{\text{obs}} - \boldsymbol{t}^{\text{the}})^{\text{T}}\ \boldsymbol{C}_t^{-1} (\boldsymbol{t}^{\text{obs}} - \boldsymbol{t}^{\text{the}}), \tag{9}$$

where $\boldsymbol{t}^{\text{obs}} = \begin{pmatrix} \boldsymbol{t}_P^{obs} \\ \boldsymbol{t}_S^{obs} \end{pmatrix}$ denotes a ($2n$ x 1) vector of observed arrival times at a number of n stations . The first n elements of vector $\boldsymbol{t}^{obs}$ are the *P*-arrival times and are denoted as $\boldsymbol{t}_P^{obs}$.

The last n elements of vector $\boldsymbol{t}^{\text{obs}}$ are the S-arrival times and are denoted as $\boldsymbol{t}_S^{obs}$. Vector $\boldsymbol{t}^{\text{the}}$ denotes respective theoretical P and S-arrival times. By definition, the covariance matrix, $\boldsymbol{C}_t$, of the data, $\boldsymbol{t}$, is equal to

$$\boldsymbol{C}_t \equiv Cov(\boldsymbol{t}) = \begin{pmatrix} Var[\boldsymbol{t}_P] & Cov[\boldsymbol{t}_P, \boldsymbol{t}_S] \\ Cov[\boldsymbol{t}_S, \boldsymbol{t}_P] & Var[\boldsymbol{t}_S] \end{pmatrix}, \tag{10}$$

where, following (1)-(8), all four sub-matrixes $Var[\boldsymbol{t}_P]$, $Var[\boldsymbol{t}_S]$, $Cov[\boldsymbol{t}_P,\boldsymbol{t}_S]$, and $Cov[\boldsymbol{t}_S, \boldsymbol{t}_P]$ are $(n\text{x}n)$ dimensional and diagonal with diagonal elements equal to

$$\{Var[\boldsymbol{t}_P]\}_{ii} = \sigma_t^2 + \frac{d_i^2}{V_P^2} q^2, \tag{11}$$

$$\{Var[\boldsymbol{t}_S]\}_{ii} = \sigma_t^2 + \frac{d_i^2}{V_S^2} q^2, \tag{12}$$

$$\{Cov[\boldsymbol{t}_P, \boldsymbol{t}_S]\}_{ii} = \{Cov[\boldsymbol{t}_S, \boldsymbol{t}_P]\}_{ii} = \frac{d_i^2}{V_P V_S} q^2, \tag{13}$$

where d_i is the distance from hypocenter to station i, and $i = 1,\ldots,n$.

The main point in developing above formalism is to show that if the P and S-arrival times are used in the location procedure of local seismic events, the covariance matrix of data, $\boldsymbol{C}_t$, contains non-diagonal elements, which are responsible for a correlation between P and S-arrival times recorded at the same station. These elements depend on the expected errors of arrival times, distances from the hypocenter and the uncertainty in P and S-velocities. The analytical form of such elements is given by equation (13). It can thus be seen that when both the P- and S-waves are used in the procedure of modelling the expected errors of epicenter location, one cannot assume that the travel time residuals of these two waves are independent variables (as can be done with the errors in arrival times). The travel-time anomalies of P and S-waves recorded at the same station are strongly correlated. This fact is a non-trivial matter and must be taken into account during the procedure of modelling of the expected location errors.