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DEFECTS, DISLOCATIONS, AND PHYSICS OF STRENGTH

Grain Size, Stress, and Creep in Polycrystalline Solids

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Abstract—If a stress σ is applied to a polycrystal of grain size L, the mode of creep deformation depends on the answers to the following questions: (I) Does σ exceed the Peierls stress σ_p; (II) Does L exceed the dislocation spacing in a Taylor lattice stabilized by σ_p; (III) Does $L\sigma$ exceed the value required for a Frank–Read or Bardeen–Herring source to operate within the grain? (IV) Does $L^{1/2}\sigma$ exceed the Hall–Petch value required for slip to propagate across a grain boundary? The (L, σ) plane is thus partitioned into regions in which different creep modes predominate. © *2000 MAIK "Nauka/Interperiodica".*

1. DIFFUSIONAL CREEP AND HARPER–DORN CREEP

In diffusional creep, transport of matter occurs by the migration of vacancies from grain boundaries roughly normal to a tensile stress to boundaries roughly parallel to this stress. The migration occurs either through the body of the grain [1, 2] or along the grain boundaries [3]. In Harper–Dorn creep, vacancies migrate from edge dislocations with their Burgers vectors roughly parallel to the tensile axis to edge dislocations with their Burgers vectors roughly perpendicular to the tensile axis. The spacing l between adjacent dislocations, which are modeled as forming a Taylor lattice, reaches an equilibrium value such that the stress each dislocation exerts on its neighbor is of the order of the Peierls stress σ_p [4, 5]. Thus,

$$b\mu/2\pi l \approx \sigma_p, \quad l \approx b\mu/2\pi\sigma_p, \tag{1}$$

and Harper–Dorn creep is possible only if $l < L$, i.e.,

$$L > b\mu/2\pi\sigma_p. \tag{2}$$

When this condition is satisfied, the diffusion paths for Harper–Dorn creep are shorter than those for diffusional creep, and Harper–Dorn creep will be faster than Nabarro-Herring creep provided that [6]

$$L/b > 7\mu/\sigma_p. \tag{3}$$

Different modes of creep will operate depending on whether the product $L\sigma$ is or is not large enough for Bardeen–Herring climb sources to operate within or on the surface of the grain. If the line tension of a dislocation is Γ, sources can operate freely if

$$b\sigma > 4\Gamma/L. \tag{4}$$

With $\Gamma \approx b^2\mu/2$, where μ is the shear modulus, this becomes

$$L\sigma > 2b\mu. \tag{5}$$

The authors of [7, 8] interpret this kind of formula in the following way. As diffusional creep occurs, edge dislocations climb along the grain boundaries. Inequality (5) represents the condition that, if these dislocations are removed, they can be replaced by new dislocations generated by Bardeen–Herring sources in the grain boundaries. This interpretation seems to be incorrect on two grounds. First, a typical large-angle boundary contains edge dislocations all of the same sign separated by distances of the order of $b/3$. If these dislocations all climbed out of the boundary and were not replaced, the total deformation would be of the order of 30%, larger than that normally observed in diffusional creep. In fact, the dislocations will not disappear, but will, statistically, continue to climb in adjacent grain boundaries. Second, it is not clear why Bardeen–Herring sources should operate preferentially in grain boundaries. When inequality (5) is satisfied, sources can operate within the grains, probably even more freely than in the grain boundaries.

Inequality (5) should rather be interpreted in the following way. The equilibrium spacing l of Eq. (1) is achieved by a balance of the multiplication of dislocations by the operation of Bardeen–Herring sources within the grain and the annihilation of dislocation pairs under their mutual attraction. This process occurs and Harper–Dorn creep is possible if inequality (4) is satisfied. If the inequality is not satisfied, dislocations climb into the grain boundaries and are absorbed, and, after a possible transient, diffusional creep, rather than Harper–Dorn creep, occurs.

2. STRESSES ABOVE THE PEIERLS STRESS

When the applied stress σ exceeds the Peierls stress σ_p,

$$\sigma > \sigma_p, \tag{6}$$

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dislocations can move freely by glide.

If, in addition, inequality (5) is satisfied, dislocations will multiply by glide within the cell much more rapidly than they can annihilate by climb. Harper–Dorn creep gives way to power-law creep [9]. If the product $L^{1/2}\sigma$ is less than the Hall–Petch stress-intensity factor k_{HP}

$$L^{1/2}\sigma < k_{HP}, \quad (7)$$

glide cannot percolate from one grain to its neighbor.

A possible mode of deformation is then that considered by Spingarn and Nix [10], which can be outlined as follows. The reduced stress is large enough to support glide on only one system in each grain. Coherence between the grains is maintained largely by sliding on the grain boundaries. This sliding is impeded by ledges formed on the boundaries by pile-ups of dislocations. The rate-controlling process is the smoothing of these ledges by the diffusion of vacancies between adjacent ledges. The distance λ between adjacent slip planes in a grain is likely to be of the order of the dislocation passing distance, given by

$$b\sigma = b^2\mu/2\pi\lambda$$

or

$$\lambda = b\mu/2\pi\sigma. \quad (8)$$

Allowing for the piling-up of dislocations, the work done by the external stress when a vacancy is transferred from the head of a pile-up is easily seen to be of the order of

$$W = Lb^2\sigma^2/2\mu. \quad (9)$$

On average, a vacancy travels a distance $\lambda/4$ to relieve the local strain, and so the thermodynamic driving force on a vacancy is

$$4W/\lambda = 4\pi Lb\sigma^3/\mu^2. \quad (10)$$

If the effective diffusion constant is D_e, the flux ϕ of vacancy is D_e/kT times the thermodynamic force, or

$$\phi = 4\pi Lb\sigma^3 D_e/\mu^2 kT. \quad (11)$$

At high temperatures, diffusion will occur through the bulk, D_e will be the bulk coefficient of diffusion D, and the flow of vacancies at each step will occur through an area of order $L\lambda/2 = Lb\mu/4\pi\sigma$. The volume V of matter transported at each step in unit time is then

$$V = Lb\mu\phi/4\pi\sigma = L^2b^2\sigma^2 D/\mu kT. \quad (12)$$

The time t taken to remove a step is

$$t = b\lambda L/8V = \mu^2 kT/16\pi L\sigma^3 D. \quad (13)$$

The shear strain is b/λ, and so the strain rate $\dot{\varepsilon}$ is given by

$$\dot{\varepsilon} = b/\lambda t = 32\pi^2 L\sigma^4 D/\mu^3 kT. \quad (14)$$

At low temperatures, D is replaced by the grain-boundary diffusion coefficient D_b, and the flux of vacancies occurs over an area of order Lb. The strain rate is then

$$\dot{\varepsilon} = 128\pi^3 L\sigma^5 D_b/\mu^4 kT. \quad (15)$$

Both processes occur within the normal range of power-law creep.

At higher stresses, several glide systems operate in each grain, and dislocation cells are formed having widths w given approximately by

$$w = 10.5b\mu/\sigma. \quad (16)$$

Under these conditions, power-law creep with an exponent 4–5 is observed. As the discussion in [11] shows, simple mechanisms of creep in this structure lead to the "natural" exponent of 3. An exponent of 5 can be obtained by assuming that diffusion occurs along the cores of dislocations, which are present with a density proportional to σ^2, but such a process would have an activation energy of only about half the observed value, which is close to that for the lattice self-diffusion. Other models of power-law creep [12, 13], which take into account the formation of dislocation cells within the grains, involve rather arbitrary assumptions.

3. POWER-LAW BREAKDOWN

When inequality (7) is not satisfied, a slip in one grain can transfer to a neighboring grain. While there is still some thermal activation of the slip process, as is shown by the slow decrease in flow stress with increasing temperature, the rate of deformation is no longer controlled by diffusion, but is a rapidly increasing function of stress. This is the domain of power-law breakdown.

4. NUMERICAL VALUES

The quantities entering this analysis are b and μ, which are well determined, and σ_p and k. Both theoretical and experimental values of the Peierls stress σ_p fall into two classes, with one class being several hundred times greater than the other. There are reasons to believe that, in problems of progressive plastic deformation, it is the values of the lower class that are relevant [14], and we use these. There are no satisfactory theoretical estimates of the Hall–Petch coefficient k, and we use values from the review by Hansen [15].

For aluminum, the relevant parameters are $b = 2.86 \times 10^{-10}$ m, $\mu = 26 \times 10^9$ Pa, $\sigma_p = 2.5 \times 10^{-5}\ \mu$, $k_{HP} = 0.53 \times 10^5$ Nm$^{-3/2}$, and the relevant diagram is shown in Fig. 1.

The present analysis does not apply to very high stresses, where the lattice may break down, or at high stresses and very small grain size, where the Hall–Petch criterion may not apply because a pile-up of sev-

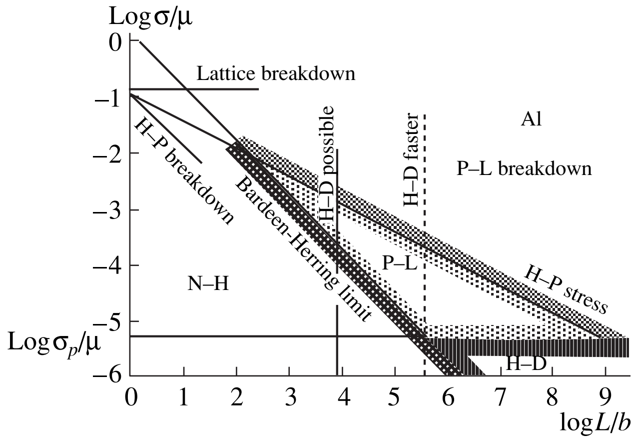

Fig. 1. The plane of grain size L and applied stress σ is divided into domains of different creep models in aluminum by four boundaries: the Peierls stress σ_p, the grain size at which Harper–Dorn creep becomes possible (or becomes faster than diffusional creep), the product $L\sigma$ above which Bardeen–Herring (or Frank–Read) sources can operate within the grain, and the Hall–Petch product $L^{1/2}\sigma$ above which glide can percolate between grains.

eral dislocations within the cell is not possible. The regions where the analysis does not apply are indicated in the figure.

When $L\sigma$ is below the Bardeen–Herring limit, dislocations cannot multiply within the grain even if $\sigma > \sigma_p$. Dislocations are swept into the grain boundaries, and only diffusional creep is possible in the steady state. Above the Bardeen–Herring limit, Harper–Dorn creep occurs when $\sigma < \sigma_p$, and the grain size is not too small, or, more precisely, for very large grain sizes where $L^{1/2}\sigma$ is large enough to allow dislocations to cross the grain boundary. For σ somewhat below σ_p, the Bardeen–Herring limit occurs at about $\log(L/b) = 6$, corresponding to $L = 290$ μm, in reasonable agreement with the value of 400 μm estimated by Mohamed [16] from the experimental data. Power-law creep occurs in the region bounded by the Bardeen–Herring limit, the Peierls stress, and the Hall–Petch stress line. The region above both the Bardeen–Herring and the Hall–Petch lines is that of a power-law breakdown.

For copper, $b = 2.56 \times 10^{-10}$ m, $\mu = 48 \times 10^9$ Pa, $\sigma_p = 10^{-5}\,\mu$, $k_{HP} = 1.6 \times 10^5$ Nm$^{-3/2}$, and the resulting diagram in the (L, σ) plane is shown in Fig. 2.

The topology of the diagram is different from that of Fig. 1. There is a region that lies below the Peierls stress, above the Bardeen–Herring limit, and at grain sizes so small that Harper–Dorn creep is either impossible or slower than diffusional creep. In this new region, diffusional creep will dominate.

5. INFLUENCE OF DISLOCATION CELLS

The discussion so far has assumed that the only obstacles to dislocation motion are the Peierls stress and the grain boundaries. However, dislocations can also assemble into cells of width $w(\sigma)$, where [11]

$$w(\sigma) \approx 10.5 b\mu/\sigma \qquad (17)$$

and usually do so provided that $w < L$. On the rather drastic assumption that the cell walls are as effective barriers to dislocation motion as grain boundaries, L must be replaced by $w(\sigma)$ in the preceding discussion. In Fig. 3, the diagram for aluminum is augmented by the line $w(\sigma)$. At large grain sizes, where the Hall–Petch line lies above the line $w(\sigma)$, the effective grain size is $w(\sigma)$, and power-law breakdown occurs at a constant stress given by the intersection of the $w(\sigma)$ and H–P stress lines. Then, as is observed, the regimes of

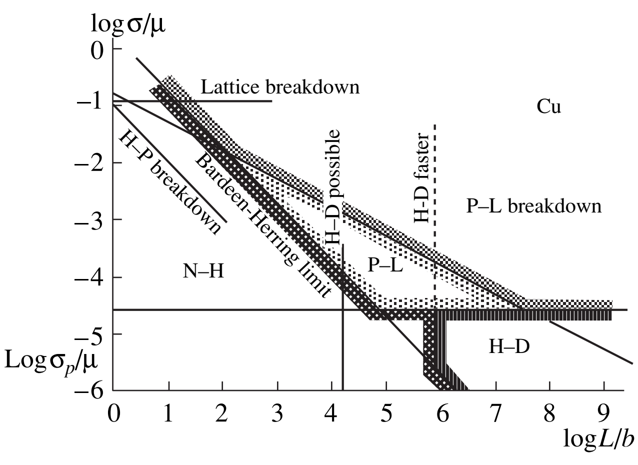

Fig. 2. A creep-mode diagram for copper similar to that of Fig. 1 for aluminum. There is a new domain, in which $\sigma < \sigma_p$. $L\sigma$ exceeds the Bardeen–Herring limit, but $L^{1/2}\sigma$ is below the value at which Harper–Dorn creep is faster than diffusional creep.

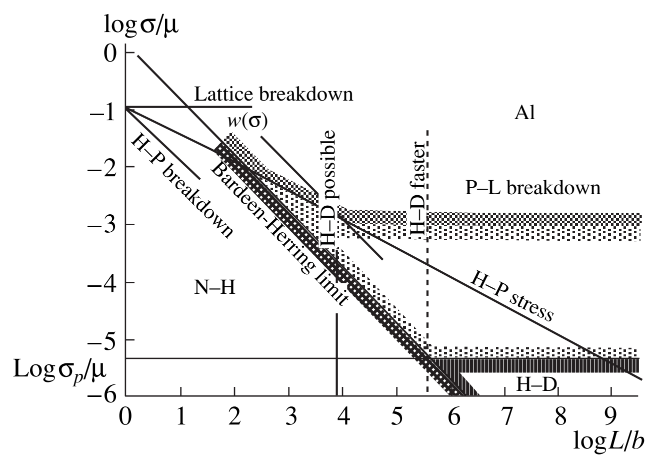

Fig. 3. Diagram for aluminum augmented by the line $w(\sigma)$ determining the dislocation cell size. Where $w < L$, power-law breakdown occurs at a stress independent of grain size.

Harper–Dorn creep and power-law breakdown are separated by a regime of power-law creep. It appears that this regime covers a factor of several hundreds in stress, in agreement with the observations reported by Wu and Sherby [17].

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