Modification of Piezoelectric Vibratory Gyroscope Resonator Parameters by Feedback Control

Philip W. Loveday and Craig A. Rogers

***Abstract*—A method for analyzing the effect of feedback control on the dynamics of piezoelectric resonators used in vibratory gyroscopes has been developed. This method can be used to determine the feasibility of replacing the traditional mechanical balancing operations, used to adjust the resonant frequency, by displacement feedback and for determining the velocity feedback required to produce a particular bandwidth. Experiments were performed on a cylindrical resonator with discrete piezoelectric actuation and sensing elements to demonstrate the principles. Good agreement between analysis and experiment was obtained, and it was shown that this type of resonator could be balanced by displacement feedback. The analysis method presented also is applicable to micromachined piezoelectric gyroscopes.**

I. Introduction

Many different vibratory gyroscope designs have been documented in the literature [1]–[17] during the last 35 years. In these sensors the effect of Coriolis forces that act on a vibrating structure, when it is subjected to a rotation rate, is used to measure the applied rotation rate. Various geometry's including, strings, beams, tuning forks, rings, discs, cylinders, and hemispheres have been used for the vibrating structure, or resonator. The resonators are forced to vibrate by actuators, and the displacement of the resonator is measured by sensors. Typically these actuators and sensors are either electrostatic, piezoelectric, or electromagnetic.

The principle of operation of these devices is based on the Coriolis coupling that occurs, between two modes of vibration of the resonator, when a rotation is applied to the structure. Many of the resonators are designed to have two vibration modes that, in the perfect resonator, occur at the same frequency. During operation, one of these vibration modes is driven at resonance at a constant amplitude. When a rotation rate is applied, the Coriolis effect couples energy from this vibration mode into the second vibration mode. The response of the second vibration mode then provides a measure of the applied rotation rate. Because the two vibration modes have the same natural frequency, the energy transfer between them is very efficient and produces a high sensitivity to applied rotations. In practice, however, small imperfections always occur during manufacture, and the two vibration modes will not have identical natural frequencies. These imperfections degrade the performance of the gyroscope [18]–[21], and the effect of these imperfections is usually minimized by a mechanical balancing procedure. This balancing procedure generally involves the removal of small amounts of mass from certain locations on the resonator in order to modify the dynamics of the resonator so that the difference between the two natural frequencies is reduced. This process is time consuming, expensive, and very difficult to perform on small micromachined designs. As this process is performed once at a single temperature, changes in the dynamics of the resonator over time or with temperature will not be accounted for. Also some of these resonators operate in a partial vacuum but are balanced at atmospheric pressures. The evacuation process also can effect the dynamics of the resonator causing an increase in the difference between the natural frequencies.

A novel method of balancing was developed for gyroscope resonators that use electrostatic actuation and sensing [5], [22]. In this method an "electrical spring" is produced by applying a DC voltage across one of the electrode gaps. The electrostatic force is proportional to the square of the gap distance; therefore, a decrease in the gap size results in an increase in the electrostatic force and vice versa. Because the variations in gap size during operation are very small, the effect of the electrostatic field may be represented (to first order) as a negative linear spring. By adjusting the value of the DC voltage across the gap the spring constant may be varied. These electrical springs are adjusted to minimize the difference in natural frequency caused by manufacturing imperfections in the resonator structure.

It has been demonstrated in the literature that the natural frequencies of cantilever beam-type structures containing piezoelectric actuators may be adjusted by feedback techniques [23], [24]. The purpose of this paper is to demonstrate that, in a gyroscope resonator with piezoelectric sensing and actuation, an "electrical spring" can be formed by using position feedback, and an "electrical damper" can be formed by applying velocity feedback. Therefore, the electrical springs can be used to decrease the effect of manufacturing imperfections on the performance of a piezoelectric vibratory gyroscope. Large feed-

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back gains would result in large signal amplitudes that are impractical. Therefore, it is desirable to be able to predict the gains required. A method of calculating the magnitude of the feedback gains required is presented. This method is applied to a cylindrical resonator with discrete piezoelectric actuator and sensor elements, and the theoretical predictions are verified by measurement.

II. Theoretical Development

The equations of motion for a resonator, excited by piezoelectric actuators and sensed by piezoelectric sensors, may be derived by application of Hamilton's principle to the coupled electromechanical system and discretization by the Rayleigh-Ritz method [25]. The resulting set of equations has the form:

$$\begin{aligned} M\ddot{r} + Kr - \Theta\nu &= B_f f \\ \Theta^T r + C_p \nu &= B_q q \end{aligned} \tag{1}$$

where M and K are the mechanical mass and stiffness matrices, r is the vector of mechanical generalized displacements, f is the applied force vector, Θ is the piezoelectric coupling matrix, C_p is the capacitance matrix, ν is the vector of electrical potential generalized coordinates, q is the vector of electric charges applied to the electrodes, B_f and B_q are generalized coordinate conversion matrices for forces and charges, respectively.

These two equations are referred to as the actuator equation and the sensor equation. The sensor equation may be partitioned to separate the voltages at the drive electrodes from the voltages at the sensing electrodes as follows:

$$\begin{bmatrix}\Theta_d \ \Theta_s\end{bmatrix}^T \{r\} + \begin{bmatrix} C_{pdd} & C_{pds} \\ C_{psd} & C_{pss}\end{bmatrix} \begin{Bmatrix}\nu_d \\ \nu_s\end{Bmatrix} = B_q \begin{Bmatrix} q_d \\ 0 \end{Bmatrix}. \tag{2}$$

This equation can be rearranged to give the sensed voltage as a function of the displacement and the applied voltage.

$$\nu_s = -C_{pss}^{-1}[\Theta_s^T r + C_{psd}\nu_d] \tag{3}$$

If we feed a combination of the sensed voltages back to the drive ceramics, this can be regarded as displacement feedback and can be represented as follows:

$$\begin{aligned} \nu_d &= G_d \nu_s \\ &= -G_d C_{pss}^{-1}\Theta_s^T r - G_d C_{pss}^{-1} C_{psd}\nu_d \\ &= -[I + G_d C_{pss}^{-1} C_{psd}]^{-1} G_d C_{pss}^{-1} \Theta_s^T r \end{aligned} \tag{4}$$

where I is the identity matrix and G_d is a matrix of displacement feedback gains. The sensed voltage can then be written as:

$$\nu_s = -C_{pss}^{-1}\left[\Theta_s^T - C_{psd}[I + G_d C_{pss}^{-1} C_{psd}]^{-1} G_d C_{pss}^{-1}\Theta_s^T\right] r \tag{5}$$

Substituting the drive and sense voltages from (4) and (5) into the actuator equation yields the undamped equations of motion for the system including displacement feedback:

$$M\ddot{r} + K^* r = 0 \tag{6}$$

where

$$\begin{aligned} K^* = K &+ \Theta_d [I + G_d C_{pss}^{-1} C_{psd}]^{-1} G_d C_{pss}^{-1}\Theta_s^T \\ &+ \Theta_s C_{pss}^{-1}\left[\Theta_s^T - C_{psd}[I + G_d C_{pss}^{-1} C_{psd}]^{-1} G_d C_{pss}^{-1}\Theta_s^T\right] \end{aligned} \tag{7}$$

Equation (7) shows that the effective stiffness of the system can be altered by feeding back signals that are proportional to the displacement of the structure. Therefore, an "electrical spring" has been constructed by using feedback control.

If there is no capacitive coupling between the drive and sense ceramic electrodes ($C_{psd} = 0$), the equations of motion simplify to:

$$M\ddot{r} + [K + \Theta_d G_d C_{pss}^{-1}\Theta_s^T + \Theta_s C_{pss}^{-1}\Theta_s^T]r = 0 \tag{8}$$

Equation (8) shows that leaving the sensing ceramics with open circuit (or high impedance) boundary conditions and feeding back signals from the sensing ceramics to the drive ceramics both contribute to the effective stiffness of the structure. Calculating the eigenvalues of this system for different feedback gains provides a method of determining the influence of the displacement feedback gains on the natural frequencies of the resonator.

If velocity feedback is included, the voltage applied to the drive ceramics may be expressed as:

$$\nu_d = G_d \nu_s + G_\nu \dot{\nu}_s \tag{9}$$

where G_ν is a matrix of velocity feedback gains.

Including an arbitrary viscous damping matrix (C) and again omitting capacitive coupling between the sensing and actuation ceramics, yields the following equations of motion:

$$\begin{aligned} M\ddot{r} &+ [C + \Theta_d G_\nu C_{pss}^{-1}\Theta_s^T]\dot{r} \\ &+ [K + \Theta_d G_d C_{pss}^{-1}\Theta_s^T + \Theta_s C_{pss}^{-1}\Theta_s^T]r = 0 \end{aligned} \tag{10}$$

From (10) it is seen that the effect of velocity feedback is to modify the damping characteristics of the system. Therefore, it is possible to construct an "electrical damper" by feeding back a signal proportional to the velocity of the structure. The natural frequencies and damping factors of this system may be calculated by transforming the equations of motion to state space and then calculating the eigenvalues.

III. Experimental Procedure

The feasibility of using feedback techniques to modify the dynamics of a vibratory gyroscope resonator was

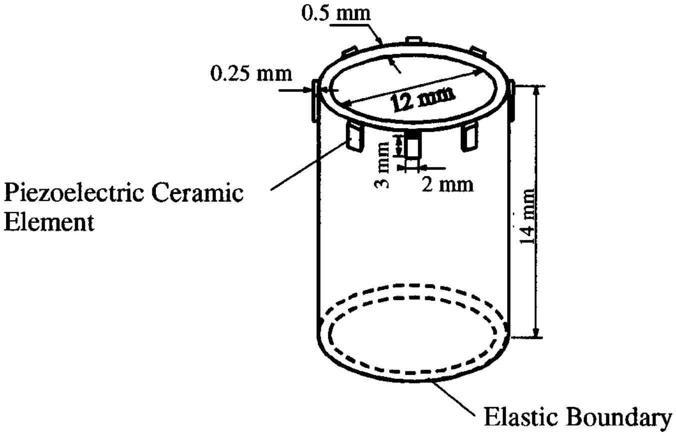

Fig. 1. Resonator geometry.

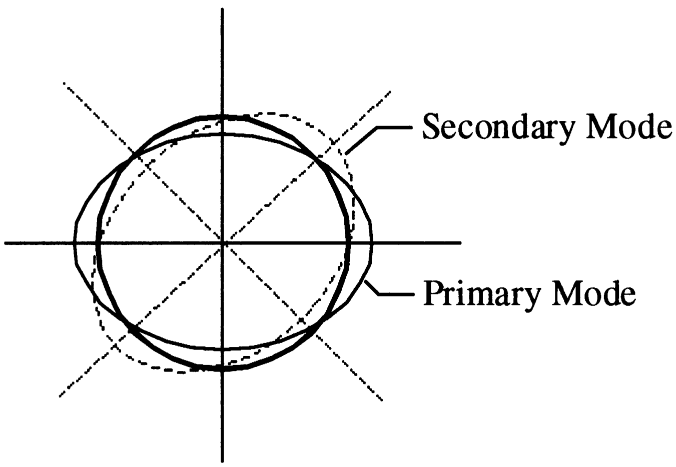

Fig. 2. Radial displacements of operational modes.

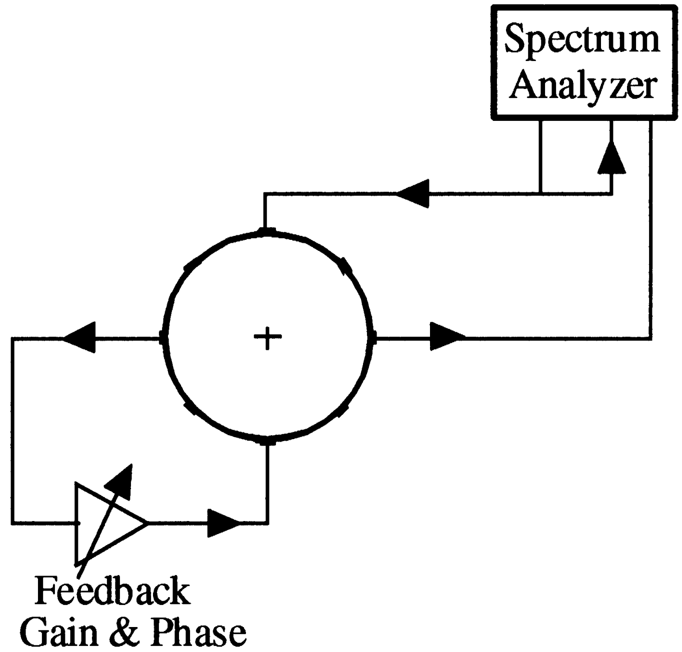

Fig. 3. Experimental set-up.

tested for a vibrating cylinder resonator. This particular resonator comprises a thin walled, steel cylinder, closed at one end, with eight discrete piezoelectric ceramics (PZT5A) bonded near the open end. The resonator is shown schematically in Fig. 1.

The radial displacement patterns of the two operational modes of vibration are shown in Fig. 2 as viewed from the open end of the cylinder. During operation the primary mode is excited to oscillate at the resonance frequency (approximately 14,500 Hz) at a constant amplitude. When a rotation is applied about the axis of the cylinder, energy is coupled from the primary mode into the secondary mode, and the vibrating pattern appears to shift relative to the cylinder. The vibration of the secondary mode is suppressed by actively damping the structure in order to increase the bandwidth of the gyroscope. In a perfect resonator, the primary and secondary mode would have identical natural frequencies. Imperfections that occur during manufacture, however, cause a difference in natural frequency and locate the mode shapes relative to the structure [18].

The experimental set-up selected to demonstrate the use of feedback to modify the dynamics of the resonator is shown in Fig. 3. A HP 3562A dynamic signal analyzer was used to measure the frequency response of the resonator by applying random noise excitation and measuring the response over a 100 Hz frequency range. The resonant frequency and the Q factor of the primary mode of vibration were then extracted from the measured frequency response function. The feedback gain and phase were varied so that displacement and velocity feedback could be investigated.

IV. Results and Discussion

The procedure for obtaining the coupled equations of motion for the vibrating cylinder resonator was applied to the resonator as described in [20]. In that work the resonator cylinder was assumed to have a clamped boundary condition at the closed end of the cylinder. This assumption resulted in the model over-predicting the natural frequencies. The model has since been extended to include flexibility in this boundary condition, thus making it possible to adjust the boundary condition until the correct natural frequencies are obtained. The piezoelectric ceramics were soldered to the cylinder, at a temperature of approximately 335°C, then polarized. The value of the piezoelectric coefficient (e_{31}) for the ceramics in this condition is not certain, so the response of the resonator at a frequency of 1 kHz was used to calibrate this parameter. Decreasing this constant, by 15% from the catalogue value of -5.4 Coulomb/m^2, gave good agreement at 1 kHz.

The experiment described in Section III was simulated using the theory presented in Section II. The predicted and measured effect of displacement feedback on the resonant frequency of the primary mode is shown in Fig. 4. The results indicate that, by varying the feedback gain from -2 to 2, a change in the resonant frequency of approximately 10 Hz can be produced. This change in frequency is larger

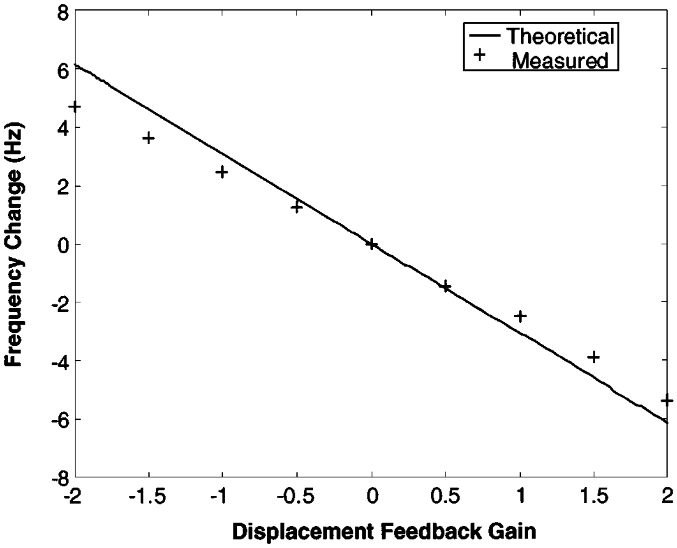

Fig. 4. Resonant frequency change caused by displacement feedback.

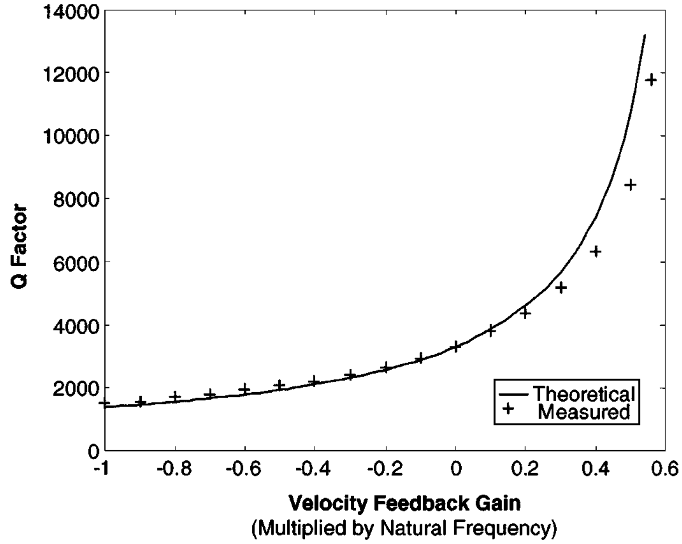

Fig. 5. Q Factor change caused by velocity feedback.

than the difference in frequency caused by manufacturing imperfections.

The slope of the curve is dependent on the square of the piezoelectric coupling coefficient of the piezoelectric ceramic material as this coefficient influences the magnitude of the sensed voltage (through matrix Θ_s) and the effect of the drive voltage (through matrix Θ_d).

Positive displacement feedback gains caused a decrease in the resonant frequency rather than an increase as would be expected. This occurs because, for the mode of interest, there is a 180° phase difference between the displacement of the structure at the sensing ceramic element and the displacement of the structure at the drive ceramic element as shown in Fig. 2.

If the feedback control loop was disconnected so that both the sense and drive ceramics had open cicuit boundary conditions, the voltages generated at these ceramics, due to displacement of the primary mode, would be equal in magnitude but would have opposite phase. This situation is identical to connecting the feedback control loop with a displacement feedback gain of -1 and, therefore, would be expected to cause the same change in resonant frequency. This experiment was performed, and it was found that changing the electrical boundary condition of the drive ceramic from closed circuit to open circuit increased the resonant frequency by 2.5 Hz. This is equal to the frequency shift that was obtained by applying a feedback with gain of -1, therefore, it is a very simple method of experimentally determining the slope of this curve without performing any closed loop experiments.

The effect of velocity feedback is to modify the damping of the system. The damping of a resonator usually is quantified by the Q factor, which is inversely related to the mechanical damping factor (ζ) by the expression $Q = 1/2\zeta$ [26]. Fig. 5 shows the effect of changing the velocity feedback gain on the Q factor of the resonator. The arbitrary viscous damping included in the model was adjusted to give agreement with the experimental values when no feedback was applied. Positive velocity feedback gains cause an increase in the Q factor (decrease in damping) because of the phase difference between the velocities of the sense and drive ceramics. As the velocity feedback gain is increased, the total damping in the system tends toward zero and the Q factor increases rapidly toward infinity. Increasing the velocity feedback gain further results in instability of the linear system. Applying a velocity feedback gain of -1 resulted in a change in Q factor from 3300 to 1500, which represents a modification in the mechanical damping factor from $\zeta = 0.00015$ to $\zeta = 0.000334$. Velocity feedback is commonly used to increase the bandwidth of vibratory gyroscopes, and the method presented here can be used to determine the velocity feedback gain required for a particular bandwidth.

The difference between the theoretical and measured results can be almost completely eliminated by decreasing the piezoelectric coupling coefficient by a further 10%. It appears that the method of calibrating this coefficient by using the response at 1 kHz has overestimated this coefficient because the contribution of other vibration modes to the response at 1 kHz has not been included in the model.

V. Conclusions

A method has been presented for calculating the modification to the dynamic characteristics of a piezoelectric resonator that can be achieved by applying displacement and velocity feedback. The method was applied to a vibrating cylinder resonator, with discrete sensing and actuation piezoelectric ceramics, and the calculated results agreed with experimental results, thereby verifying the method. The resonator considered here showed a change in resonant frequency of approximately 2.5 Hz per unit displacement feedback gain. This is sufficient for balancing of this type of resonator to be performed using displacement feedback instead of the conventional mechanical mass removal. It also was demonstrated that a simple experiment can be performed to determine the effect of displacement feedback on the natural frequency of a resonator without the use of any feedback electronics.

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Craig A. Rogers a native of Barre, Vermont, began his tenure as the Dean of the College of Engineering at the University of South Carolina in August 1996. A scientist as well as an educator, he came from Virginia Polytechnic Institute and State University at which he held joint appointments in both the College of Engineering and the College of Education.

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A mechanical engineer by profession, his impact on the University of South Carolina's College of Engineering during his short tenure has been widely recognized for innovative educational approaches to engineering education. One of his latest innovations is the formation of the National Collegiate Association for Racing, an association dedicated to enhancing the educational opportunities at colleges of engineering throughout the nation. Through this expansion of the motor sports program, students are able to apply the theory and research learned in the classrooms and labs with practical hands-on experience that will give them valuable experiences upon graduation.

Another topic Dr. Rogers has made a priority is the introduction of engineering concepts into the science curriculum in the elementary schools. His belief is that children are natural inventors and designers, and they are the key to the future. Incorporating engineering principles within elementary science classes is a natural program to stimulate young and creative minds.

In 1992 Dr. Rogers received the Young Inventor's Award from the Office of Naval Research and later a NSF Presidential Young Investigator Award. In 1993 he was the first recipient of the American Society of Mechanical Engineers Adaptive Structures and Material Systems prize in recognition of lifetime contributions to the field of intelligent material systems. He has received international recognition as an author and lecturer, traveling throughout the United States and abroad.