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# STRESS-DRIVEN GRAIN GROWTH

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## Introduction

Consider a two-dimensional polycrystal of a material with a rectangular unit cell with lattice spacings $b$ and $b\ (1 + \varepsilon)$, subjected to a uniform external stress $\sigma$. Consider a grain in which the lattice vector of length $b\ (1 + \varepsilon)$ is parallel to $\sigma$, embedded in a grain in which the lattice vector $b\ (1 + \varepsilon)$ is transverse to $\sigma$. If the embedded grain grows at the expense of its matrix, the source of the stress will do work, and therefore the presence of this stress will drive the growth of the embedded grain. We estimate the rate of this process, and discuss an apparently anomalous consequence of this estimate. The process involved is distinct from that of diffusional creep, but, because the two are related, we begin with a summary of the theory of diffusional creep.

## Diffusional Creep

We consider a two-dimensional model of a polycrystal under a uniform tensile stress $\sigma$. The unit cell is square, of side $b$. A square grain of side $L$ lies with two edges transverse to $\sigma$ and two lateral edges parallel to $\sigma$. Let the equilibrium volume concentration of vacancies in the absence of stress be $c_o$. The stress will not affect the equilibrium concentration near the lateral interfaces. However, if a vacancy is formed at a transverse interface, the external stress does work $\sigma b^3$. The enthalpy of formation of a vacancy at a transverse interface is reduced by this amount, and so the equilibrium concentration of vacancies near a transverse interface is $c_o \exp(\sigma b^3/kT)$. There is thus a gradient in the concentration of vacancies of order

$$L^{-1}c_o[\exp(\sigma b^3/kT) - 1] \approx c_o\sigma b^3/LkT$$

If the diffusion coefficient of vacancies is $D_v$, the flux of vacancies is of order

$$D_v c_o \sigma b^3/LkT \approx D\sigma b^3/LkT \tag{1}$$

where $D = D_v c_o$ is the coefficient of self-diffusion by the vacancy mechanism.

This is the rate at which volume is added to or subtracted from unit area of the cell boundary. The strain rate is therefore given by

$$\dot{e} = D\sigma b^3/L^2kT. \tag{2}$$

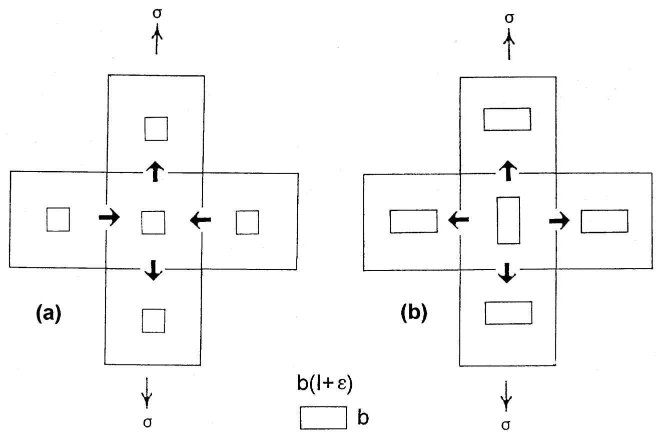


Figure 1. (a) Grain boundary motion during diffusional creep. (b) Grain boundary motion during stress-driven grain growth. (Inset) Lattice parameters in case (b).

If diffusion occurs along the grain boundaries, with a diffusion coefficient $D_b$, this rate is multiplied by a factor of order $D_b b/L$.

## Stress-Driven Grain Growth

The process of diffusional creep is illustrated in Fig. 1(a), where the heavy arrows indicate the directions of motion of the interfaces. Now consider the situation of Fig. 1(b), where the unit cells are no longer squares of sides $b \times b$, but rectangular of sides $b \times b(1 + \varepsilon)$. If a cell in the embedded grain changes its orientation to that of the cells in the matrix grain, the external forces do work $\sigma b^3 \varepsilon$, and so the presence of the external stress encourages the growth of the embedded grain. We now consider the kinetics of this process. The directions of motion of the interfaces are again indicated by heavy arrows.

The lower transverse interfaces contain misfit dislocations such as that shown in Fig. 2(a). The dislocations are separated by $b/|\varepsilon|$. The interface will move down by a distance of about $b$ if each of these dislocations climbs down by one interatomic spacing. The configuration of Fig. 1(a), containing 10 atoms, has become that of Fig. 2(b), containing 11 atoms. The region has emitted a vacancy. Similarly, when the right-hand lateral interface of Fig. 1(c) moves by about $b$ to the right, a configuration containing 10 atoms has become that of Fig. 1(d), containing 9 atoms. The region has absorbed a vacancy. The process is thus similar to that of diffusional creep, and is controlled by the diffusion of vacancies from the transverse to the lateral interfaces. It differs from diffusional creep in two respects: (i) the movement of the interface by a distance $b$ requires the emission or absorption of only one vacancy on a length $b/|\varepsilon|$ of the interface, and (ii) the movement of the interface by a distance $b$ causes the external forces to do only a fraction $\varepsilon$ of the work they do when interfaces move by a distance $b$ in diffusional creep. The work done in transferring a single vacancy from a transverse to a lateral interface is the same in both cases, and the creep rates are the same.

If the growing grain is favourably oriented, but the matrix is randomly rather than unfavourably oriented, the creep rate will be halved. If the long lattice vector of the grains cell makes an angle $\theta$ with the tensile axis, its rate of growth will be multiplied by a factor $2\cos^2\theta - 1$, on, in three dimensions, $3\cos^2\theta - 1$.

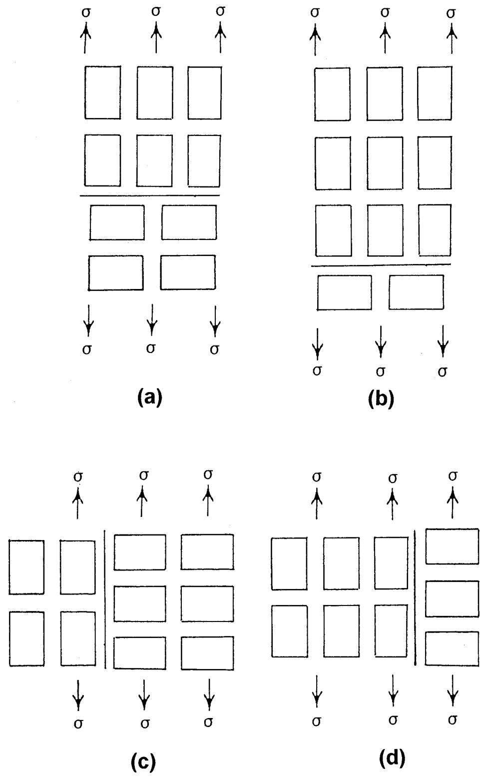


Figure 2. (a) Misfit dislocation on lower transverse boundary of embedded grain. (b) The embedded grain has grown by one atomic layer. (c) Misfit dislocation on right lateral boundary of embedded grain. (d) The embedded grain has grown by one atomic layer.

## Discussion

The analysis correctly shows that the growth or shrinkage of the embedded grain is determined by the sign of $\varepsilon$. It has the unexpected feature that the rate of growth is independent of the magnitude of $\varepsilon$. This clearly cannot be true when $|\varepsilon|$ is very small. The anomaly arises from an unstated approximation in the analysis. It has been assumed that the impedance to the diffusional flow of vacancies is that of a channel of length $L$ and of width $L$. This will be true only if the misfit dislocations which are the sources and sinks of vacancies have spacings much less that $L$. These spacings are $b/|\varepsilon|$. The calculation is valid only if $L \gg b/|\varepsilon|$. Then, from eq. (2), the strain rate is

$$\dot{e} \ll D\sigma b\varepsilon^2/kT, \qquad (3)$$

which correctly tends to zero as $|\varepsilon|$ tends to zero. A more complete calculation would allow for the impedance of the diffusional "bottlenecks" near the individual misfit dislocations.

## Acknowledgment

The process discussed in this communication was suggested to me by Heidi de Villiers Lovelock.