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EULDEP: A PROGRAM FOR THE EULER DECONVOLUTION OF MAGNETIC AND GRAVITY DATA

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Abstract—Measurements of the geomagnetic field can be used to help determine the structure of the Earth, since rocks often contain magnetic minerals. Measurements of the strength of the Earth's gravitational field can also be used, since the more dense the rocks, the stronger the field. The interpretation of large amounts of data can be aided by "automatic" interpretation techniques such as Euler deconvolution. This considers the anomalies to be caused by many relatively simple sources (such as dipoles or lines of dipoles) and produces the positions and depths of these sources. This data can then be used as a basis for a more detailed interpretation. The software runs under DOS and is available free via ftp from ftp.cs.wits.ac.za in directory/pub/general/geophys. © 1998 Elsevier Science Ltd. All rights reserved

Code available at http://www.iamg.org/CGEditor/index.htm

Key Words: Geophysics, Magnetics, Gravity, Deconvolution, Euler.

INTRODUCTION

Measurements of the magnetic and gravity field of the Earth are used extensively to explore its structure, particularly in the search for gold, oil, diamonds and other substances of economic value. Magnetic data are frequently collected by aircraft that can cover substantial amounts of ground in a short time, resulting in large amounts of data that require interpretation. Euler deconvolution can assist the interpreter by indicating portions of the data of interest, which can then by modelled in detail. It does not assume any particular geological model (such as a dyke or contact), but uses a range of elementary magnetic or gravity distributions such as point poles and dipoles as the source of the anomalies. It was first discussed by Hood (1963) and developed further by Thomson (1982) and Reid and others (1990). In South Africa it has been used extensively (Durrheim, 1983; Corner and Wilsher, 1989).

Euldep reads in the observed field magnetic data from an ASCII file and produces a plot of the solutions onscreen. Euldep calculates the horizontal and vertical gradients of the field, as well as the pole reduced field (for magnetic data), and these are also displayed. Figure 1 shows a sample screenshot of a magnetic data profile and hardcopy printer output is also available. An output file containing the solutions is written to disk. The maximum number of data points that can be processed is 2048. It was written using Microsoft Fortran 5.0 and runs under DOS on computers that have at least an 80286 processor.

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EULER DECONVOLUTION

A contrast in magnetization can be represented as a distribution of magnetic poles at the interface, although magnetic monopoles do not actually exist. The intensity of the magnetic anomaly from an arrangement of poles or dipoles has a characteristic decay with distance. The field due to a distribution of magnetic poles can be written as

$$f(x, y, z) = M/r^N \tag{1}$$

where r is $\sqrt{(x^2 + y^2 + z^2)}$, M is proportional to magnetization and N is the structural index, which can assume values between zero and three (see Table 1 and Fig. 2).

The user can choose which structural indices are to be located by Euldep and the solutions are then plotted using different symbols, as shown in Table 2.

So for example the solutions corresponding to the first structural index used (which could be any value, e.g. 2.0) would be plotted using the + symbol, whereas those corresponding to the second structural index used would be plotted using the × symbol.

The magnetic field due to a point source such as a pole or dipole at a position (x_0, y_0, z_0) is of the form;

$$\Delta T(x, y) = f((x - x_0), (y - y_0), z_0) \tag{2}$$

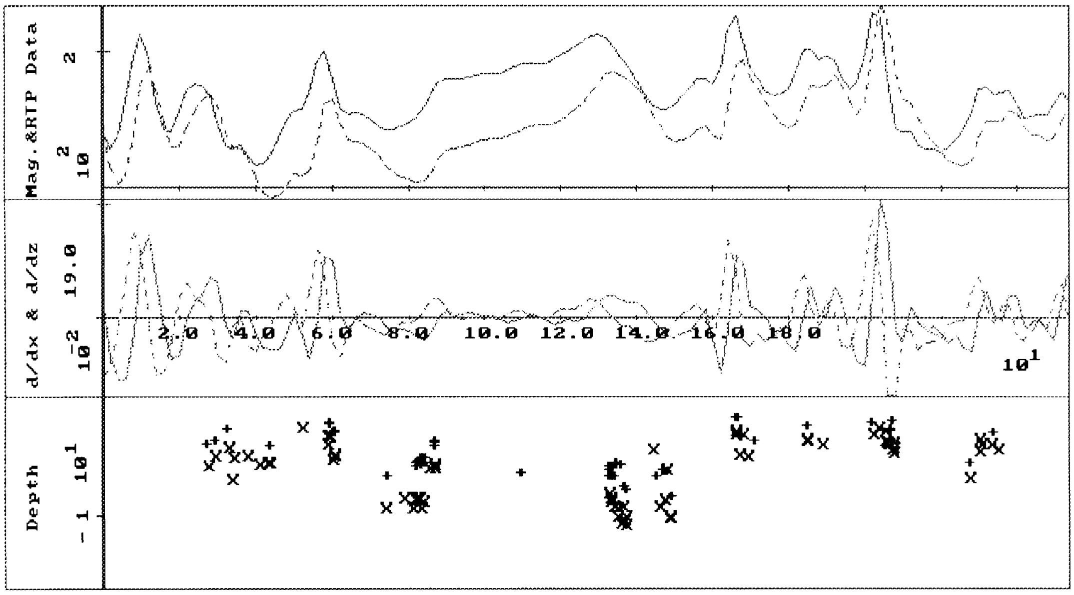

Figure 1. Sample output from Euldep. Upper portion of plot shows original magnetic data and pole reduced magnetic data (drawn in dashed linestyle). Middle plot shows horizontal and vertical gradients (latter being drawn in dashed linestyle). Lower plot shows different solutions. See Table 2 for explanation of symbols used

A function $f(x, y, z)$ is homogenous of degree n if

$$f(tx, ty, tz) = t^n f(x, y, z). \qquad (3)$$

If a function f is homogenous of degree n then it satisfies Euler's equation, given by (Thomson, 1982):

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + z\frac{\partial f}{\partial z} = nf \qquad (4)$$

or (Blakely, 1995, p. 243);

$$r \cdot \nabla f = -nf. \qquad (5)$$

Equation (1) is homogenous of order $-N$ and so Equation (2) can be rewritten as

$$(x - x_0)\frac{\partial \Delta T}{\partial x} + (y - y_0)\frac{\partial \Delta T}{\partial y} + z_0\frac{\partial \Delta T}{\partial z} = -N\Delta T(x, y). \qquad (6)$$

Euldep uses profile data and assumes that the field is symmetric transverse to the profile, so $\partial \Delta T/\partial y = 0$. The total field can be considered to be the sum of a regional field and the anomaly due to the point source, i.e.

$$T(x) = \Delta T(x) + B \qquad (7)$$

where B is the regional field. Then, from Equation (6)

$$x_0\frac{\partial T}{\partial x} + z_0\frac{\partial T}{\partial z} + N \cdot B = x\frac{\partial T}{\partial x} + N \cdot T. \qquad (8)$$

So the position x_0, depth z_0 and anomaly base level B of a specific magnetic source can be solved if the total magnetic field and the horizontal and vertical gradients are known at three points along the profile. Because real magnetic and gravity bodies are more complex than simple poles and dipoles, and because real data is contaminated by noise, seven data points are used. This yields an overdetermined set of equations which are then solved by least-squares inversion using (Menke, 1989, p. 153)

$$S = (A^T A)^{-1} A^T G \qquad (9)$$

where A is a 7×3 matrix that contains seven data points from the profile for horizontal and vertical gradients and the structural indices being searched for and G is a 7-point matrix containing the right hand side of Equation (7). S is the solution matrix that contains the depths and horizontal positions of the solutions.

The process is repeated for different values of the structural index N. The seven point window is moved along the profile, solving for x_0, z_0 and B at each position for a range of structural indices. A source is regarded as being localized when solutions for x_0 and z_0 at consecutive window positions are within a user-specified distance of each other. Depth limits can be input to Euldep and any solutions which fall outside these ranges are rejected.

Table 1. Structural indices for magnetic data

Model	Structural index
Line of poles	1.0
Point pole	2.0
Line of dipoles	2.0
Point dipole	3.0

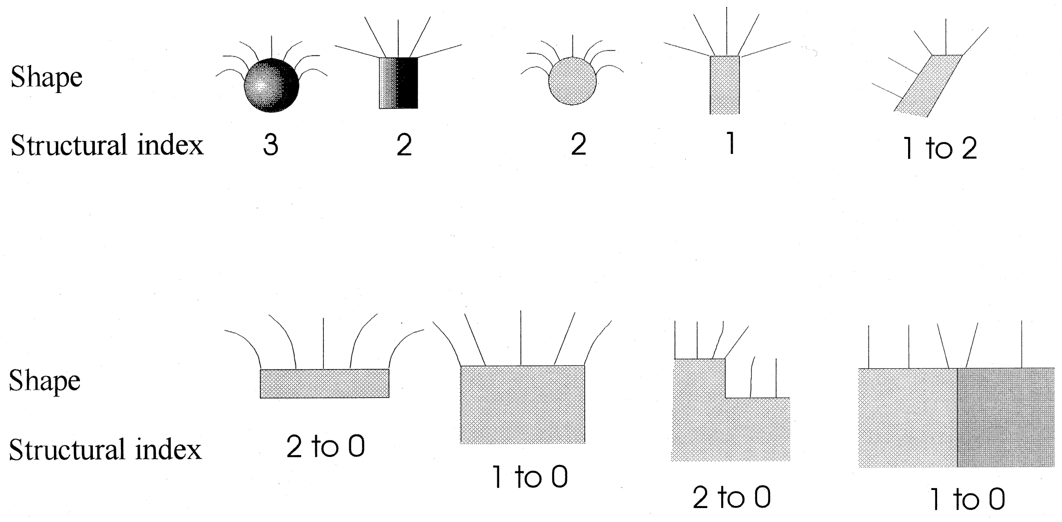

Figure 2. Different structural indices are shown together with simple geological structures that they are associated with (after Breiner, 1973). All shapes shown are of infinite strike extent in and out of paper, except for sphere and cylinder shown at upper left

For Euldep to provide accurate solutions, the 7 point window must adequately sample each anomaly on the profile. This occurs when the window size is about half the anomaly size. Since problems will occur if the window size is greater than twice the anomaly size, or less than half the anomaly size, several passes are made through the data looking for magnetic anomalies that occur on different length scales. This is done by using either every data point (resulting in a window size of six times the data sampling interval), or every Nth data point (resulting in a window size of $N-1$ times the data sampling interval). When this is done the data is upward continued by a distance of N times the data sampling interval to reduce the high frequency components of the data which will appear as noise to the larger window size. Solutions produced by different window sizes are plotted onscreen in different colours.

If the horizontal and vertical gradient data along the profile are not available, then they are calculated by Euldep from the total field using frequency domain methods. The first-order horizontal gradient is given by (Gunn, 1975)

$$A'(f) = A(f) \cdot \mathrm{i}f \tag{10}$$

where $A(f)$ is the amplitude at a frequency f before filtering, $A'(f)$ is the corresponding amplitude afterwards and $\mathrm{i} = \sqrt{-1}$. Similarly, the first vertical derivative is given by (Gunn, 1975);

$$A'(f) = A(f) \cdot f. \tag{11}$$

Table 2. Symbols used to plot structural indices

Index used	Symbol
1st	+
2nd	x
3nd	@
4th	*
5th	^

If the data are noisy, then the horizontal and vertical gradients will be even more noisy and this will affect the performance of Euldep. For that reason a low-pass filter can be applied to the data by Euldep to smooth it before the gradients are calculated. The filter is given by

$$\text{for } f \le f_c \quad A'(f) = A(f)*1.0$$

$$\text{for } f > f_c \quad A'(f) = A(f)*\mathrm{e}^{(-k(f-f_c)^2)}. \tag{12}$$

Thus there are two parameters that must be estimated before the data can be filtered; the cutoff wavenumber f_c and the attenuation factor k, which determines the steepness of the exponential decay of the filter is. The values of these parameters depends on the dataset being processed, and can be determined by inspection of its power spectrum.

It has been observed that depth estimates that are obtained from magnetic data are more accurate if the pole-reduced magnetic field is used (Thomson, 1982). Pole reduction takes the magnetic anomalies, as measured at any latitude, and transforms them into that which would have been measured if the body had lain at the magnetic pole, i.e. the region here the inclination is vertical. The operator is (Spector and Parker, 1979)

$$A'(f) = \frac{A(f)}{(\sin(I) + \mathrm{i}\cos(I)\sin(D))^2} \tag{13}$$

where I is the geomagnetic inclination and D is the angle between the profile bearing and magnetic North. However the method assumes that no remnant magnetization is present (because the remnant component of the anomaly is independent of the

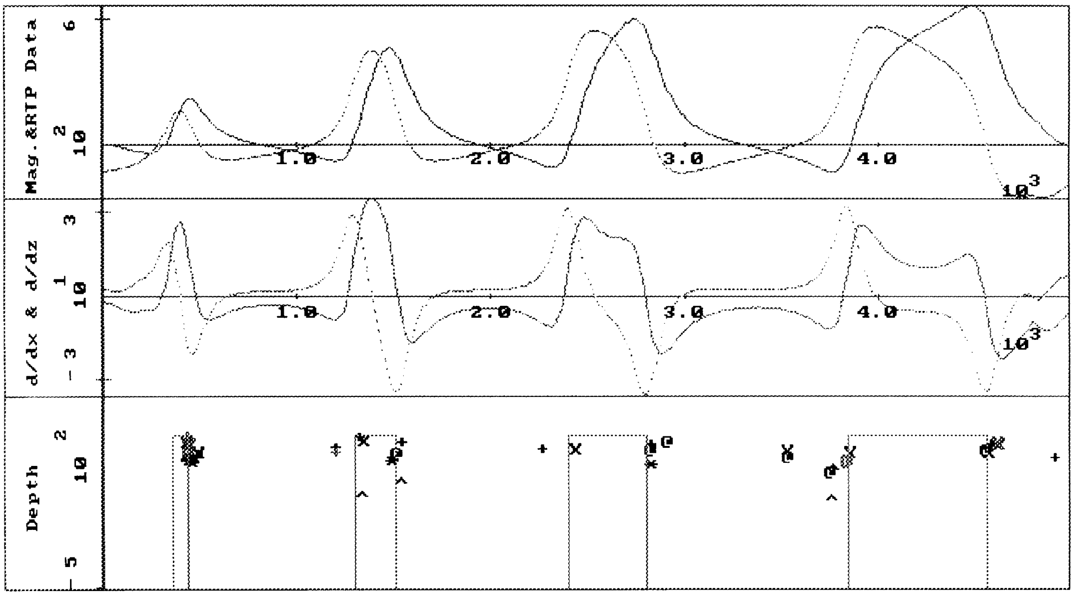

Figure 3. Results of running Euldep over magnetic data profile. Symbols used are as for Figure 1. Rectangular causative bodies, which have infinite strike extent and are orthogonal to profile direction, are superimposed for comparison with results from Euldep

current geomagnetic field and would hence have the same form at the pole as at the point of measurement) and it does not work well at low latitudes where the field is almost horizontal and the vertical component is small. Reduction of such data results in a noisy profile containing spurious anomalies which will obviously be detrimental to the performance of Euldep.

EXAMPLES OF THE USE OF EULDEP

Figure 3 shows magnetic anomalies from three dykes, with the causative magnetic bodies superimposed on the results from running Euldep. The dykes are two dimensional, i.e. their strike direction is orthogonal to that of the profile, and are of infinite strike extent. For a shallowly buried ribbon-like source the strike length should be at least 20 times the source width for this criteria to be valid (Blakely, 1995, p. 191). As can be seen, the edges of the dykes were well located by the method in this example. A small number of spurious solutions were generated however, and these can be caused by the overlap or interference of the anomalies from adjacent bodies. Thomson (1982) found that the lower structural indices were the better depth estimators and this is the case for the model shown here.

Euler deconvolution can be applied to any field that is homogenous, such as the analytical signal of magnetic data. The analytical signal is a combination of the horizontal and vertical gradients of magnetic data. It is obtained by setting the amplitudes of the coefficients of the negative frequencies of the data to zero, and doubling the amplitudes of the coefficients of the positive frequencies (Blakely, 1995, p. 352). It is often used in place of pole reduction for data from low magnetic latitudes (as discussed previously). Figure 4 shows the result of using Euler deconvolution on analytical signal data from the model shown. The structural indices are 1.0 greater than those obtained from the magnetic data (Reid, 1995).

Reid (1995) applied Euler deconvolution to gravity data and found a structural index of 1.0 gave the best results. Figure 5 shows the result of applying Euler deconvolution to gravity data generated by three rectangular bodies. The edges of the bodies in the model have been detected and the depth estimates given by the smaller structural indices used are close to the true values.

SOURCES OF ERROR

Euler deconvolution can predict sources incorrectly for several reasons. Firstly noise (as mentioned previously) will both distort the shape of existing anomalies and appear to Euldep as the result of the presence of a magnetic source. Noisy data should therefore either be filtered prior to the application of Euldep, or the low-pass filter within Euldep should be used.

If a window size is used that is large enough to encompass two or more neighbouring anomalies then they will be considered as one anomaly by Euldep, which will produce an incorrect result. However since a range of window sizes are used, the smaller sizes will resolve the individual anomalies better and hence produce a more accurate source location.

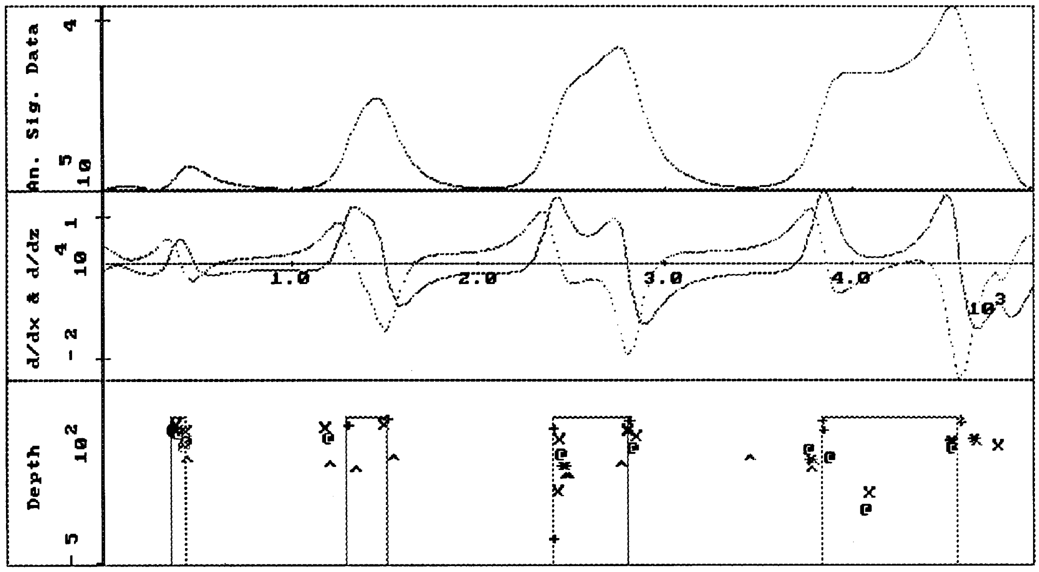

Figure 4. Results of running Euldep over analytical signal data profile. The symbols used are as for Figure 1. Rectangular causative bodies, which have infinite strike extent and are orthogonal to profile direction, are superimposed for comparison with results from Euldep

In addition, if any of the assumptions about the data that are made by Euldep are incorrect, namely that the causative bodies are two-dimensional and lie orthogonal to the profile direction and that the magnetic anomalies contain no remanent magnetization, then incorrect solutions will result.

CONCLUSIONS

Euldep can be used to process magnetic and gravity profiles quickly, providing a range of starting models that can be refined by further modelling. This can be invaluable when large amounts of data are to be interpreted. The interpretation of geophysical data is fundamentally ambiguous however, for example even complete, noise-free, gravity data cannot distinguish between two buried spherical bodies with equal mass but differing radii (Burkhard and Jackson, 1976). Hence Euldep cannot be applied blindly and the user must select the filter parameters carefully. Where possible the results must be integrated with other geophysical data to provide a complete interpretation.

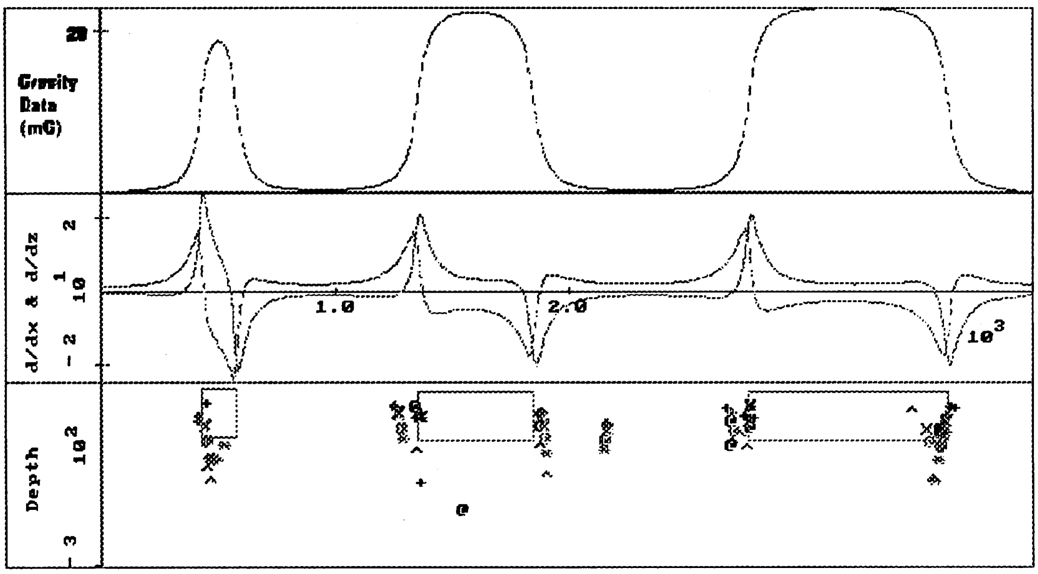

Figure 5. Results of running Euldep over gravity data profile. Symbols used are as for Figure 1. Rectangular causative bodies, which have infinite strike extent and are orthogonal to profile direction, are superimposed for comparison with results from Euldep

REFERENCES

Blakely, R. J. (1995) *Potential Theory in Gravity and Magnetic Applications*. Cambridge University Press, New York, 435 pp.

Breiner, S. (1973) Applications manual for portable magnetometers. GeoMetrics corporation 395 Java Drive, Sunnyvale, California 94089, 58 pp.

Burkhard, N. and Jackson, D. D. (1976) Application of stabilized linear inverse theory to gravity data. *Journal of Geophysical Research* **81**(8), 1513–1518.

Corner, B. and Wilsher, W. A. (1989) Structure of the Witwatersrand basin derived from the interpretation of the aeromagnetic and gravity data. In *Proceedings of Exploration '87: Third Decennial International Conference on Geophysical and Geochemical Exploration for Minerals and Groundwater*, ed. G. D. Garland, p. 960.

Durrheim, R. J. (1983) Regional-residual separation and automatic interpretation of aeromagnetic data. M.Sc. Thesis, University of Pretoria, South Africa, 117 pp.

Gunn, P. J. (1975) Linear transformations of gravity and magnetic fields. *Geophysical Prospecting* **23**(2) , 300–312.

Hood, P. J. (1963) Gradient measurements in aeromagnetic surveying. *Geophysics* **30**(1), 891–902.

Menke, W. (1989) *Geophysical Data Analysis: Discrete Inverse Theory*, 2nd edn. Academic Press, New York, 285 pp.

Reid, A. B. (1995) Euler deconvolution: past, present and future. A review. Abstract Society of Exploration Geophysicists, 1995.

Reid, A. B., Allsop, J. M., Granser, H., Millet, A. J. and Somerton, I. W. (1990) Magnetic interpretation in three dimensions using Euler deconvolution. *Geophysics* **55**, 180–191.

Spector, A. and Parker, W. (1979) Computer compilation and interpretation of geophysical data. *Geophysics and geochemistry in the Search for Metallic Ores*. In *Proceedings of Exploration '77*, ed. P. J. Hood, Economic Geology Report 31, pp. 527–544. *Geological Survey of Canada*.

Thomson, D. T. (1982) Euldph: A new technique for making computer assisted depth estimates from magnetic data. *Geophysics* **47**(1), 31–37.