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VAN SUID-AFRIKA

FUEL RESEARCH INSTITUTE
OF SOUTH AFRICA

ONDERWERP:
SUBJECT: PREDICTING THE PERFORMANCE OF A COAL WASHER WITH THE AID OF A MATHEMATICAL MODEL

AFDELING:
DIVISION: ENGINEERING

NAAM VAN AMPTENAAR:
NAME OF OFFICER: T C ERASMUS

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THIS REPORT IS INTENDED TO BE READ AT THE 7TH INTERNATIONAL COAL PREPARATION CONGRESS IN SYDNEY, AUSTRALIA, IN MAY, 1976

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TITLE : PREDICTING THE PERFORMANCE OF A COAL WASHER WITH THE AID OF A MATHEMATICAL MODEL

CO-WORKERS : -

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INVESTIGATION REQUESTED BY : -

SECTION : ENGINEERING

SYNOPSIS

The data of a hundred and forty beneficiation tests, conducted in the cyclone washer, were statistically processed and the quasi-constants, affording analytical descriptions of the washer performance, correlated with the cut point, ranging from 1,36 to 1,68 relative density.

The correlations facilitate performance prediction, and were used to indicate :

(i) that considerable variation in performance is possible even under close control;

(ii) that the relative density range spanned by the partition curve, the Ecart probable (moyen), the Ecart Mayer, and the error area all increase with increasing cut-point;

(iii) that the partition curve becomes progressively more asymmetrical as the cut-point increases.

1. The Fuel Research Institute of South Africa has on record the data of a large number of coal beneficiation exercises covering both a variety of coal and a wide range of cut-points. The data generally include the coordinates of the partition curve, the corresponding yields, washability data, and in most instances also the ash distributions. The objective of this paper is the correlation of the performance of the cyclone washer with the cut-point.

The Tromp partition curve graphically illustrates the performance of a coal washer. However, being a non-linear curve, it does not readily lend itself to correlation. Descriptive parameters, formulated to express characteristics of the partition curve, numerically, are also unsatisfactory in this respect. This is because the use of these entails the partial loss of detailed information. For instance, the Ecart probable (moyen) takes no cognizance of the asymmetry of the partition curve. Therefore, identical values do not necessarily imply identical performance. For correct interpretation, these parameters must be read in conjunction with the partition curve.

In an earlier publication[1], it was shown that the partition curve may be linearised without loss of detail. This linearisation then makes it possible to subject experimentally determined partition curves to a basically simple statistical analysis. In the transformed state, the partition coefficient is related to the corresponding relative density as follows :

$$\text{Arctan}\,(k(s-c))=t_2 - (t_2 - t_1)D/100 \qquad (1)$$

wherein D is the partition coefficient, expressed as a percentage;
S is the corresponding relative density;
t_2 is a constant;
K, C, and t_1 are quasi-constants.

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1) ERASMUS, T.C., COAL, GOLD AND BASE MINERALS 21(4), 63 - 67, June, 1973.

Although k, C and t_1, are independent of the relative density, it will be shown at a later stage that their respective values depend on the cut-point, and are therefore termed quasi-constants.

k, C, t_1, and t_2 constitute a short-hand description of the partition curve, and therefore of the performance of a washer. The complex problem of correlation is thus simplified, entailing correlation of the four constants with the cut-point.

Equation 1 also allows the analytical evaluation of the various descriptive parameters. For instance the cut-point, S_c, is the relative density corresponding to a partition coefficient of fifty per cent. Thus, by assigning the value 50 to D in equation 1, it yields upon simplification :

$$S_c = c + (\mathrm{Tan}(0,5(t_1 + t_2)))/k \qquad (2)$$

The Ecart probable (moyen), E, is defined as half the difference between the relative densities corresponding to partition coefficients of 75 and 25, respectively. The Ecart probable follows from equation 1 and is given by :

$$E = (\mathrm{Tan}(0,75t_2 + 0,25t_1) - \mathrm{Tan}(0,25t_2 + 0,75t_1))/2k \qquad (3)$$

The Ecart probable will be used, at a later stage, to indicate how the shape of the partition curve varies with the change in the cut-point. In order to obtain a more general picture, three supplementary parameters will be used. These are :

(i) The Ecart Mayer[2], E', defined as the difference between the relative densities corresponding to partition coefficients of 90 and 10, respectively, and given by :

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2) MAYER, F.W., AUFBEREITUNGS-TECHNIK, NR. 12, 1967

$$E' = (\mathrm{Tan}(0{,}9t_2 + 0{,}1t_1) - \mathrm{Tan}(0{,}1t_2 + 0{,}9t_1))/k \qquad (4)$$

(ii) The parameter, E'', defined as the difference between the relative densities corresponding to partition coefficients of 95 and 5, respectively :

$$E'' = (\mathrm{Tan}(0{,}95t_2 + 0{,}05t_1) - \mathrm{Tan}\ (0{,}05t_2 + 0{,}95t_1))/k \qquad (5)$$

(iii) The relative density range spanned by the partition curve, E''', which is the difference between the relative densities corresponding to partition coefficients of 100 and 0, respectively. E'''is given by :

$$E''' = (\mathrm{Tan}\ t_2 - \mathrm{Tan}\ t_1)/k \qquad (6)$$

Hitherto, the error area had to be determined by cumbersome graphical techniques. In order to show how the error area varies with the change in the cut-point, the following equations, determined by integration techniques, were derived :-

$$\text{Error area} = (A \ln X)/k \qquad (7)$$

$$A = 100/(t_2 - t_1) \qquad (8)$$

$$X = (1 + B - F)/2B \qquad (9)$$

$$B = \mathrm{Cos}\ t_1\ \mathrm{Cos}\ t_2 \qquad (10)$$

$$F = \mathrm{Sin}\ t_1\ \mathrm{Sin}\ t_2 \qquad (11)$$

Exact determination of the error area is possible because the limits of the relative density range spanned by the partition curve are precisely defined.

Knowledge of k, C, t_1, and t_2 not only allows the construction of the partition curve, but also enables the evaluation of parameters describing the important characteristics of the curve.

4/2 ....

2. Evaluation of k, C, t_1, and t_2 from the observed coordinates of the partition curve is based on the following : The relationship between D and arctan (k(s - c)) will approach optimum linearity as k and C tend to the respective optimum values. The degree of linearity is conveniently monitored by the coefficient of linear correlation.

Essentially, various combinations of k and C are utilized in conjunction with the observed coordinates, S and D, of a particular test to maximise the correlation coefficient. The optimum values of k and C are those corresponding to the maximum correlation coefficient. The values of t_1 and t_2 follow from the regression line based on the optimum values of k and C.

The analysis of the recorded data entailed the following :-

(i) The evaluation of k, C, t_1, and t_2, for the different tests.

(ii) The correlation of these with the cut-point.

(iii) The indication of the accuracy of the resulting correlations.

(iv) The use of the correlations to indicate the variation in performance with the change in the cut-point.

The analysis of the data is based on equation (1) and it is therefore necessary to indicate the accuracy of this equation. As an indication, it can be stated that linear correlation coefficients in excess of 0,999 are not uncommon. A more comprehensive illustration is given in Table 1, wherein the observed partition coefficients of a typical test run are compared with the corresponding computed values.

5/Table 1 ....

TABLE 1

Observed versus computed partition coefficients

Relative density	Partition coefficient	
	Observed	Computed
1,31	94,3	96,2
1,33	93,3	93,8
1,35	90,1	89,0
1,37	74,7	74,3
1,39	34,9	31,6
1,41	9,5	11,0
1,43	2,3	5,0
1,45	1,4	2,3
1,47	1,3	0,7
1,49	1,8	0

3. Initial attempts in relating the optimum values of k, C, t_1, and t_2 to the cut-point were unsatisfactory. Trends could be discerned, but the scatter of the points was excessive.

It was, however, found that utilization of the optimum value of C allowed relatively large variation in k, t_1, and t_2 without seriously affecting the coefficient of linear correlation. Plotting the "acceptable" range, rather than the optimum values of k, t_1, and t_2, would therefore offer greater insight. All those values, resulting in correlation coefficients in excess of 0,995, constitute the "acceptable" range of a particular constant at a given cut-point. Other values constitute the "unacceptable" range.

The respective plots are reproduced in Figures 1 to 3, inclusive. In these the vertical lines represent the "unacceptable" range. It will be noted that the alignment of the "acceptable" ranges is imperfect. Subjective curve fitting was based on the principle of minimum interception of the vertical lines. In this way the following relationships were established :

6/t_2..........

$$t_2 = \text{constant} = 1{,}4 \qquad (13)$$

$$t_1 = -2{,}2 + 0{,}59\ S_c \qquad (14)$$

$$k^2 = 729/(S_c - 1{,}23) \qquad (15)$$

At any given cut-point, S_c, the values of k, t_1, and t_2 can be had, using these equations :
These values are then used to determine the remaining quasi-constant, C, from equation 1. For this purpose equation 1 is rearranged thus :

$$C = S_c - (\mathrm{Tan}(0{,}5(t_2 + t_1)))/k \qquad (16)$$

4. The accuracy of equation 1, based on the optimum values of k, C, t_1 , and t_2, has been shown in section 2, and has been proved time and again in the past. It remains to show what accuracy can be expected when using the values derived from equations 13 to 16, inclusive. For this, use is made of the error, defined as the difference between the computed and the corresponding observed partition coefficients.

The observed partition coefficients are determined as mean values of the respective relative density intervals,* and thus located on the centre lines of the intervals. Error determination can therefore only be had at discrete points on the partition curve. To obtain an overall view, eleven points of comparison were selected.

The partition curves, and therefore the errors, of the different tests are superimposed so that the relative density intervals containing the cut-points coincide. As a result, the intervals can no longer be identified by the relative density and for convenience these are

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* At the Institute the relative density intervals are standardised at 0,02 units.

7/numbered ......

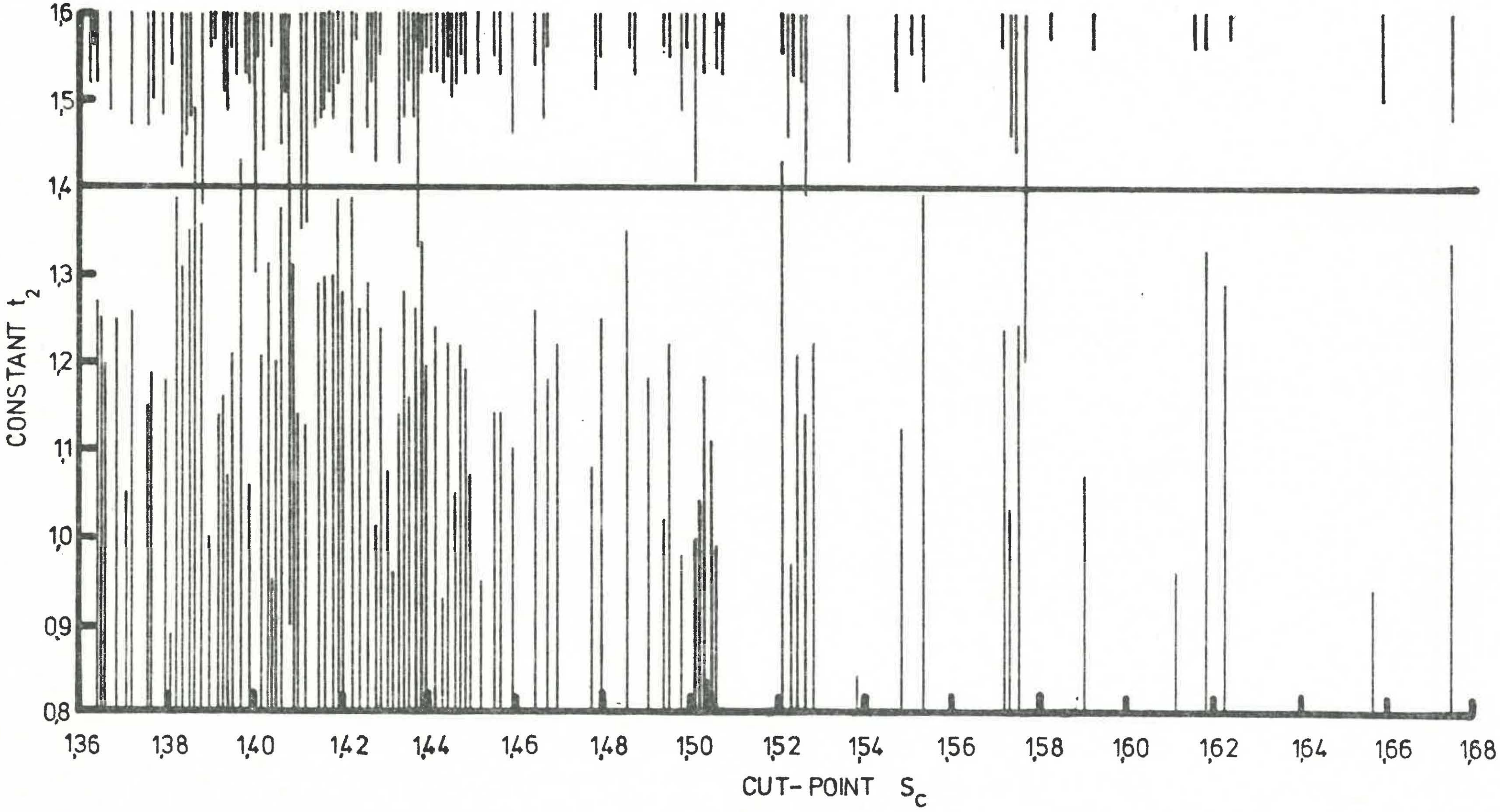

FIGURE 1 CONSTANT t_2 VERSUS CUT-POINT S_c

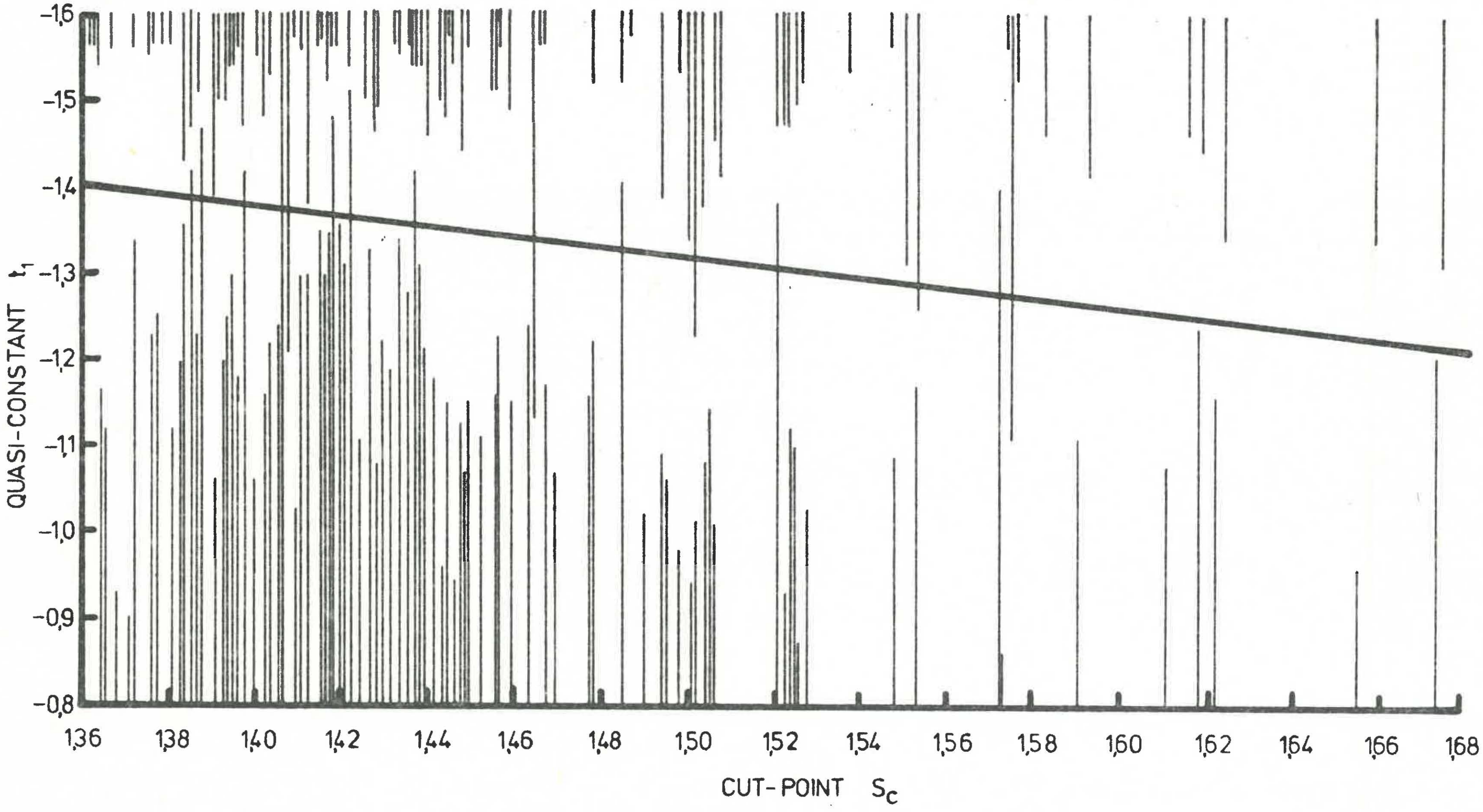

FIGURE 2. QUASI-CONSTANT, t_1, VERSUS CUT-POINT, S_c

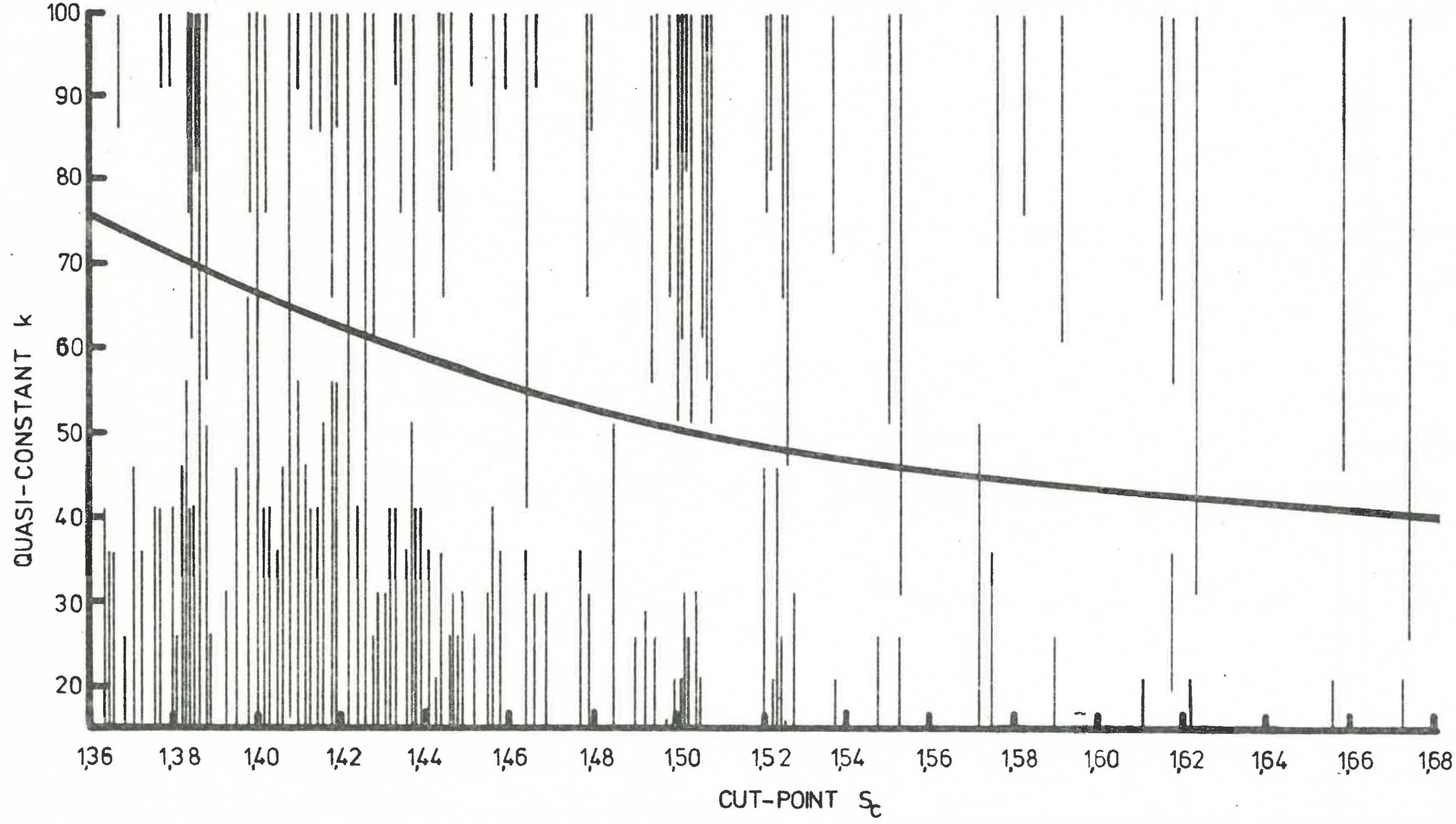

FIGURE 3. QUASI-CONSTANT k VERSUS CUT-POINT S_c

numbered 1 to 11 as shown in Figure 4. For each of the eleven intervals the error distribution was determined for subsequent use, and the results are reproduced in Table 2.

TABLE 2

ERROR DISTRIBUTION

Relative density interval	Percentage of total number of errors within the error interval									
	>4	3 to 4	2 to 3	1 to 2	0 to 1	-1 to 0	-2 to -1	-3 to -2	-4 to -3	< -4
1	3,9	3,9	21,5	43,1	27,4	0	0	0	0	0
2	7,7	12,5	27,8	39,4	12,5	0	0	0	0	0
3	15,0	6,7	21,0	30,0	22,5	2,2	2,2	0	0	0
4	8,6	9,3	9,3	12,2	40,3	10,8	4,3	0,7	2,1	2,1
5	13,2	6,6	5,1	13,2	30,9	11,8	8,1	2,9	4,4	3,6
6	1,4	1,4	7,1	17,1	32,1	18,6	12,8	7,1	0	2,1
7	26,8	9,4	9,4	12,3	17,4	7,2	6,5	2,9	2,9	5,1
8	23,9	11,6	19,6	15,2	13,8	2,9	4,3	3,6	0,7	4,3
9	2,3	4,6	12,9	17,5	42,0	7,6	3,0	3,8	0	6,1
10	0	0	0,8	4,9	45,9	22,1	11,5	5,7	2,4	6,5
11	0	0	0	0	30,9	28,6	15,5	9,5	6,0	9,5

The standard deviation of the errors affords a concise illustration of the accuracy of equation 1, when based on k, C, t_1, and t_2 values obtained from equations 13 to 16. The respective standard deviations, based on the total number of tests, are reproduced in Table 3.

The performance tests are unevenly distributed throughout the cut-point range. (See figure 1). The highest concentration occurs within the interval 1,40 to 1,44 relative density. The effect of good correlation in regions of high concentration may be so overwhelming that poor correlation at low concentration may go undetected. The existence of localized poor correlation could be disproved by grouping the tests into different cut-point intervals and determining the standard deviation of the different test groups. The results are included in Tables 3 and 4.

8/Table 3 ....

TABLE 3

THE STANDARD DEVIATION OF THE ERRORS

Relative density interval	Standard deviation of errors within the cut-point range										
	1,36 to 1,68	<1,38	1,38 to 1,40	1,40 to 1,42	1,42 to 1,44	1,44 to 1,46	1,46 to 1,48	1,48 to 1,50	1,50 to 1,52	1,52 to 1,54	1,54 to 1,56
1	1,8	-*	-	1,7	2,9	1,7	1,4	-	1,8	0,8	1,0
2	2,5	-	2,6	2,4	2,8	2,2	2,1	2,4	2,4	1,5	1,9
3	2,6	3,1	2,2	2,8	2,7	1,7	1,8	2,4	2,1	3,1	3,0
4	2,4	2,4	1,9	1,8	2,0	2,9	2,7	2,2	3,0	4,0	2,9
5	2,9	2,8	2,8	2,9	2,2	3,3	2,4	3,3	3,5	4,4	1,5
6	2,0	2,5	2,5	2,1	1,5	1,9	1,4	2,4	2,2	1,7	1,2
7	3,7	1,9	2,9	4,3	4,0	4,3	3,5	3,1	3,2	4,0	4,7
8	3,4	2,2	2,8	3,8	3,4	2,8	1,9	3,0	4,2	4,7	5,2
9	2,2	1,0	1,3	1,8	2,4	2,5	2,6	2,7	5,2	3,6	3,3
10	2,1	1,7	2,2	1,1	1,9	3,0	2,6	2,7	2,4	2,2	4,7
11	2,6	2,8	2,0	1,9	3,0	3,9	2,6	3,4	-	2,3	-
Number of tests	140	13	24	29	17	16	7	5	7	7	5

<u>Notes</u> *No experimental data are available in the particular relative density range.

For cut-points in excess of 1,56, the number of tests contained within the selected cut-point ranges are not sufficient to allow accurate determination of the standard deviation. These are nevertheless reproduced in Table 4 to illustrate the agreement between thoory and practice.

9/ Table 4 .....

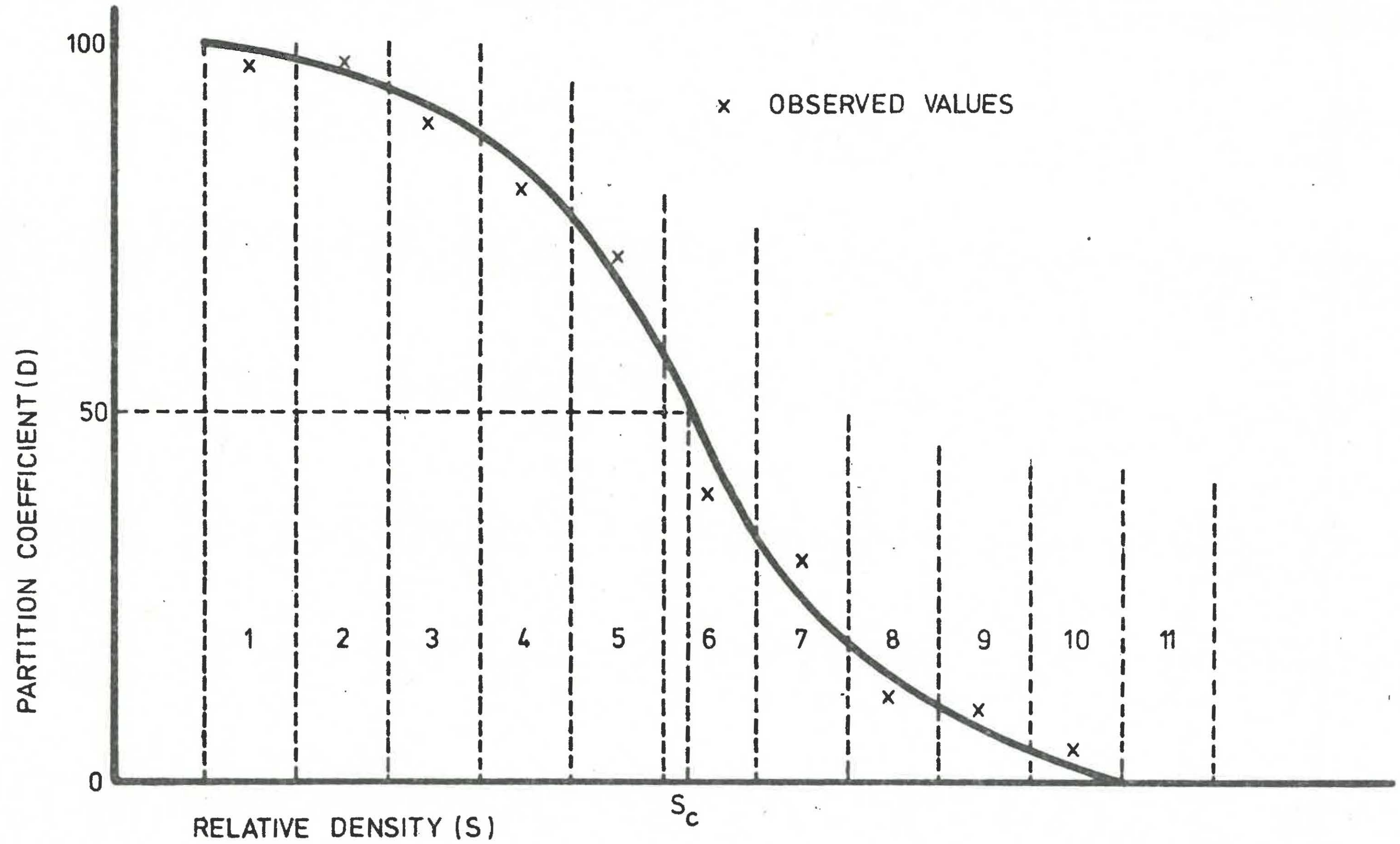

FIGURE 4 IDENTIFICATION OF RELATIVE DENSITY INTERVALS

TABLE 4

STANDARD DEVIATION OF ERRORS AT CUT-POINTS IN EXCESS OF 1,56 r.d.

Relative density interval	Standard deviation of error in cut-point range				
	1,56 to 1,58	1,58 to 1,60	1,60 to 1,62	1,62* to 1,64	1,64 to 1,66
1	1,1	-	1,1	4,1	0,8
2	2,5	2,3	1,9	5,9	1,4
3	1,9	2,0	2,6	7,9	2,8
4	0,7	0,4	2,5	6,3	0,5
5	2,1	0,9	1,9	15,4**	0,7
6	0,7	1,0	1,1	2,7	0,8
7	5,8	3,7	1,8	1,7	0,5
8	4,6	3,9	2,6		0,4
9	1,6	0,6	1,0		2,5
10	0,4	2,4	1,8		1,4
11	1,8	4,2	-		-

Note *The standard deviations reported in this column are based on a single test.

**In the light of standard deviations reported in adjacent columns, it must be concluded that excessively large standard deviations reflect an experimental error of excessive magnitude rather than an inaccurate correlation.

With reference to Tables 3 and 4, it follows that there is a good agreement between theory and practice and that the agreement is unaffected by variation in the cut-point.

Neither the yield nor the amount of "near-gravity" material* affects the accuracy of equation 1. This is substantiated by the plots of the error versus the yield and the amount of "near-gravity" material, respectively. The plots are reproduced in Figures 5 and 6.

10/ To ....

*"Near-gravity" material is defined as the percentage of the unwashed material contained within the relative density range: cut-point ± 0,01.

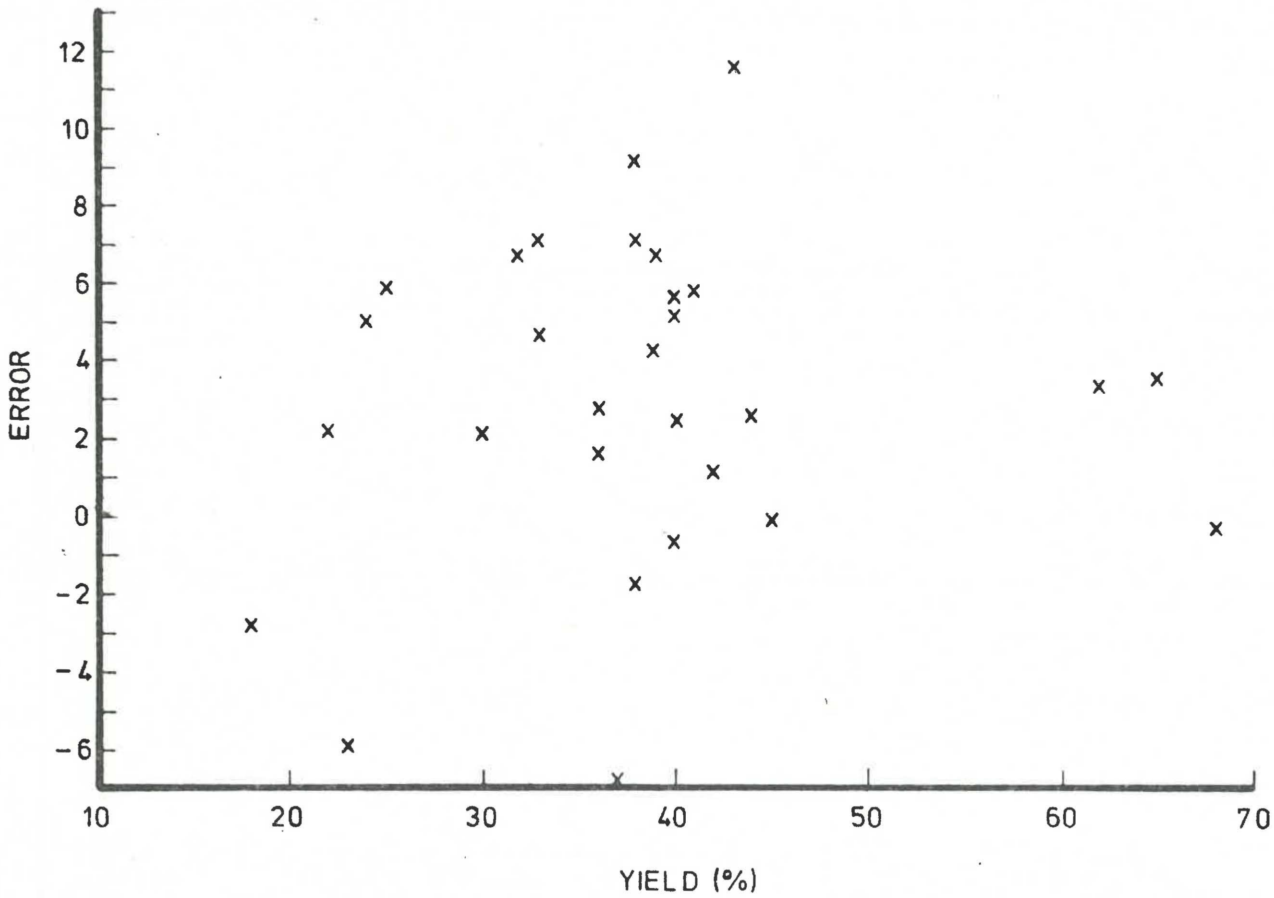

FIGURE. 5. ERROR (7TH. RD. INTERVAL) VERSUS YIELD.

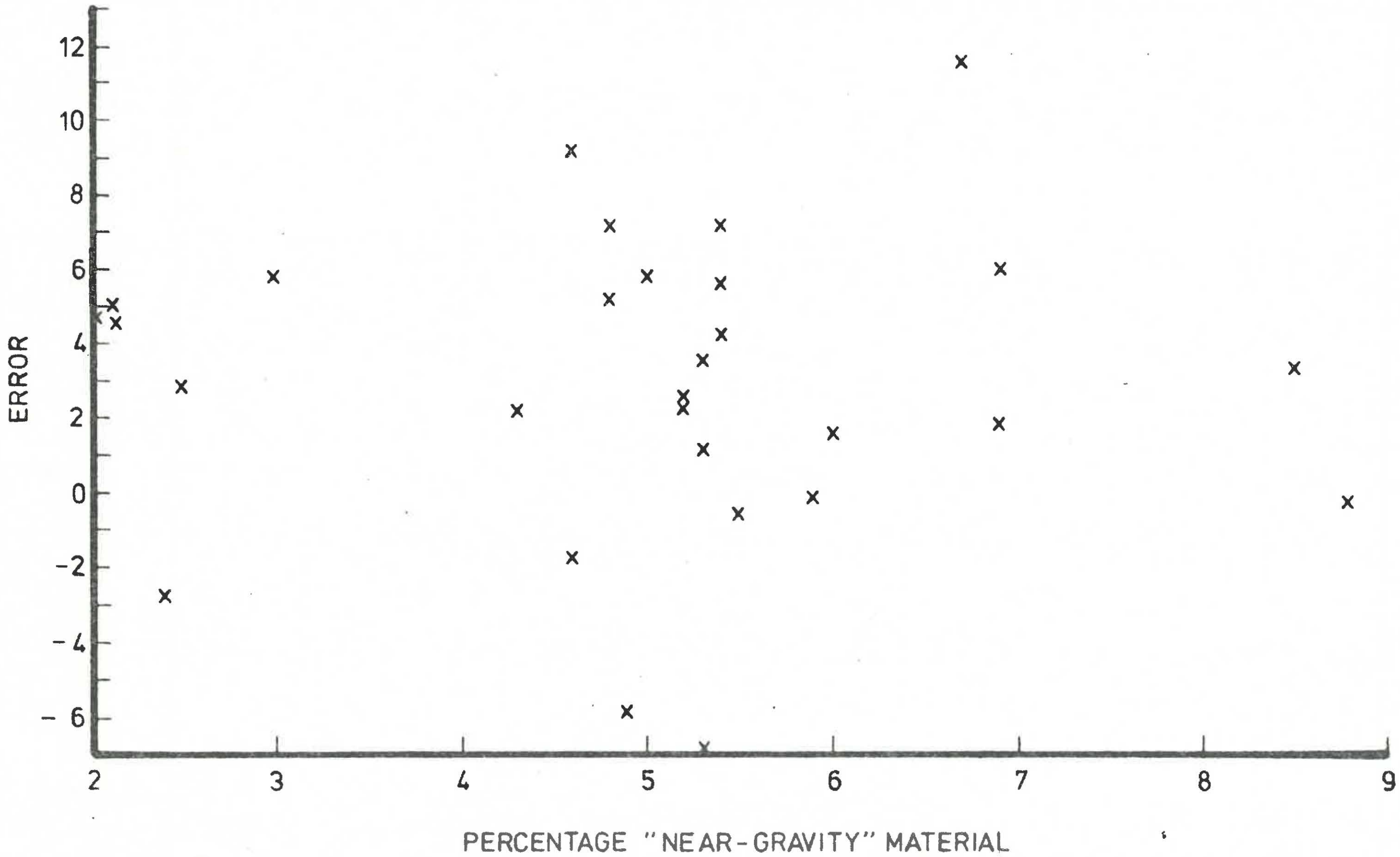

FIGURE 6. ERROR (7TH R.D. INTERVAL) VERSUS NEAR-GRAVITY MATERIAL.

To avoid possible effects due to large variation in the cut-point, only tests within the cut-point interval 1,40 - 1,42 relative density were used. This interval contains the largest number of tests. Errors within the seventh relative density interval (see table 2) were used to take advantage of the maximum spread of the error.

5. The shape of the partition curve changes as the cut-point is changed. The normal tendency is for the separation efficiency to deteriorate as the cut-point increases. The order of magnitude of the deterioration can be had from equations 3,4,5 and 6, and is summarised in Tables 5 and 6.

TABLE 5

VARIATION IN THE SHAPE OF THE PARTITION CURVE WITH CHANGE IN CUT-POINT

Cut=point	Relative density range	Ecart probable	Half Ecart* Mayer	Half E''*
	E'''	E	E'/2	E''/2
1,40 1,48 1,56 1,64	0,167 0,182 0,195 0,206	0,013 0,015 0,017 0,019	0,031 0,036 0,040 0,043	0,046 0,052 0,057 0,062

11/Table 6 ....

*In order to facilitate comparison with the Ecart probable (moyen), the half values of the Ecart Mayer and the parameter, E'', are reproduced.

TABLE 6

ERROR AREA AS A FUNCTION OF THE CUT-POINT

Cut= point	Error area (%)
1,40	1,89
1,48	2,17
1,56	2,39
1,64	2,59

The partition curve tends to become more asymmetrical as the cut-point increases. This tendency can best be illustrated by the ratios $(S_c - S_{75})/(S_{25} - S_c)$ and $(S_c - S_{95})/(S_5 - S_c)$.
The results are reproduced in Table 7. (The relative densities S_5, S_{25}, S_{75}, and S_{95} correspond to the partition coefficients 5, 25, 75, and 95, respectively).

TABLE 7

THE ASYMMETRY OF THE PARTITION CURVE AS A FUNCTION OF THE CUT-POINT

Cut= point	$\frac{(Sc - S75)}{(S25 - Sc)}$	$\frac{(Sc - S95)}{(S5 - Sc)}$
1,40	0,95	0,94
1,48	0,86	0,84
1,56	0,78	0,75
1,64	0,71	0,68

The tendencies illustrated must not be interpreted as hard-and-fast. In fact, it will be shown that contradictory results are possible.

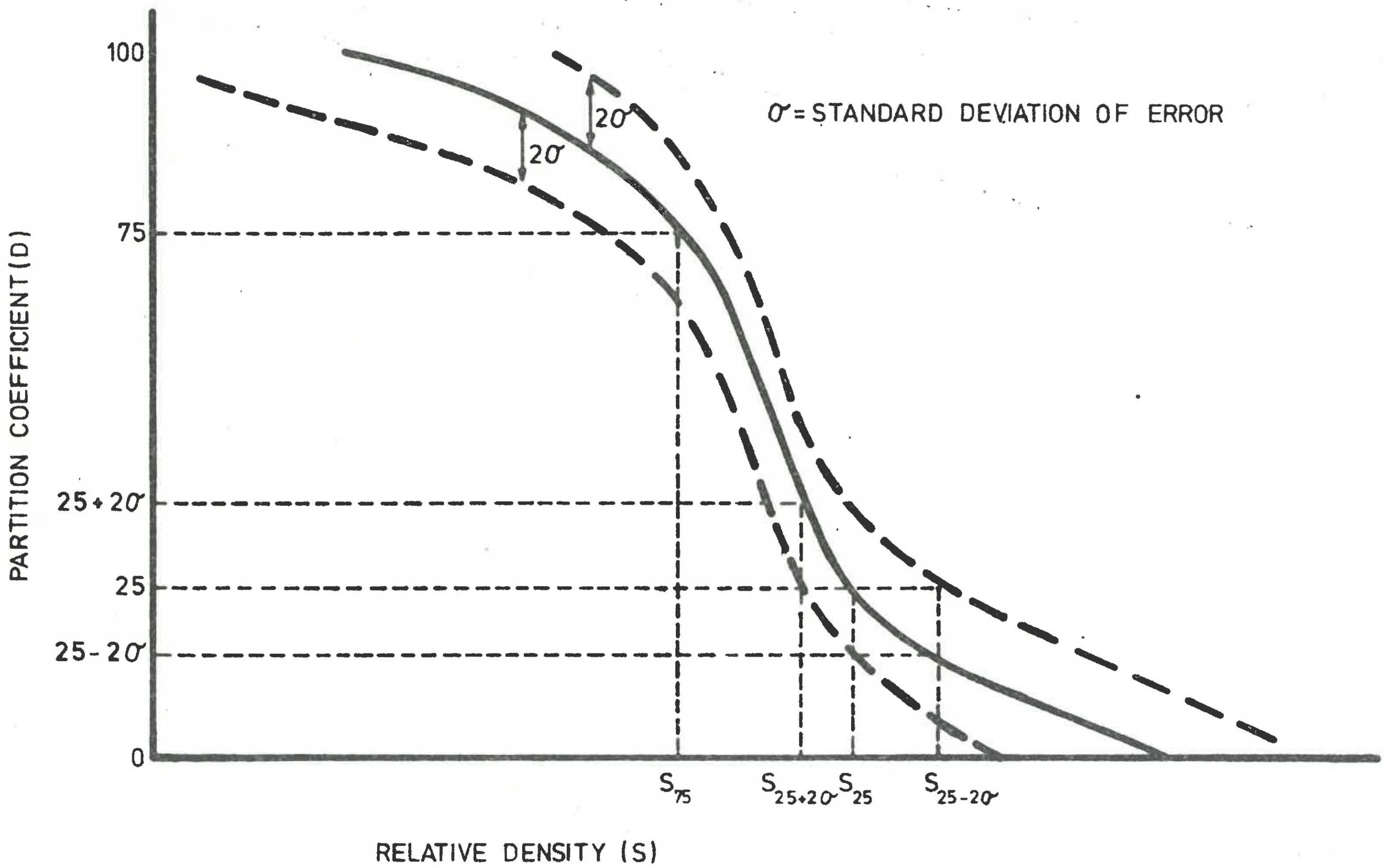
σ = STANDARD DEVIATION OF ERROR
2σ
2σ
100
75
25+2σ
25
25-2σ
0
PARTITION COEFFICIENT (D)
S75
S25+2σ
S25
S25-2σ
RELATIVE DENSITY (S)

FIGURE 7

Ninety-five per cent of the population of the normal distribution ~~curve do not~~ deviate from the central tendency ~~by~~ more than two standard deviations. Extension of this principle to the predicted partition curve allows the definition of a region which will contain the majority of the observed curves. The boundaries of this region are illustrated by the broken lines in Figure 7.

The value of the Ecart probable (moyen) is dependent on the relative densities, S_{25} and S_{75}, corresponding to partition coefficients of 25 and 75, respectively. It can be seen from Figure 7 that both S_{25} and S_{75} can vary relatively widely. For instance, S_{25} can assume any value within the range $S_{25} + 2\sigma$ to $S_{25} - 2\sigma$. By substituting the values of the corresponding partition coefficients into equation 1, the limiting values of the Ecart probable (moyen) can be calculated. The results are reproduced in Table 8. The variation in the Ecart Mayer can be evaluated similarly. For the purpose of comparison, Table 8 also shows the variation in the half Ecart Mayer.

TABLE 8

VARIATION IN ECART PROBABLE (MOYEN) AND THE HALF ECART MAYER AS A FUNCTION OF THE CUT-POINT

CUT=POINT	Variation range	
	Ecart probable	Half Ecart Mayer
1,40	0,009 - 0,018	0,022 - 0,046
1,48	0,010 - 0,021	0,026 - 0,052
1,56	0,012 - 0,023	0,029 - 0,057
1,64	0,013 - 0,026	0,032 - 0,062

It can be seen that it is possible to obtain an Ecart probable, at high cut-points, which is superior to that corresponding to low cut-points. The variation possible in the half Ecart Mayer is

13/generally .....

generally greater than that of the corresponding Ecart probable. The variation in the half value of the parameter, E'', will be even greater.

The cut-point is also subject to variation. However, due to the relatively large slope of the partition curve at the cut-point, its variation is relatively small, as indicated in Table 9.

TABLE 9
VARIATION OF THE CUT-POINT AT VARIOUS LEVELS

Cut= point	Cut-point Variation range
1,40	1,40 ± 0,0025
1,48	1,48 ± 0,003
1,56	1,56 ± 0,0035
1,64	1,64 ± 0,004

6. Conclusions

The equation originally developed to allow the fitting of a smooth curve to observed partition coordinates, permits a short-hand description of washer performance in terms of the constant t_2, and the quasi-constants k, C and t_1.

The data of a hundred and forty cyclone washer performance tests were processed and k, t_1, and t_2 correlated with the cut-point. The correlations were proved sufficiently accurate and were subsequently used to indicate the following:

(i) The performance is independent of both the yield and the amount of "near-gravity" material.

(ii) The separation efficiency deteriorates as the cut-point increases. More specifically, the Ecart probable (moyen), the Ecart Mayer, and the error area all increase as the cut-point is increased.

(iii) The partition curve becomes more assymmetrical as the cut-point is increased.

It was also demonstrated that considerable variation is possible in the Ecart probable (moyen), the Ecart Mayer, and the Parameter E'', and that this is so even under close control, such as is normally possible only in a preparation pilot plant. Furthermore, the variation in the Ecart probable is less than that of the half Ecart Mayer, which in turn is less than that of half the parameter E''. In view of this, the importance of taking the utmost care during acceptance testing of plant cannot be over-emphasized.

PRETORIA.
11th April, 1975.
TCE/mr

T.C. ERASMUS
CHIEF RESEARCH OFFICER.