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Mode, Skewness and Kurtosis of Fibre Distributions and some Practical Applications

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ABSTRACT

By quantifying the non-symmetrical distribution of fibre diameter and length of different wool lots, it has been shown that the measures of the mode, skewness and kurtosis can play a significant role in the spinning performance of yarns and in determining the regularity and strength properties.

INTRODUCTION

Two important properties of wool fibres are their fineness and their length. Fineness used to be expressed using the 'count' system which was based on the length of yarn or number of hanks of standard length that could be spun from a given mass of fibre and staple crimp played an important role in estimating the count [1,2]. Length was often expressed in terms of staple length and reflected how long the wool had been growing before it was shorn, e.g. six months wool or eight/ten months.

Over the years, measurement techniques have developed considerably. The airflow system and the projection microscope are now widely used to measure the diameter of fibres [3]. Many fibres (usually more than three hundred) are measured and their average value is taken as representative of the fineness of the sample. Similarly, the lengths of individual fibres are measured and averaged [4].

For merino wools the average values for fineness and length can be found in the range of about 17 μm to 27 μm and 40 to 100 mm, respectively.

Examination of the data on a large number of fibres (say three hundred) from a sample of wool will reveal that not all the fibres have the same diameter or length. For an average of 20 μm and 70 mm, individual fibres could range between, say 12 μm and 28 μm and 20 mm to 120 mm . These distributions of dimensions are characterised by calculating the standard deviations (s.d.) and hence the coefficient of variation (which is the s.d. expressed as a % of the mean). Values of CV for diameter and length are commonly about 20% and 40%, respectively.

Experience has shown that the CVs vary from wool to wool, e.g. CV's of diameter range, say, from 18% to 26% and of length from 30% to 50%. Although the mean values are of overriding importance, it has also been shown that the actual values of CV play some role in determining the

The distributions can be reduced by the Gauss curve to two numbers, the mean and the CV, and hence their influence on the processing properties of different wools can be ascertained.

The goodness of fit of this curve to the data can be measured by the Chi-squared test. A value for Chi-squared is calculated from the observed values of the number of fibres in each group and their expected value from the equation according to the formula:

$$\text{Chi-squared} = \frac{\Sigma (o - e)^2}{e}$$

where o = observed value
e = expected value.

Because the few values for 'o' in the tails can lead to a large inaccurate chi-squared value, only the data in groups 7 to 14 are used. This sample gave a value for Chi-squared of 146.

Statistical tables for degrees of freedom of 7 (8 groups less 1) give a value of 24,3 at $\rho = .001$. The test says, if the curve really represents the data then we should expect a low value for Chi-squared. However, chance errors can give a high value, a value as high as 24,3 can be expected once in a thousand times. Once in twenty we can accept, and could then take the curve to represent the data. Our Chi-squared of 146 is far too high to accept the notion that the mean and standard deviation are fair statistical representations of the data. The operative word here is "statistical". We are still able to work on the assumption that the mean and CV represent the data, as we have done of for many years.

A different mathematical curve based on the Pearson system of curves has been fitted to the data of Table 1 and gave a Chi-squared value of 33. Again, this was not a good value; the curve was a better fit than the Gauss curve but was still not statistically acceptable. However, for a series of 88 diameter samples, and making the test more strict by accepting observations as few as 10 instead of 20, more than one half were statistically significant at $\rho = .05$. Only two of the Gauss curves had these low Chi-squared values.

These curves based on the Pearson[6,7] system of frequency curves are evolved from the formula:

$$\frac{d(\log y)}{dx} = \frac{x + a}{b_0 + b_1x + b_2x^2}$$

where y is the number of members having a value of x.

In order to calculate the parameters of the Gauss curve the first moment and the second moment, about the mean of the data have to be calculated.

These are:

(1) the mean $= \frac{\Sigma x}{N}$ = 1st moment $= \bar{x}$

(2) (standard deviation)^2 $= \frac{\Sigma (x - \bar{x})^2}{N} =$ 2nd moment $= V_2$

The Pearson system requires the 3rd and 4th moments where:

(3) $\frac{\Sigma (x - \bar{x})^3}{N} =$ 3rd moment $= V_3$

(4) $\frac{\Sigma (x - \bar{x})^4}{N} =$ 4th moment $= V_4$

Because the raw data set is data counted in groups, Sheppard's corrections have to be made. These corrections are made to the 4th and 2nd moments in turn:

$$U_4 = V_4 - \tfrac{1}{2} V_2 + 7/240$$

$$U_3 = V_3$$

$$U_2 = V_2 - 1/12$$

Consequently:

$$\sigma \text{ (standard deviation)} = \sqrt{U_2}$$

Two further terms are derived from these values, namely:

$B_1 = U_3^2/U_2^3$

and $B_2 = U_4/U_2^2$

Now G_1 $(= \sqrt{B_1})$ is called skewness and has a negative value when U_3 is negative and G_2 $(= B_2)$ is called kurtosis.

From B_1 and B_2 we calculate K, where

$$K = \frac{B_1 (B_2 + 3)^2}{4 (4B_2 - 3B_1)(2B_2 - 3B_1 - 6)}$$

The numerical value of K determines the particular curve

If $K < 0$ the curve is type I
If $0 < K < 1$ the curve is type IV
If $K < 1$ the curve is type VI
If $K = 0$ the curve is the Gauss curve or type II or VII;
Type II when $B_2 < 3$ and type VII when $B_2 > 3$.

A few points concerning skewness and kurtosis can be made. Skewness is related to the 3rd moment and kurtosis to the 4th moment. A histogram of Table 1 would show more steps for fibres thicker than the average and fewer for those less than the average. The distribution is not symmetrical. As the Gauss curve is always symmetrical, it can not be entirely suitable. If the long tail of the asymmetrical distribution is to the coarser diameter end then the skewness is positive as is the third moment.

Kurtosis describes the sharpness or breadth of the distribution. The Gauss curve has $B_2 = 3$. If B_2 is greater than 3 the distribution is more sharply peaked and less than 3 reflects a broader curve. The equations for these Pearson curves, using the mean as the origin for mathematical convenience, are given in the Appendix.

This discussion has been in terms of fibre diameter. The curves can similarly be applied to fibre length.

It has been suggested earlier that the Pearson curves give a better fit to the data than the usual Gauss curve. For instance the diameter at the mode is probably a better parameter than the mean to characterise a sample. The CV value obtained from the Gauss curve gives an indication of the spread of results. It is not ideal but it is one measure that is available. The Pearson curves have no equivalent to this standard deviation measurement. One approach to obtain an equivalent is to consider certain attributes of the standard deviation.

Between the mean and one standard deviation below the mean is 68% of the number of fibres whose dimensions are less than the mean and similarly for the area above the mean. The ordinate or y-value or the number of fibres having a dimension of mean less one standard deviation, is 0,61 of the number at the mean. (Note the ordinate $Z = .399$ at $x = 0$ and $Z = .242$ at $x = 1$, for a normalised distribution).

Equivalent parametres from the Pearson curves could be:

(a) the fibre dimensions at 68% of the area below the mode and above it

(b) the fibre dimensions where the number of fibres are 61% of those at the mode, above and below again.

Both (a) and (b) will give a range. For instance, the sample quoted above gave:

mean = 20,2 μm, standard deviation = 4,37 μm and MODE = 18,0 μm

The diameters at 68% areas were 15,5 μm and 22,0 μm and those at 61% of the mode were 14,0 μm and 24,5 μm .

Hence where the Gauss curve gave a range of 2 x 4,37 μm or 8,74 μm the Pearson curve gave 6,5 μm or 10,5 μm .

If a processing problem requires knowledge of the numbers of coarse (or fine, or long or short) fibres then the Pearson curve areas can be determined appropriately. These curves tend to fit the tails of the distributions better than does the Gauss curve.

EXPERIMENTAL

Included in SAWTRI's data bank is information on 88 samples collectively known as BR wools. These were collected for a study on different breeds of sheep and have been previously reported[9]. For this current study the test data for fibre diameter, by the projection microscope method, and for single fibre length of wool tops was used. The number of fibres measured for each sample was up to four thousand, the lowest number being about six hundred.

Physical properties of the fibres in the tops which were considered were:

Mean fibre diameter	X_d
CV of diameter	CV_d
Diameter Mode	M_d
Diameter Skewness	G_{1d}
Diameter Kurtosis	G_{2d}

plus the corresponding fibre length values, identified by the suffix ℓ, and measured by the single length method. Crimp of the raw wool was also included. The number of fibres in the cross-section of the yarn was denoted by Z. Tex was calculated from the formula:

$$Tex = Z.X_d\uparrow 2[1 + (CV_d/100)\uparrow 2]/972$$

Spinning Potential

The objectives of the investigation were to assess the importance of the skewness and kurtosis parameters of fibre diameter and length distributions in explaining observed variations in the mean spindle speed at break (MSS). Could higher spinning speeds be obtained by judicious choice of starting material? This objective was pursued by means of regression equations, the criterion being that if an increased % fit was obtained by a certain model then a better prediction of the spinning performance could be made, the relevant parameters identified and their influence assessed.

The choice of models was guided by previous work[10] the best equation which was based on 251 wool lots giving an 80,9% fit.

The following Table 2 briefly identifies the various starting models used in this investigation and indicates their degree of success by the % fit obtained.

Note that the parameters "means", etc., refer to both diameter and length. Tables 3—7 give the various best fit regression equations and various predictions of MSS.

Yarn Properties

Many of these samples were made into yarns and have been reported[11].

Seven yarn properties, namely:

Irregularity (CV%)
Thin places (per 1 000 m)
Thick places (per 1 000 m)
Neps (per 1 000 m)
Breaking Strength (cN)
Elongation at Break (%)
Hairiness (Hairs/m)

were regressed (log-log) against mean fibre diameter and length and their CV's and crimp for each of three yarn types, S380, S640 and Z610, which had nominal tex values of 50, 50 and 25. These are called "Old" regression models.

The regressions for these data sets were repeated using as independent variables, the Mode, CV, G_1 and G_2 for diameter and for length plus crimp, and are identified as "New" regressions. Significant regression equations were also obtained when various range values were included. These did not give as good or a better fits than G_1 and G_2 and are therefore not discussed further. Table 8 lists the % fits given by the two models for the different yarns and their properties. It will be seen that the new models which incorporate the mode, G_1 and G_2 give, in general, better fits.

Table 9 gives the details of the best fit log-log regression equations for the S380 yarns. The neps property has been excluded because the fit was still low. The hairiness property is not given because the new parameters did not feature. Crimp appeared to be the determining factor for hairiness.

The values of the fibre properties are given in Table 10. Table 11 shows the equations for the linear and interactions model for S380 yarns.

DISCUSSION

Of the 88 diameter distributions, 49 were type I, 16 type IV and 23 were type VI. All had positive tails i.e. positive third moments. Hence the mode was always less than the mean. On average it was 1,75 μm lower ranging from 0,6 μm to 3,2 μm lower. The range of skewness was from 0,1 to 1,3, the average being 0,7. Average kurtosis was 3,9, ranging from 2,8 to 6,8. The mean diameter was 22,9 μm .

The fraction of fibres finer than the mode was, on average, 36%.

Eightynine length distributions were analysed. Seventyfour were of Type I, 10 of Type II, 3 of Type VI and 1 each of Types VI and VII. Sixtyone had negative tails. The position of the mode could be up to 29 mm longer or 16 mm shorter than the mean. By way of illustration, three wools whose mean lengths were 78 mm, had their modes at 69, 88 and 100 mm, respectively. The skewness ranged from −0,8 to +0,5, the average being −0,1. Kurtosis averaged 2,6 ranging from 1,9 to 4,0. The average length was 93 mm .

The fractional area shorter than the mode ranged from 40% to 65%.

Spinning Potential

Computational restrictions imposed a limit on the number of terms which could be considered in a starting model. It is possible that not all useful interactions have been considered. For instance, no interactions with crimp were used. However, the regressions enable the usefulness of the influence of skewness and kurtosis to be assessed.

The log-log regressions tended to give an inferior fit to the data compared with the linear regressions, as the Table 2 shows.

This table also indicates that the "modes" models tend to be marginally better than the "means" models, although the differences in fit are not large.

TABLE 2

% FITS OF VARIOUS MODELS

Model	Parameters	Linear	Log-log
1 & 5	Z + means	75,2	67,4
3 & 7	Z + Modes	75,6	71,5
2 & 6	Tex + Means	71,6	68,8
4 & 8	Tex + Modes	69,7	71,4

Linear Models, CV versus Modes, etc.

Table 3 gives the coefficients of the significant terms for regressions 1,2, 3 and 4.

Models 1 and 3 show that longer wools with more fibres in the yarn cross-section can be spun at higher speeds. The influence of fibre diameter and its distribution is more complex.

TABLE 3

COEFFICIENTS OF LINEAR REGRESSION EQUATIONS FOR MODELS 1, 2, 3 AND 4 FOR MSS

Parameter	Model 1		Model 2		Parameter	Model 3		Model 4	
	Coeff.	% Contribution	Coeff.	% Contribution		Coeff.	% Contribution	Coeff.	% Contribution
Z	590	26			Z	473	28		
Tex			857	38					
X_ℓ	264	16	174	15	$M_d.Z$	−12,1	9		
X_d	NS		−464	7	CV_d	−66,9	1	−149	1
$Z.X_d$.	−16,1	10			G_{2d}	−2124	6	NS	
$Tex.X_d$.			−12,2	4	$M_\ell.\ CV_\ell$	0,9	25	1	11
$Tex.X_\ell$			−3	3					
$X_\ell.CV_d$	−9,5	9	−1,5	1	$M_d.G_{2d}$	96,2	5	−122	17
CV_d	22,5	6	NS		M_dG_{1d}	NS		−141	1
					$Tex.M_d$			−8.8	9
					$Tex.G_{2d}$			143	29
Crimp	−438	7	−410	3	Crimp	−298	3	−250	1
Constant	−10981		5797		Constant	3260		16747	
% fit		75,2		71,6	% fit		75,6		69,7

The equation of model 1 suggests that short coarse wools give higher MSS values if the CV is high, even though a high CV usually worsens the MSS. The equation of model 3 suggests that increasing the diameter of wools, or their diameter kurtosis, will lower the MSS. However, when high values for both mode and kurtosis occur together in a wool, a better MSS can be obtained. The following tables (4 and 5) calculated from these equations illustrate the effects.

Regression 1 suggests that coarse (23 to 28 μm), short (50 mm) wools having a high CV of diameter (26) are odd in that, contrary to expectations they give a higher MSS. Similarly, regression 3 suggests that coarse wool of high diameter kurtosis (5,5) will give a higher MSS. To reconcile these findings is not easy. It is suggested that although a high CV of diameter is loosely related to a low kurtosis (breadth of distribution), in fact the opposite appears to be the case. Examination of the predictions from the equation previously published[10] shows that an increase in CV of diameter from 18 to 26 will increase the MSS by about 1 200 rev/min for wool of 50 mm length but will descrease it by 300 and 1 800, respectively, at 70 mm and 90 mm length. The effect therefore appears to be consistent. On the other hand, examination of the experimental data shows a sparsity of data points in these regions. A suitably designed experiment involving artificial blends could perhaps resolve this apparent anomaly.

TABLE 4

VALUES OF MSS PREDICTED FROM MODEL 1 REGRESSION

CV_d		18	26
Diameter	**Length**		
18μm	50mm 90mm	11 260 14 980	10 700 11 380
23μm	50mm 90mm	10 060 13 780	10 400 11 080
28μm	50mm 90 mm	9 350 13 070	10 230 10 907

Calculated at Z = 40

TABLE 5

VALUES OF MSS PREDICTED FROM MODEL 3 REGRESSION

Kurtosis of diameter	2,5	5,5
Diameter Mode 15	12 373	10 340
Diameter Mode 20	11 166	10 566
Diameter Mode 25	9 948	10 792

Calculated at Z = 40, CV_d = 22, CV_ℓ = 40, crimp = 3,9, M_ℓ = 50.

Log-log Models, Means and CV versus Modes, Skewness and Kurtosis

Table 6 gives the coefficients of the significant terms in the regression equations for models 5 to 8

TABLE 6

COEFFICIENTS OF LOG-LOG EQUATIONS FOR MODELS 5—8 FOR MSS

Parameter	Model 5		Model 6		Model 7		Model 8	
	Coeff.	% Contribution	Coeff.	% Contribution	Coeff.	% Contribution	Coeff.	% Contribution
Z	0,815	49			0,773	54		
Tex			0,811	34			0,758	32
X_d			1,837	25				
M_d					−0,081	1	−1,402	27
X_ℓ	0,487	18	0,464	8				
M_ℓ					0,355	14	0,340	7
CV_d			−0,272	1			−0,429	3
$G_{1\ell}$					0,088	4	0,099	3
Crimp			0,114	1				
Constant	1,857		5,034		2,281		4,846	
% fit		67,4		68,8		71,5		71,4

The number of fibres in the cross-section and the length of the wool are of major importance in determining the MSS; more of each give higher MSS. High length skewness also contributes to higher MSS values. Thus, of two samples having the same length mode and spun to the same number of fibres in the cross-section, the sample having a positive skewness of length (relatively more long fibres) will give a higher MSS than the one with a lower skewness. A 10% increase in MSS can be expected for a change in $G1_\ell$ from −0,5 to +0,5.

Models 6 and 8 which replaced Z by Tex, correspond very closely to models 5 and 7 when the relation between Z and Tex, is considered.

The effect on MSS of changing the values of the parameters in these log-log regressions is illustrated by the following table 7.

TABLE 7

CHANGE IN MSS FOR GIVEN CHANGES IN FIBRE PROPERTIES

Property	Change from	Increase in MSS
Mean diameter or mode	25 to 20 μm	50% or 40%
Mean length or mode	50 to 90 mm	30% or 20%
CV diameter	25 to 20	about 8%
Crimp or Length Skewness	7 to 3 −0,5 to +0,5	10% 10%
Tex	10 to 30	about 140%

The skewness of length factor shows that an improvement in MSS can be made by having a positively skewed length distribution. This can be explained by the fact that more long fibres supplement the effect of fibre length. Thus if a small quantity of a longer wool lot is blended with a shorter wool lot a better spinning performance should be obtained. This suggests that the principle of a long fibre carrier as used in short staple processing can also be useful in the worsted processing system. It will be noted that the linear equations and log-log equations are not consistent. However, the new parameters do appear in both models.

Yarn Properties

Table 8, which gives the % fits of the best log-log equations, shows that inclusion of the skewness and Kurtosis terms gave better fits than means and CVs alone. Hence these must be relevant properties to consider. Further, linear models plus certain interaction terms, gave even better fits for the S380 yarn.

Table 9 gives the coefficients of the log-log regressions and % contribution of the independent variables for the S380 yarns while Table 10 shows the linear plus interactions equations.

TABLE 8

% FIT OF OLD MODELS AND NEW MODELS (LOG-LOG) INCORPORATING ROOT G_1 and G_2

Yarns	S380 50 tex			S640 50 tex		Z610 25 tex	
Regression Models	Original	New	Multi linear	Original	New	Original	New
Yarn Properties: Irregularity	79,8	84,5	85,1	86,0	89,6	89,9	88,7
Thin places	58,6	62,1	78,7	59,0	67,4	84,4	83,5
Thick places	56,8	61,0	68,4	55,9	60,2	77,1	78,0
Neps	16,9	32,8	35,6	15,7	15,7	26,8	47,3
Breaking Strength	48,9	58,6	64,3	56,7	61,7	70,8	68,6
Extension	66,1	68,1	73,1	76,5	77,1	69,0	71,1
Hairs	79,5	77,8	72,9	64,0	60,0	88,8	87,3

Irregularity

The following equations were found for irregularity:

Irregularity = 0,94 diameter$^{.863}$ x crimp$^{.097}$ (79,8% fit)
and Irregularity = 1,11 diameter mode $^{.888}$ x diameter skewness$^{.121}$. (84,5% fit)

Interpreting these equations we can say that the mean or mode play similar roles. Of more interest, however, is the second term, crimp and diameter skewness. The first equation suggests that more crimpy wools, of the same diameter give worse irregularity. At the mean diameter, increasing crimp from 2 to 7,5 change the irregularity from 14,9 to 16,9, not a large effect. The second equation suggests that the skewness of the diameter distribution is more important than crimp; crimp was rejected in favour of G_1. At the average

TABLE 9

S380 YARNS LOG-LOG, REGRESSION EQUATIONS-COEFFICIENTS AND THEIR % CONTRIBUTION TO THE FIT

Yarn Property	Diameter					Crimp	Length					A_0	%
	Mean	Mode	CV	G_1	G_2		Mean	Mode	CV	$G_1 + 1$	G_2		
Irregu-larity	0,863 75%	—		—	—	0,097 5%		—		—	—	−0,028	79,8
	—	0,888 75%		0,121 9%			—					0,046	84,5
Thin Places	16,214 50%	—		—	—	2,843 8%		—		—	—	−22,895	58,6
	—	17,463 42%		3,219 9%	−3,914 4%	2,188 6%	—			—		−20,753	62,1
Thick Places	6,356 54%	—		—	—		−1,429 3%	—	—	—	—	−4,776	56,3
	—	8,432 48%		1,416 9%			—	−1,255 4%		—		−7,323	61,0
Breaking Strength	−0,798 34%	—	−0,295 4%	—	—	−0,220 11%		—		—	—	4,100	48,9
	—	−0,772 37%		−0,134 7%			—	0,207 10%		—	0,176 4%	3,061	58,6
Extension	−2,269 42%	—		—	—	−0,665 17%			−0,660 7%	—	—	5,764	66,1
	—	2,716 3.5%		−0,380 8%		−0,613 17%	—		−0,752 8%	—		6,323	68,1

NOTE: Blanks in the columns indicate that that property did not contribute to the best equation. The "dash" indicates that the property was not included in the model.

diameter mode, a skewness change from 0,28 to 1,49 increased the irregularity from 14,3 to 17,5. The correlation between crimp and diameter skewness in this data set was 0,5 which should not cause confusion.

Hence the results from the log-log models suggest that although at first sight it appeared that crimp could affect irregularity, it is more likely to be due to the influence of the distribution of the fibre diameters. The more skewed the distribution, the more coarse fibres present, the worse is the irregularity. However, the linear plus interactions model (see Table 12) showed that both diameter skewness and crimp can affect the irregularity of a yarn.

Thin Places

The characteristics of the distribution of fibre diameter affect the number of thin places, length was not important. From the equation given in Table 9

TABLE 10

MEANS AND STANDARD DEVIATION OF THE LOGS OF DATA VALUES OF THE INDEPENDENT VARIABLES

Property	Mean	Standard Deviation
Diameter:		
Mean	1,355	0.065
Mode	1,325	0.070
CV	1,337	0.039
G_1	−0,190	0.181
G_2	0,577	0.143
Crimp	0,591	0.143
Length:		
Mean	1,845	0.065
Mode	1,877	0,082
CV	1,608	0,055
$G_1 + 1$	−0,113	0,163
G_2	0,401	0,064

NOTE: Because some of the G_1 values for the length data were negative, one was added to the value before its log was taken. All the diameter values had positive tails; positive G_1 values.

the following equation represents the number of thin places at the average diameter (mode) of 21 μm and crimp of 3,8.

Thin places = $4771 . G_{1d}^{3,219} G_{2d}^{-3,914}$

Low skewness and high kurtosis give very few thin places, while a highly skewed distribution having low kurtosis will give about 380 thin places for this S380 yarn.

Thus a widely spread distribution of diameter with a coarse tail will give far more thin places than a narrow symmetrical distribution. Perhaps this is related to the "coarse edge" which is a term used in the trade.

TABLE 11

EQUATIONS FOR "LINEAR AND INTERACTIONS" MODELS FOR S380 YARNS: COEFFICIENTS OF SIGNIFICANT TERMS AND THEIR % CONTRIBUTION

	Irregularity	Thin	Thick	Breaking Strength	Extension
M_d	0,612 72%	18,33 25%	9,27 60%	−14,2 44%	−0,56 3%
$M_d.G_{1d}$	0,715 10%	−25,4 18%		−3,41 7%	−2,3 25%
Crimp	0,297 2%			−13,8 8%	−2,46 19%
G_{1d}		859 19%	55,9 9%		39,5 19%
$G_{1d}.G_{2d}$		−70,7 10%			
$M_d.G_{2d}$		3,1 7%			
$G_{1\ell}$					−3,53 4%
$M_d.M_\ell$				0,031 6%	
$G_{2\ell}$					2,34 3%
Constant	−0,84	−652	−209	691	38,3

Thick Places

The model which includes G_1 and G_2 terms gave:

$$\text{Log(Thin Places)} = 8{,}342\ \text{Mode}_d - 1{,}255\ \text{Mode}_\ell + 1{,}416\ G_{1d} - 7{,}323$$

The influence of diameter and length is as in earlier models which showed that coarse short wools can cause many more thick places than fine long wools. The skewness of the diameter distribution also affects the number of thick places; the coarser diameter tail can increase the number from 5 to 50 at the average mode values.

Breaking Strength

The influence of each independent variable on breaking strength is illustrated in the following Tables 12 and 13 by considering the lowest and highest values of a variable while the others are held at their mean values.

TABLE 12
PREDICTED VALUE OF BREAKING STRENGTH FOR DIFFERENT LEVELS OF THE INDEPENDENT VARIABLES

Level of Property	Mean Diameter	Crimp	CV of Diameter
$\bar{x} - 2\sigma$, low	421	384	349
$\bar{x} + 2\sigma$, high	261	287	316

Fine, low crimped wools having a low CV gave the best breaking strength values.

The equation having a higher fit is illustrated by the following values:

TABLE 13
PREDICTED VALUE OF BREAKING STRENGTH FOR DIFFERENT LEVELS OF THE INDEPENDENT VARIABLES

Level of Property	Diameter Mode	Length Mode	Diameter Skewness (G_{1d})	Length Kurtosis ($G_{2\ell}$)
$\bar{x} - 2\sigma$, low	427	308	298	316
$\bar{x} + 2\sigma$, high	260	360	373	351

Fine wools with few coarse fibres (low skewness) gave the better breaking strength as did long wools from a sharp (high kurtosis) length distribution. Coarse wools with more coarse fibres in the tail plus short wools of broad distribution gave a low breaking strength.

A lower diameter skewness would give more fibres in the cross-section and more friction while a higher length kurtosis would suggest a better interlocking during twisting by fibres having a more uniform length and hence produce a stronger yarn.

Extension at Break

Greater extension is obtained from fine, low crimped wools having a low CV of length. The introduction of skewness and kurtosis into the model revealed that diameter distributions which have a longer positive tail adversely affected the extension at break. The equations suggest that, at the mean value for the other variables, the effect over the range of skewness in the data is to halve the extension (about 23% to 12%).

A multilinear model with interaction terms gave an improved fit (73,1% c.f. 68,1% for log-log) and the effect of skewness was similar (about 21% to 11%). Note, the measured extensions in the data spanned the range 8% to 28%.

SUMMARY AND CONCLUSIONS

Wool fibres in a lot vary over a wide range of diameter and length. For many years the Normal or Gauss curve has been used to characterise the distribution of diameter and length in a wool lot, the mean and coefficient of variation being the appropriate measures. The Gauss curve represents a symmetrical distribution while very few of the wool lots show this symmetry. Hence the Pearson system of frequency curves have been considered. These are more general, non-symmetrical curves of which the Gauss curve is a special case. They are characterised by the values for the mean and second, third and fourth moments of the distribution. The latter three terms combine to give a measure of skewness (non-symmetrical tail) and kurtosis (sharp or flat distribution). In this work detailed procedures are given to determine (a) the skewness and kurtosis of the fibre diameter and length distribution and (b) the parameters of the appropriate Pearson curve which fits these experimental data better than does the more usual Normal or Gauss curve.

This work has shown that mode, skewness and kurtosis of the diameter and length distributions play a role in determining the spinning potential of these particular wool lots. The diameter properties tend to decrease, and length properties to increase the spinning potential. They also help to account for variations in yarn properties.

Of the new distribution parameters considered for a 50 tex S380 yarn structure, the analysis showed that the irregularity, thin and thick places, the breaking strength and extension were each influenced to varying degrees by the mode skewness and kurtosis.

The results of this study indicate that a positive tail in the diameter distribution (coarse edge) generally has an adverse effect on spinning performance and yarn properties while a positive tail in the fibre length distribution (i.e. a relatively higher proportion of long "carrier" fibres) tended to have a beneficial effect.

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APPENDIX

The following equations are based on Elderton and Johnsons[7] treatment.

Type I

$$y = y_o (1 + x/A_1)\uparrow M_1.(1 - x/A_2)\uparrow M_2$$

The curve extends from $-A_1$ to $+ A_2$

NOTE: the symbol ↑ is used to denote the raising to a power, e.g. x↑2 means x to the power 2.

Type VI is somewhat similar:

$$y = y_o (1 + x/A_1)\uparrow -Q_1. (1 + x/A_2)\uparrow Q_2$$

The curve extends from $A_1 - A_2$ to infinity.

If U_3 is negative, the curve extends from minus infinity to $A_1 - A_2$.

Type IV

$$y = y_o 1 + (^{X}/A - V/R)\uparrow 2\uparrow(-M).e\uparrow(V\tan^{-1}(^{X}/A - V/R)) \ldots (1)$$

The curve has unlimited range in both directions.

Type II

$y = y_o [1 - (^{X}/A)\uparrow 2]\uparrow M$ and the range is between $-A$ and $+A$..(2)

Type VII

$y = y_o [1 + (^{X}/A)\uparrow 2]\uparrow (-M)$ and has unlimited range ....(3)

Preliminary calculations are required from the test data before the particular curve parameters are determined.

Calculate U_2, U_3 and U_4 (which include Sheppard's corrections). Use values of μm or mm and not the group number. Calculate B_1 B_2 and K as indicated above.

$$\text{Calculate } R = 6\,(B_2 - B_1 - 1)/(6 + 3B_1 - 2B_2).$$

Some values of B_2 and B_1 can give a very large value for R. This occurs when B_2 is very nearly equal to three. Now B_1 and B_2 are subject to experimental error. If B_1 equals approximately zero and B_2 approximately 3 then values of 0 and 3 should be used and hence K = 0 gives the Gauss curve. To check the values of B_1 and B_2 in this respect statistical tables are used[8]. This process can be incorporated into a calculation programme as an automatic feature by taking the 5% point from the tables for root B_1 or G_1 and for B_2 together with the N (number of fibres) and obtaining log regressions.

These are:

$$\text{absolute } G_1 = 0{,}416 - 0{,}0425 \log_e N \qquad r = 0{,}987$$
$$\text{absolute } (B_2 - 3) = 0{,}863 - 0{,}0885 \log_e N \qquad r = 0{,}982$$

Values of G_1 less than about 0,01 and of B_2 between about 2,8 and 3,2 are assumed to be zero and 3 respectively, i.e. a Gauss curve.

If root B_1 or G_1 was not significantly different to zero but B_2 was significantly different to 3, then curves or Type II or VII would be applicable; Type II for B_2 less than 3 and Type VII for B_2 greater than 3. This differentiation between types II and VII and Gaussian is justified on the grounds that lower Chi-squared values are obtained by their use.

For non-zero values of K, the appropriate Pearson curve parameters are determined.

Type I

Condition $K < 0$

$$A_1 + A_2 = \tfrac{1}{2}\,\text{SQR}\,[U_2\,(B_1(R + 2)^2 + 16\,(R + 1)\,)]$$

$$M_1 \text{ and } M_2 \text{ from } \tfrac{1}{2}\left\{R - 2 \pm R\,(R + 2)\,\text{SQR}\left[B_1/\left(B_1(R + 2)^2 + 16\,(R + 1)\right)\right]\right\} \quad (4)$$

M_2 is the positive root when U_3 is positive.

$$\text{MODE} = \text{MEAN} - 1/2.U_3/U_2.(R + 2)/(R - 2)$$

$$(M_1 + 1)/A_1 = (M_2 + 1)/A_2 = (M_1 + M_2 + 2)/(A_1 + A_2) = R/(A_1 + A_2) \; \ldots. (5)$$

The final term to be calculated is y_o, the expected number of fibres at the mean. This calculation is somewhat involved; GAMMA (T) functions are used. The expression for y_o is:

$y_o = N/(A_1 + A_2).(M_1 + 1)\uparrow M_1.(M_2 + 1)\uparrow M_2/(M_1 + M_2 + 2)\uparrow(M_1 + M_2).G$
where $G = T(M_1 + M_2 + 2)/T(M_1 + 1)/T(M_2 + 1)$

Because the various parts of this expression can involve very large numbers it is better to evaluate y_o in log terms.

An adequate approximatation for the GAMMA function is:

$$\log_{10} T(x + 1) = \log_{10}\sqrt{2}\pi + (x + .5)\log_{10}x - (x - 1/12\,x)\log_{10}e$$

The error when x = 2,4 is about 0,5%; at 4,4 the error is 0,004%.

Type IV

Condition $0 < K < 1$

Put $R = -R$
$M_1 = (R + 2)/2$
$V = -R(R - 2)SQR[B_1/(16(R-1) - B_1(R - 2)\uparrow 2)]$
$A = SQR\,[U_2/16(16(R - 1) - B_1(R - 2)\uparrow 2)]$

If V and U_3 have the same signs, both positive or both negative then the sign of V is reversed.

Y_o is calculated from:-

$$Y_0 = \frac{N}{A.F\,(R,V)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (6)$$

where $F(R,V) = e\uparrow(-V\pi/2).\,G(R,V)$

and $G(R,V) = \int_0^{\pi} \sin^R\theta.\,\theta^{v\theta}.d\theta$

Summatation by computer in steps of 0,1 radians is adequate.

$$\text{i.e. } \log F(R,V) = \sum_{0,1}^{3,1} [R.\log_{10}\sin\theta + \log_{10}(e\uparrow V(\theta - \pi/2))] - 1 \quad ..(7)$$

MODE = MEAN − 1/2. U_3/U_2. (R − 2)/(R + 2)

Type VI

Condition $K > 1$
$Q2 = (R - 2)/2 + R(R + 2)/2\ SQR\ [B_1/(B_1(R + 2)\uparrow 2 + 16(R + 1))]$
$Q_1 = Q_2 + 2 - R$
$A = 1/2\ SQR\ [U_2.(B_1(R + 2)\uparrow 2 + 16(R + 1))]$ .... (8)

If U_3 is negative then change the sign of A to negative.

$A_1 = A\ (Q_1 - 1)/(Q_1 - Q_2 - 2)$
$A_2 = A\ (Q_2 + 1)/(Q_1 - Q_2 - 2)$
$Y_o = N/A\ (Q_2 + 1)\uparrow Q_2(Q_1 - Q_2 - 2)\uparrow(Q_1 - Q_2)/(Q_1 - 1)\uparrow Q_1.G$
where $G = T(Q_1)/T(Q_1 - Q_2 - 1)/T(Q_2 + 1)$
$MODE = MEAN - 1/2.U_3/U_2.(R + 2)/(R - 2)$

GAUSS Curve

$B_1 = 0;\ B_2 = 3\ ;\ K = 0$
$Y_o = N/(2\pi U_2)^{0,5}$ .... (9)
$Y = Y_o.e\uparrow(-\ x\uparrow 2/2U_2)$ – for origin at the mean

Type II

Condition $B_1 = 0$, B_2 less than 3.
Calculate:

$M = (5B_2 - 9)/2\,(3 - B_2)$
$A^2 = 2\ U_2B_2/(3 - B_2)$
$Y_o = N/A/\sqrt{\pi}\ .\ T.(M + 1,5)/T(M + 1)$

The curve extends from minus A to plus A, about the mean.

Type VII

Condition is $B_1 = 0$, B_2 greater than 3.
Calculate:

$M = (5B_2 - 9)/2/(B_2 - 3)$
$A^2 = 2U_2B_2/(B_2 - 3)$
$Y_o = N/A\sqrt{\pi}.T.(M)/T(M - 1/2)$

The curve has unlimited range in both directions.

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