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FUEL RESEARCH INSTITUTE
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I N D E X

FUEL RESEARCH INSTITUTE OF SOUTH AFRICA

REPORT NO. 69 OF 1977

STREAM TRANSPORT OF PARTICLES IN FULL SUSPENSION. THE ENERGY DISSIPATION FUNCTION AS AN INVARIANT OF THE CONCENTRATION

SUMMARY

The friction factor (or the Chezy coefficient) of a stream carrying a fully suspended load of particles is analysed.

Friction factors (or Chezy coefficients) of the suspension coincide with those of the pure liquid, if referred to the actual fluid velocity, but are less than those relative to the cumulative velocity, i.e. less than the friction factors pertinent to a volumetrically equivalent stream of pure liquid.

In the case of small particles the stated difference between the two values of the friction factor becomes negligible.

SCOPE OF THE INVESTIGATION

Streams carrying suspended particles are common phenomena of our physical world.

In the very small range of particle sizes and settling velocities, the stream can suspend particles almost ideally, i.e. the different phases in flow behave as an almost homogeneous fluid.

To this group of suspensions belong emulsions, fogs, smoke, turbid water, etc.

For greater particle sizes the settling velocity increases, although still remaining inside the laminar regime.

Particles of this kind are easily transported by streams, but settle if not continuously supported by the fluid turbulence.

To this group of suspensions belong slurries, sand storms, natural streams when carrying sand, etc.

A further increase in particle diameter produces settling velocities lying inside the region of the turbulent regine.

This kind of particles become fully suspended only at high transport velocities.

High velocity streams with suspended particles often find industrial application in solid transport either by air or by water.

It is the purpose of this report to investigate the aspect of energy-dissipation of these solid liquid systems, having particles fully suspended.

Anticipating a rather surprising result, such systems dissipate the same energy as the fluid above would do, if streaming without particles.

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In other words the conveyance of particles is done at no extra energy cost.

However, this free transport condition is only apparently favourable, because suspending velocities may be so high as to make the energy dissipated excessive, i.e. unbearable from an economical point of view.

It is the purpose of this report to discuss certain aspects of the mechanics of these streams and to produce results which agree with the experimental evidence, as provided by the technical literature on the subject.

1. INTRODUCTION

In this paper some hydraulic phenomena are discussed which are relevant to the hydraulic transport of particles in a fully suspended state.

If a liquid, e.g. water, flows in a horizontal pipe with a sufficiently high velocity, the solid particles are conveyed as fully suspended.

A case of particular interest is that of particles of a density close to that of the liquid.

In such systems the effects of the force due to gravity are practically removed and the hydraulic phenomena of interest become more accessible to investigation.

Experiments using such almost neutral particles have been described by C Elata and T Ippen (ref. 1) for open channel flow by J W Daily and T K Chou (ref. 2), by C P Roberts and J F Kennedy (ref. 3) and by G K Batchelor, A M Binnie and O M Phillips (ref. 4) for pipes respectively.

In the present work some of their results are reported and used to explain the hydraulic phenomena of interest.

Using capital letters to express average quantities relative to the stream, let us denote with:

Q, the total flow rate (solid plus liquid)
V, the mean velocity of the mixture
Q_w, the flow rate of the liquid phase (water)
V_w, the mean velocity of the liquid phase (water)
Q_p, the flow rate of the solid phase (particles)
V_p, the mean velocity of the solid phase (particles)

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Moreover introducing a friction factor

f' is relative to the mean velocity of the mixture V (solid plus liquid),

f_w is relative to the mean velocity of the liquid phase V_w (water),

expressing with g the acceleration due to gravity, with d and D the particle and pipe diameters, one can write the Darcy Weissbach equation and express the hydraulic gradient i in two equivalent forms:

$$i = f' \frac{V^2}{2gD} \text{ relative to the flow of the mixture} \qquad (1)$$

$$i = f_w \frac{V_w^2}{2gD} \text{ relative to the flow of the liquid phase.} \qquad (2)$$

Consequently for the same hydraulic gradient i measured, two different friction factors can be defined in function of the selected stream velocities V and V_w respectively as per equs (1) and (2).

In Figures 1, 2, 3 and 4 are represented the experimental results obtained by the above-mentioned authors in a plot f_w, RE_w, where

$$RE_w = \frac{V_w D}{\nu} \qquad (3)$$

is the Reynolds number of the pipe, referred to the mean velocity of the liquid phase (water) and ν is the kinematic viscosity of the pure liquid (water), at the temperature of the experiment.

In Figures 1 and 2 the reduction of the experimental results to RE_w has been carried out by the author, while Figures 3 and 4 are reproductions of the original graphs of Robert and Kennedy.

In Figure 1 the friction factor of the channel is given as a Chezy coefficient ($C_w/g^{\frac{1}{2}}$), i.e. referred to the velocity of the liquid.

The main parameters of interest of the various experiments reported have been grouped in Table 1.

From Figures 1, 2, 3 and 4 it appears that the experimental points fall according to the representation adopted along the clear liquid line (water), i.e. the friction factor f_w (or $C_w/g^{\frac{1}{2}}$) relative to a clear liquid velocity V_w, is the same as the experimental friction factor represented by the points.

This statement is valid for the results of Figure 1 only in a first degree approximation.

Let us also emphasize that the stated coincidence is not affected (in the regions of the graph where it exists) either by changes in volumetric concentration x or by variation in the particle/pipe diameter ratio $\frac{d}{D}$, i.e. experimental points fall along the clear liquid locus irrespective of variations of these two quantities.

2. THE EQUATION OF CONTINUITY FOR THE FLOW OF A SOLID SUSPENSION

With the notation already established one can express the flow rate as the sum of the two partial flow rates i.e.

$$Q = Q_w + Q_p \tag{1}$$

Introducing the discharge concentration x, defined as the volume of particles present in a certain volume of mixture collected, let us write:

$$Q_p = xQ \tag{2}$$

$$Q_w = (1-x)Q \tag{3}$$

Further, if we express the total pipe area as A and the areas occupied by the liquid and by the particles with A_w and A_p respectively, we get:

$$A = \frac{Q}{V} \tag{4}$$

$$A_w = \frac{Q_w}{V_w} \tag{5}$$

$$A_p = \frac{Q_p}{V_p} \tag{6}$$

with

$$A = A_w + A_p \tag{7}$$

hence one gets:

$$\frac{Q}{V} = \frac{Q_p}{V_p} + \frac{Q_w}{V_w} \tag{8}$$

The elimination of Q_p and Q_w from (2), (3) in (8) yields:

$$V_w = V\left(\frac{V_w}{V_p}x + 1 - x\right) = V(1 - \alpha x) \qquad (9)$$

where one has put:

$$y = \frac{V_p}{V_w} \qquad (10)$$

and

$$1 - \frac{1}{y} = \frac{y - 1}{y} = \alpha \qquad (11)$$

y is related to the "linear concentration" x_ℓ inside the pipe, by the following

$$\frac{x}{y} = x_\ell \qquad (12)$$

where for y = 1, i.e. for $V_w = V_p$: $x_\ell = x$

For a prefixed value of x_ℓ, the distribution of the solid phase may vary, for instance particles may proceed uniformly distributed or in a more or less centered pattern and still satisfy the condition of continuity.

Considering now a mixture, which for a prefixed volumetric concentration x collected at the discharge and a total flow rate Q, is such that

$$V = \frac{Q}{A} = \text{constant} \qquad (13)$$

one gets from equ (9)

$$\frac{V_w}{V} = 1 - \alpha x = K_r \qquad (14)$$

being K_r a parameter.

Equs (12) and (13) state that for certain prefixed flow rates Q, Q_w, Q_p, (i.e. for a prefixed solid concentration x_1 collected at the discharge) many velocity profiles are possible, all satisfying the equation of continuity and corresponding to the values:

K_1^1, K_1^2 ... K_1^n, this for $x = x_1$.

The same argument can be repeated for other values of solid concentration x_2, x_3, ... x_n, so that the following ordered sets of velocity ratios y can be formed:

$$\left.\begin{array}{lllll} \text{for } x = x_1 & y_1(x_1), & y_2(x_1) & \ldots & y_n(x_1) \\ \text{for } x = x_2 & y_1(x_2), & y_2(x_2) & \ldots & y_n(x_2) \\ \ldots & \ldots & \ldots & & \ldots \\ \text{for } x = x_n & y_1(x_n), & y_2(x_n) & \ldots & y_n(x_n) \end{array}\right\} \quad (15)$$

Considering now the hydraulic gradient of the stream in its expressions (1.1) and (1.2) one can write, on account of (13):

$$f_w = \frac{f'}{(1 - \alpha x)^2} \quad (16)$$

where now the quantities f' and α are unknown.

Moreover with reference to the set of values (15) one can choose from the first row set, i.e. for $x = x_1$ a velocity configuration $y_{m1}(x_1)$ such that the energy dissipated by the stream is a minimum relative to all the other y values.

Analogously from the second row one may select a velocity configuration $y_{m2}(x_2)$ producing minimum energy dissipation for the concentration x_2 and so on down to the n row.

By making the generic interval $x_{i+1} - x_i$ of the concentration small enough, one can define in principle from the sequency of values y_{1m}, y_{2m} ... y_{nm} a continuous function $y = y_m(x)$ which renders the function $\frac{f'}{(1 - \alpha x)^2}$ an extremum (minimum).

The condition of extremum relative to equ (16) is equivalent to the following:

$$\frac{f'}{(1 - \alpha x)^2} - f_w = 0 \qquad (17)$$

Anticipating a later result an explicit relationship can be obtained between α and x and between α and $\frac{d}{D}$ satisfying equ (17).

3. THE DISSIPATIVE FUNCTION ESTABLISHED AS AN ENERGY EQUATION

In the development that follows, streaming particles are treated as if they were centres of energy dissipation.

The work done in conveying the suspension is the sum of the work required to convey the pure liquid and the work required to convey the particles.

The sum of the various powers (rate of work) can be expressed thus

$$W' = W_p + W_w \qquad (1)$$

where W_p is the power dissipated by the liquid in the immediate surrounding of the particle, because of the particles' presence and W_w is the power dissipated by the liquid due to its flow as if the particles were absent.

W' is the power sum of the two powers just defined, i.e. the power actually required to convey the mixture.

The power required to convey the liquid phase only is

$$W_w = i_w \rho g Q_w L = i_w \rho g (1-x) QL \qquad (2)$$

where i_w is the hydraulic gradient, ρ the density of the liquid and L the length of the pipe.

Using the Darcy-Weissbach equation (1.2) one can redefine a friction factor f_w relative to the liquid phase and write i as i_w, i.e.

$$i_w = f_w \frac{V_w^2}{2gD} \qquad (3)$$

W_p can be expressed as the product of an average drag force multiplied by an average particle velocity V_p times the number of particles present inside a section of pipe of length L.

If n is the number of particles per unit length of pipe one can write

$$W_p = F_d \, V_p \, nL \tag{4}$$

For a drag force on a particle one can write:

$$F'_d = \tfrac{1}{2} \, C'_d \, \frac{\pi d^2}{4} \, \rho \, v^2_{rel} \tag{5}$$

where C'_d is a still undefined particle drag coefficient, d the particle diameter and v_{rel} a certain relative velocity between the particle and the surrounding portion of liquid.

Let us assume that the average drag force F_d be also described by the following expression (with non-accented symbols):

$$F_d = \tfrac{1}{2} \, C_d \, \frac{\pi^{\,2}}{4} \, \varepsilon^2 \, V_w^{\,2} \, \rho \tag{6}$$

In equ (6) εV_w represents a small relative velocity between a particle having the behaviour of the average and the surrounding fluid (i.e. with ε acting as slip coefficient).

Since the solid flow rate is

$$Q_p = \frac{\pi d^3}{6} \, n \, V_p \tag{7}$$

one gets:

$$V_p \, n = \frac{6Q}{\pi d^3} \tag{8}$$

By elimination of nV_p from (4) and (8), the following is arrived at:

$$W_p = \tfrac{3}{4} \, C_d \, L \, \rho \times \varepsilon^2 \, V_w^{\,2} \, Q \tag{9}$$

The work done in conveying the mixture can be expressed in terms of the actually measurable hydraulic gradient i, as follows:

$$W = i \rho g Q L \qquad (10)$$

where i is given by equ (1.1)

Substitution of (2), (3), (9) and (10) into (1) yields:

$$f' = (f_w (1-x) + b \varepsilon^2 x) (1 - x + \frac{x}{y}) \qquad (11)$$

where

$$b = \frac{3}{2} \frac{D}{d} C_d \qquad (12)$$

With the position:

$$\frac{y - 1}{y} = \alpha \qquad (13)$$

the following is obtained:

$$\frac{f'}{(1 - \alpha x)^2} = f_w (1-x) + b \varepsilon^2 x \quad \text{as} \qquad (14)$$

object of discussion in the next chapter.

4. THE PARTICLE COEFFICIENT OF DRAG

The following physical situations are discussed:

a) Single particle transported in a stream;

b) particle in an assembly of particles naturally settling with a settling velocity v'_{se}.

Let the hydraulic gradient of the stream be i, the density of the liquid ρ, that of the solid ρ_s.

Equilibrium of the forces in the vertical direction in cases a) and b) leads to the following expressions:

$$(\rho_s - \rho)\ \frac{g\pi d^3}{6}\ i = C_d\ \rho\ \frac{v^2_{rel}}{2}\ \frac{\pi d^2}{4} \qquad (1a)\ (X)$$

$$(\rho_s - \rho)\ \frac{g\pi d^3}{6} = C'_{dse}\ \rho\ \frac{v'^2_{se}}{2}\ \frac{\pi d^2}{4} \qquad (2b)$$

Division of (1) by (2) yields

$$i = \frac{C_d}{C'_{dse}}\ \left(\frac{v_{rel}}{v'_{se}}\right)^2 \qquad (3)$$

Consider now the ratio

$$\frac{C'_{dse}}{C_{dse}} = \Phi \qquad (4)$$

between drag coefficients relative to a particle settling in a crowded condition (accented) and as a solitary particle (non-accented).

For a solitary particle one can rewrite the condition of equilibrium in the vertical direction:

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X The first member of equ (1a) expresses the gravitational component along the slope of value i.

$$(\rho_s - \rho)g \frac{\pi d^3}{6} = C_{dse}\, \rho \frac{v_{se}^{\ 2}}{2} \frac{\pi d^2}{4} \qquad (5)$$

From (5) the (non-accented) drag coefficient can be defined as follows:

$$C_{dse} = \frac{4}{3} \frac{Gr}{Re^2} \qquad (6)$$

where the Reynolds and Grashof numbers of the particle are:

$$Re = \frac{v_{se} d}{\nu} \qquad (7)$$

$$Gr = \frac{\rho_s - \rho}{\rho} \frac{gd}{\nu^2} \qquad (8)$$

With reference to Figure 5 the locus $\Phi = 1$ plots Gr against Re for a solitary particle in naturally settling conditions (cf. Ref. 5).

The actual construction of the locus $\Phi = 1$ is obtained by calculation, from the experimentally determined well known function $C_d = f(Re)$, which for $Re < 1$ becomes Stokes law:

$$C_d = \frac{24}{Re}$$

The locus $\Phi = 1$, written as ratio between two different Reynolds numbers, e.g. at points P_1 and P_2 acquires the form

$$\frac{Re1}{Re2} = \left(\frac{Gr1}{Gr2}\right)^n \qquad (a)$$

where n is an exponent such that

$n = \frac{1}{2}$ in the fully turbulent region	$Re > 10^4$
$n = 1$ in the fully laminar region	$Re < 1$
$\frac{1}{2} < n < 1$ in the transitional region	$1 < Re < 10^4$

The inverse $\frac{1}{n}$ provides the slope of the locus $\Phi = 1$ at the particular Re number considered.

In the specific situation where particle size, density of the medium of the particle and viscosity are constant, the Grashof number is also constant.

Horizontal lines in Figure 5 (i.e. lines of constant Gr number) are thus lines of constant body force and because of dynamic equilibrium, also lines of constant superficial force, while vertical lines (i.e. lines of a constant Re number) are also lines of constant velocity.

In the case of an assembly of particles, experimental evidence has shown that equilibrium in the assembly occurs at lower velocities (or Reynolds numbers) than those applying to an identical solitary particle.

In a fluidisation column one could reword this concept by saying that the face velocity able to support the assembly is less than the settling velocity of the solitary particle.

In other words equilibrium conditions in the case of an assembly will be found in a zone to the left of the curve $\Phi = 1$.

As indicated earlier, in the case of equilibrium, horizontal lines are lines of constant drag force on the individual particles under various conditions of crowding, i.e. for various values of the void fraction.

These considerations lead to the concept that the diagram may be extended to illustrate the more general case of equilibrium in an assembly.

Referring to a fluidisation column, in which the mass of particles is kept in suspension, although the actual fluid velocity v' is unknown, the apparent velocity (face velocity) v and the apparent Reynolds number $Re = \frac{vd}{\nu}$ are known.

Let an isolate particle be in equilibrium at point P_1 with co-ordinates Re1, Gr1.

If other identical particles are now introduced into the system, the point of equilibrium shifts to lower velocities or Reynolds numbers along the line Gr = Gr1 (as the body force or Gr remains constant) say to the point P with co-ordinates Re2, Gr1.

According to equ (6)

$$C'_{dse} = \frac{4}{3}\,\frac{Gr1}{R'e^2} \qquad \text{(particle in an assembly)}$$

or as

$$R'e = Re2$$

$$C'_{dse} = \frac{4}{3}\,\frac{Gr1}{Re^2 2}$$

with equ (4) the factor Φ has been defined as the ratio $\frac{C'_{dse}}{C_{dse}}$ at identical face velocities.

Thus in this case, comparison must be made with the drag coefficients at point P_2, i.e. the coefficient of drag at P_2 is:

$$C_{dse2} = \frac{4}{3}\,\frac{Gr2}{Re2}$$

and so

$$\Phi = \frac{Gr1}{Gr2} \qquad (9)$$

One may now plot in Figure 5 the Φ = constant loci (e.g. for Φ = 4, 16, 64, etc.)

As the diagram is drawn on a logarithmic scale, all these curves may be obtained by shifting the basic curve Φ = 1 vertically over a distance corresponding to lg Φ.

Then one obtains for a known value of Φ

$$C'_{dse} = C'_{dse1} = \Phi\, C_{dse2} \qquad (10)$$

From equ (6) written for points P_1 and P_2 respectively, from equs (a) and (9) one gets

$$C_{dse2} = C_{dse1} \frac{Gr2}{Gr1} \left(\frac{Re1}{Re2}\right)^2 = \Phi^{2n-1}\, C_{dse1} \qquad (11)$$

Moreover writing equ (6) as under

$$C_{dse1} = \frac{4}{3} \frac{Gr1}{R^2e1} = \frac{4}{3} \frac{d_1 g}{v^2_{se1}}$$

C_{dse1} substituted into (11) yields

$$C_{dse2} = \frac{4}{3} \frac{d_1 g}{v^2_{se1}} \quad \Phi^{2n-1} \qquad (12)$$

Returning now to equ (3) with the position

$$v_{rel} = \varepsilon\, V_w$$

one obtains with (3) and (10):

$$v^2_{rel}\, C_d = \varepsilon^2 V^2_w C_d = iv'^2_{se}\, C'_{dse} = i\, v'^2_{se1}\, C'_{dse} = iv'^2_{se1}\, \Phi\, C_{dse2} \quad (13)$$

Expressing C_{dse2} by means of equ (12) and writing for the particle in condition as at P (cf. Figure 5).

$$\frac{v'_{se1}\, d_1}{\nu} = Re'1 = Re2$$

$$\frac{v_{se1}\, d}{\nu} = Re1$$

With the position $d = d_1$ in Re1 and further because of (a) and (9), one gets:

$$\frac{R'^2_{e1}}{R^2_{e1}} = \frac{Gr2}{Gr1} = \left(\frac{Re2}{Re1}\right)^{-\frac{1}{n}}$$

Finally from the expressions written above:

$$v'^2_{se1} = v^2_{se1} \frac{R'^2_{e1}}{R^2_{e1}} = v^2_{se1}\ \Phi^{-2n} \qquad (14)$$

By substituting (14) into (13) the following is obtained:

$$C_d = i\ \frac{4}{3}\ \frac{dg}{\varepsilon^2 V_w^{\ 2}} \qquad (15)$$

Then introducing for i the expression (1.2) one gets from (15) and (3.13)

$$b = \frac{f_w}{\varepsilon^2} \qquad (16)$$

Substitution of (16) into (3.14) yields

$$\frac{f'}{(1 - \alpha x)^2} = f_w\ (1 - x) + f_w x \qquad (17)$$

The term $f_w x$ expresses in equ (17) energy dissipation due to particles presence.

This term is equal and opposite to the amount by which the friction factor has been reduced because of reduced turbulence inside the stream (on account of the solid phase presence).

Then let us write equ (17) in the following equivalent form

$$\frac{f'}{(1 - \alpha x)^2} - f_w = 0 \qquad (18)$$

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Equ (18) is in agreement with the experimental results of Figures 1, 2, 3 and 4 so far the experimentally produced points fall along the clear liquid line.

One should also notice in this respect that the ratio $\frac{d}{D}$ has not been brought into discussion yet, i.e. the results of these figures are valid irrespective of the values acquired by $\frac{d}{D}$, at least within the limits of the experiments (in Figure 3 the points plotted for x = 0,30 are outside these limits, i.e. they do not fall along the clear water line).

5. THE DISSIPATED ENERGY AS A FUNCTION INVARIANT OF THE CONCENTRATION

A discussion of equ (4.18) follows:

1) When only decrete particles are introduced into the system, i.e. for $x \to 0$ one is in a situation described by Figure 6 (cf. ref. 4).

   The experimental points and the theoretical curves of this figure prove that a particle proceeds faster than the liquid, thus the greater the ratio $\frac{d}{D}$.

   In the case of a very small particle introduced, i.e. for $\frac{d}{D} \to 0$, the average particle velocity approaches that of the liquid i.e.

   $$y = \frac{V_p}{V_w} \to 1 \qquad \text{or } \alpha \to 0$$

2) For particles of small diameter introduced in finite concentration ($\frac{d}{D} \to 0$, $x \neq 0$) experiments prove a uniform distribution of the particles across the pipe section, i.e.

   $$y \to 1 \qquad \text{or } \alpha \to 0$$

   Then the condition of invariance can be derived from equ (4.18) as follows:

   $$\begin{cases} f' = f_w & (1) \\ & \left(\frac{d}{D} \to 0 \quad x \neq 0\right) \\ \alpha x = 0 & (2) \end{cases}$$

   Equs (1) states that the friction factor relative to the mixture, i.e. referred to the mixture velocity V is equal to that of the pure liquid proceeding with velocity V_w.

   Equ (2) expresses the condition of invariance for (4.18): this implies $V = V_w$.

3) For particles of finite size ($\frac{d}{D} \neq 0$), introduced in finite concentration ($x \neq 0$) equ (4.18) can be written as follows:

$$f' = (1 - \alpha x)^2 f_w \qquad (3)$$

Then one gets:

$$\begin{cases} f' = f_w \ (1-h)^2 & (4) \\ & \left(\frac{d}{D} \neq 0,\ x \to 0\right) \\ h = \alpha x & (5) \end{cases}$$

The friction factor is now reduced by the ratio $(1 - h)^2$ in respect of f_w.

In Figure 7, α has been plotted versus x for values $\frac{d}{D}$ = 0,067 and 0,029 respectively, using the experimental results of Roberts and Kennedy (ref. 3).

The two loci intersect the α axis at two points which are close to those determined from Figure 5.

Information of this kind was not available in the experimental material of the remaining authors (ref. 1 and 2) and so Figure 7 contains only two $\frac{d}{D} = k$ loci.

Two hyperbolae of the family corresponding to equ (5) have also been shown.

Figure 7 in a more complete representation, i.e. with many loci $\alpha x = h$ and $\frac{d}{D} = k$ drawn, would produce a reticulate consisting of the intersection of the two families of curves, with each point of the plane characterized by four values α, x, h, k.

Of these only two are the independent ones, i.e. necessary for the physical definition of the problem.

The "minimizing" function $y_m(x)$ introduced in Chapter 2, which fulfils either equ (2.17) or (4.18), can now be defined:

It is merely the experimental locus $\frac{d}{D} = k$ of Figure 6.

The $\frac{d}{D} = k$ loci can be substituted for small value of the concentration x by their geometrical tangent at the origin.

Taking as an example the locus $\frac{d}{D} = 0{,}067$ the tangent equation is

$$\frac{\alpha}{0{,}0475} - \frac{x}{0{,}22} = 1 \qquad (\frac{d}{D} = 0{,}067) \qquad (6)$$

Elimination of α and h from (4) and (5) by means of (6) produces the friction factor f' in function of the variable x only i.e.

$$\frac{f'}{f_w} = (1 - (0{,}0475\ x + 0{,}216\ x^2))^2 \qquad (\frac{d}{D} = 0{,}067) \qquad (7)$$

$$(0 < x < 0{,}10)$$

which is the wanted expression.

6. CONCLUSIONS

Experimental results of Figures 1, 2, 3 and 4 prove that friction factors relative to clear liquid and suspension (solid & liquid) are equal if referred to the same volume of streaming liquid.

Equality is unaffected by variations of particle concentration and of particle pipe diameter ratio, this at least within the limits of the mentioned experiments.

Considerations based on the equation of continuity support the hypothesis that in a conveyance of particles the dissipation of energy is an extremum function of the concentration, i.e. invariant against a concentration change.

Assuming that particles in transport be centres of energy dissipation, an equation has been set up (3.14), in which the total energy dissipated has been considered to be the sum of the energy relative to the particles and that relative to the clear liquid, respectively.

Considerations about the drag coefficient relative to a particle, when settling inside an assembly of particles and when transported in suspension inside a stream has led to the following conclusions:

That the energy dissipated at the particle is equal to the amount which would be dissipated inside the volumetrically equivalent portion of liquid, available, if the particle were not present (4.17).

Further the following cases have been discussed:

1) Particles conveyed in discrete number ($x \to 0, \frac{d}{D} \neq 0$).

2) Very small particles transported in finite concentration ($x \neq 0, \frac{d}{D} \to 0$).

3) Particles transported in finite concentration ($x \neq 0, \frac{d}{D} \neq 0$).

In case 1, one has simply reported results of previous research work (ref. 4), proving that isolated particles travel faster than the average liquid mass.

In case 2 one has re-obtained the well-known geometrical configuration of even distribution of particles in the stream cross section.

The friction factor referred to the flow of the mixture has been found in this case to coincide with that of the pure liquid flow.

In case 3 one has obtained a particle velocity which is always greater than the liquid velocity and a friction coefficient relative to the flow of the mixture, which is always lower than that of the stream when conveying an equivalent volume of pure liquid ((5.4) and (5.5)).

Finally the minimizing function $y = y_m(x)$ introduced in Chapter 2 has been identified with the loci $\frac{d}{D} = k$ represented in Figure 7.

Concluding, the condition of invariance of equ (4.18), i.e. of the expression

$$\frac{f'}{(1 - \alpha x)^2} - f_w = 0$$

has been resolved into the elementary condition

$$f' = f_w (1 - \alpha x)^2$$

This has found physical and analytical justification in Figures 1, 2, 3 and 4 and in the development of Chapter 4 respectively.

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7. NOMENCLATURE

(SI system of units)

A, A_w, A_p	total pipe area, area occupied by the liquid (water) and by the solid (particles) respectively
C_d, C'_d	drag coefficient relative to a solitary particle and to a particle in an assembly respectively
C_w	Chezy coefficient
d	particle diameter
D	pipe diameter
f'	friction factor of pipe for the flow of a mixture (solid & liquid)
f_w	friction factor of pipe for a pure liquid flow (water)
F_d, F'_d	average drag force, drag force on an individual particle of an assembly, respectively
g	acceleration due to gravity
Gr	particle Grashof number: equ (8)
h	a parameter: equ (5.6)
k	a parameter ($k = \frac{d}{D}$)
K_r	a parameter relative to certain pipe velocity profile
i	hydraulic gradient
n	number of particles existing in a unitary length of pipe

Q, Q_w, Q_o	flow rate relative to the mixture (solid & liquid) to the liquid (water) and to the solid (particles), respectively
Re	particle Reynolds number: equ (7)
v, v'	face velocity and true velocity in a fluidisation column, respectively
v_{rel}	relative velocity between particle and surrounding liquid
V, V_w, V_p	average velocity of the mixture (solid & liquid), of the liquid (water) and of the solid (particles), respectively
W, W_w, W_p	hydraulic power dissipated by the mixture (solid & liquid), by the liquid (water), by the solid (particles), respectively
x, x_ℓ	volumetric concentration at the discharge and linear concentration inside the pipe, respectively
y	ratio of expression (2.10)
α	ratio of expression (2.11)
ν	kinematic viscosity of liquid (water)
ρ, ρ_s	density of liquid and solid phase respectively
ε	velocity slip coefficient

Subscripts:
d means drag
ℓ means linear
p means particle
s means solid
w means water

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8. LITERATURE REFERENCE

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2. J W Daily & T K Chu: Rigid particle suspensions in turbulent shear flow; some concentration effects. Technical report No. 48. October 1961. Hydrodynamics Laboratory, Department of Civil Engineering, Massachusetss Institute of Technology, Cambridge Mass. USA.

3. C P Roberts and J F Kennedy: Particle and fluid velocities of turbulent flows of suspensions of neutrally buoyant particles. Report No. 103. Jan. 1967. Hydrodynamics Laboratory, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge Mass. USA.

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TABLE 1

SUMMARY OF EXPERIMENTAL CONDITIONS RELATIVE TO TESTS REPORTED

Literature reference	Authors	Conduit & sizes	Particle size	Part. rel. density or settling velocity in water	Particle pipe diameter ratio d/D	Solid volumetric conc. investigated
1	C Elata T Ipen	Flume 765 mm wide 280 mm high	85% particles from 0,10 to 0,155 mm dia.	1,05	-	0 to 10% 10 to 20% 20 to 30%
2	J W Daily T K Chu	Pipe D = 51 mm	Particle sieved between 1,14 and 1,63 mm	Between 15 and 25 $(\frac{mm}{s})$	$\frac{1,34}{51} = 0,026$	5; 10; 15; 20%
3	C P Robert J F Kennedy	Pipe D = 51 mm	Cubes, diam. of equiv. sphere d = 3,4 mm	1,015	$\frac{3,4}{51} = 0,067$	0; 10; 20; 30%
4	Ditto	Ditto	Cylinders, dia. of equiv. sphere d = 1,48 mm	1,052	$\frac{1,48}{51} = 0,029$	0; 15; 20; 30%

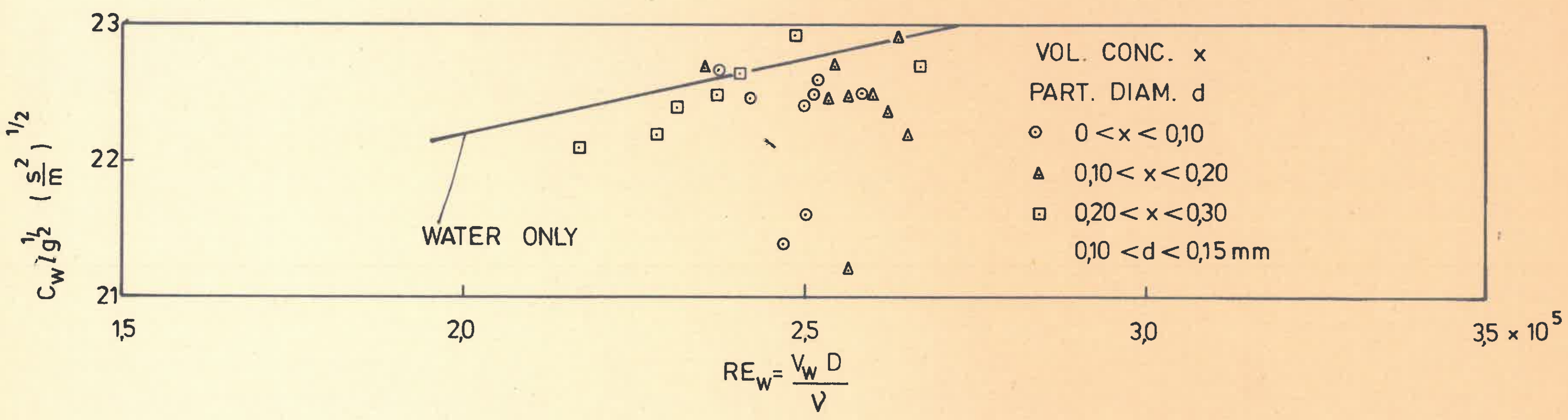

FIGURE 1: CHEZY COEFFICIENT VERSUS FLUME REYNOLDS NUMBER REFERRED TO WATER VELOCITY (TRANSFORMED FROM REF. 1 : FIGURE 17a)

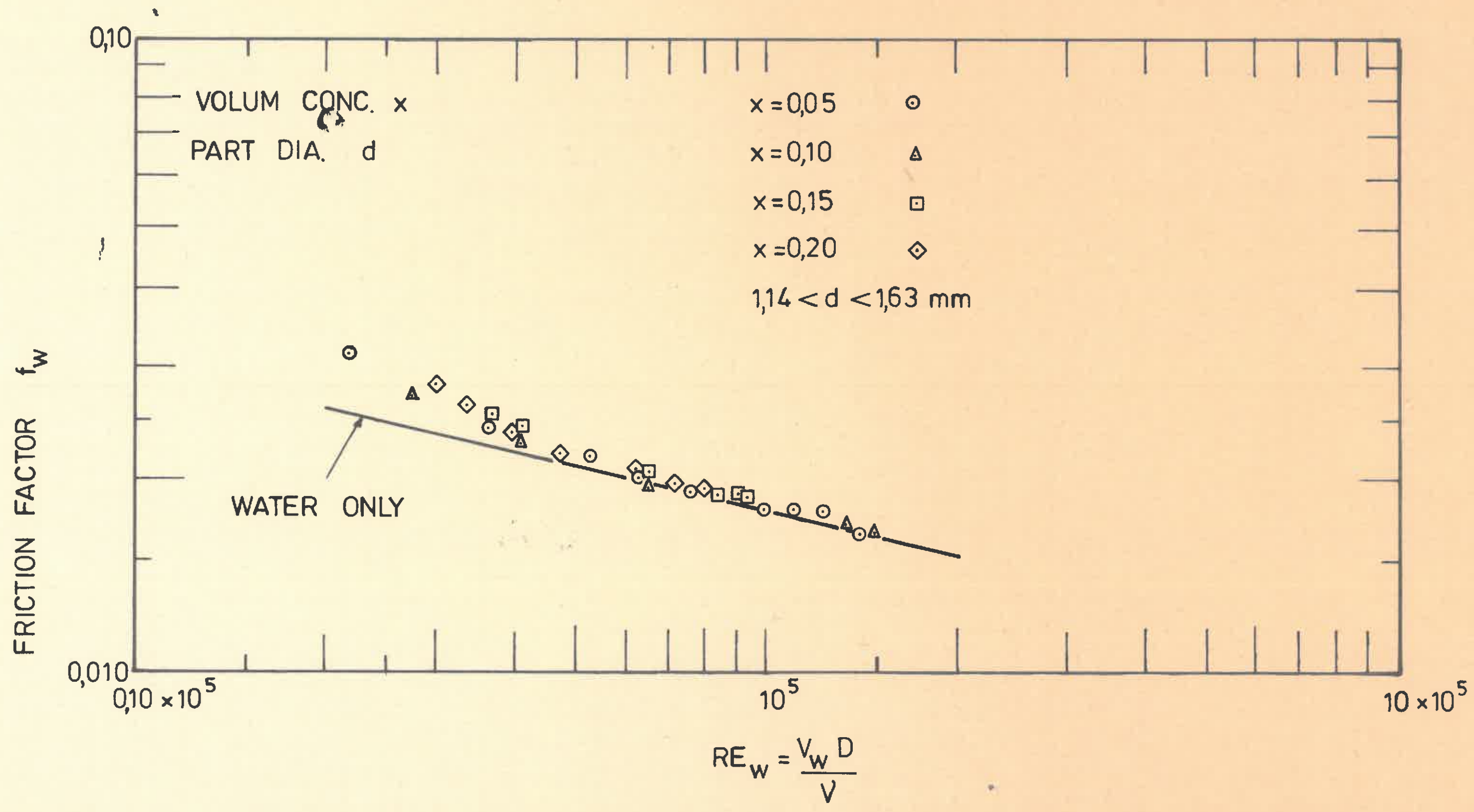

FIGURE 2 : FRICTION FACTOR VERSUS PIPE REYNOLDS NUMBER REFERRED TO WATER VELOCITY (TRANSFORMED FROM REF. 2 : FIGURE 14)

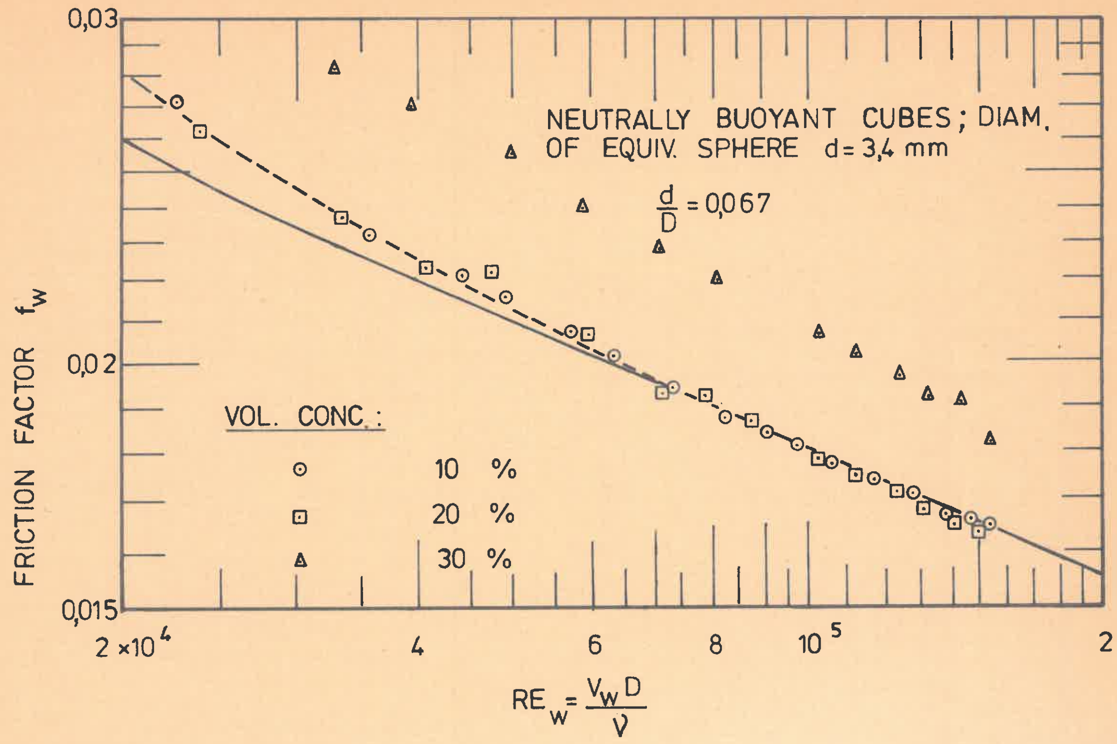

FIGURE 3 FRICTION FACTOR VERSUS PIPE REYNOLDS NUMBER REFERRED TO WATER VELOCITY (REPLOTTED FROM REF. 3)

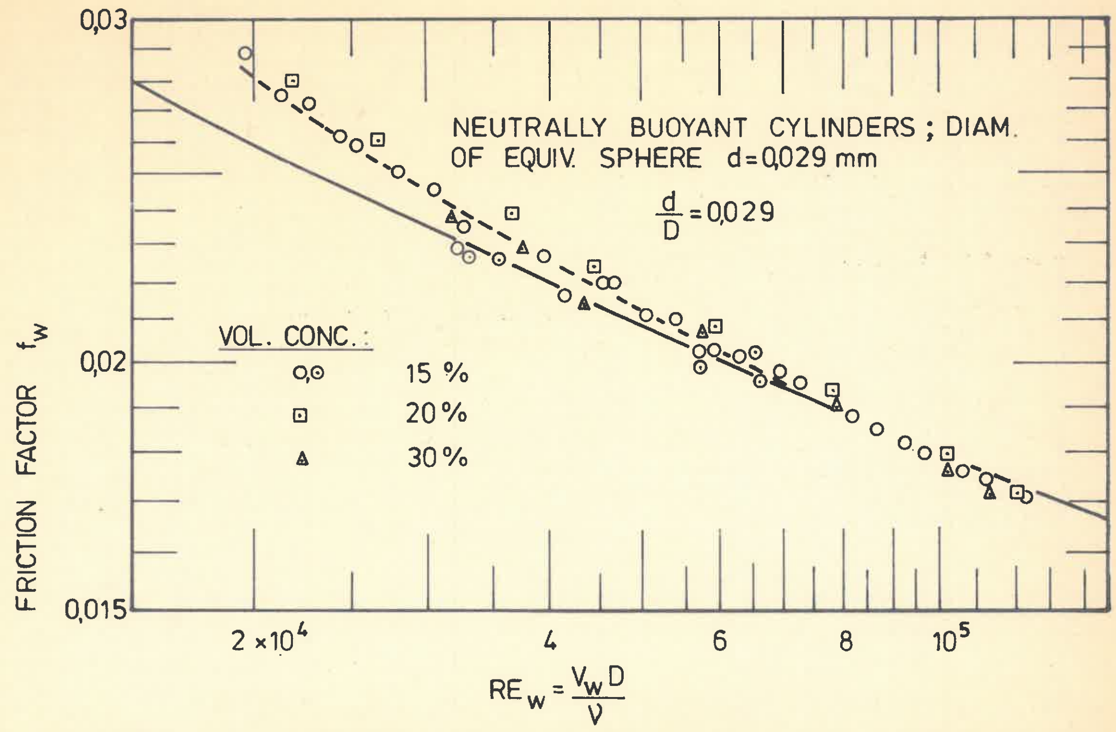

FIGURE 4 FRICTION FACTOR VERSUS PIPE REYNOLDS NUMBER REFERRED TO WATER VELOCITY (REPLOTTED FROM REF. 3)

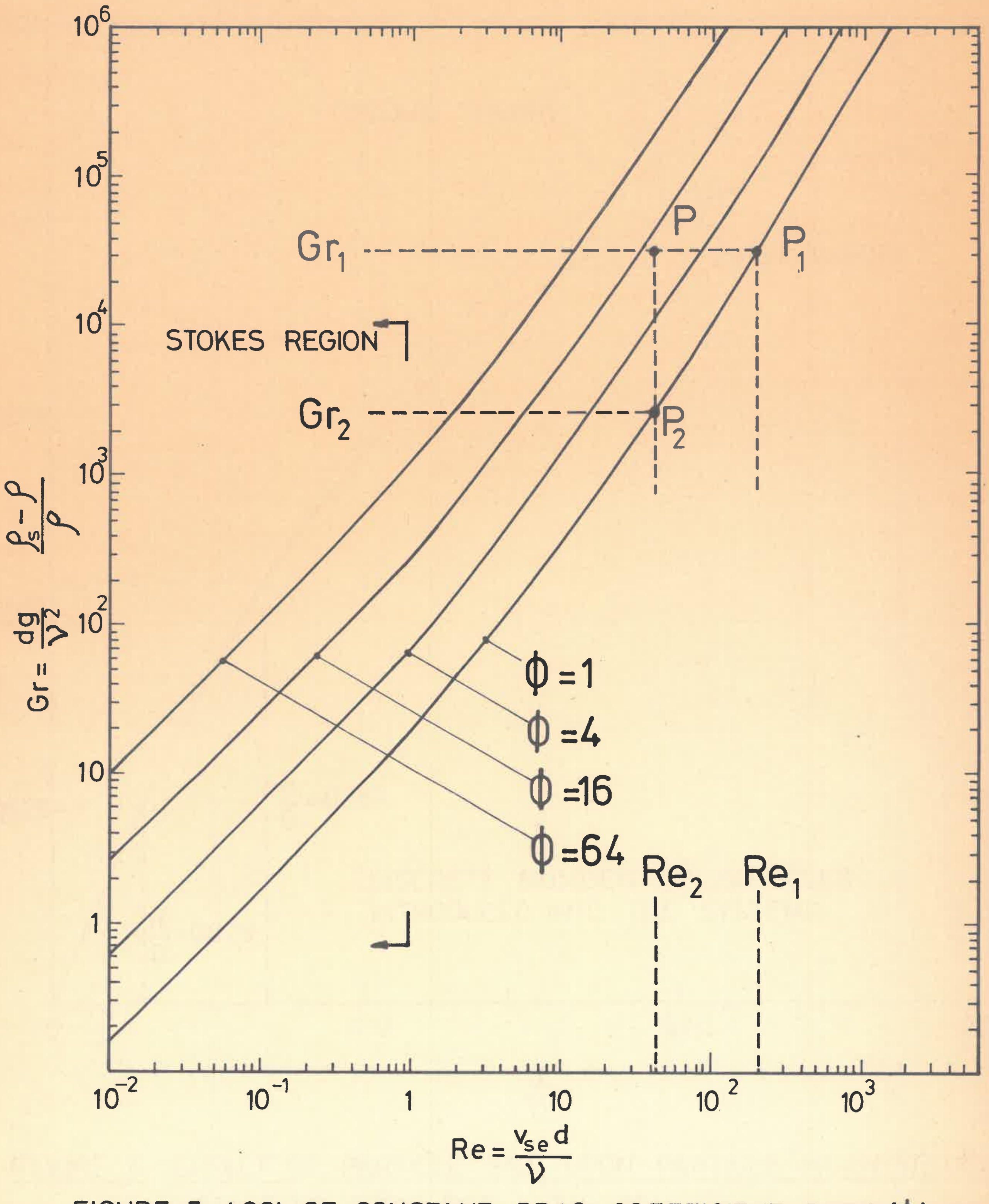

FIGURE 5 LOCI OF CONSTANT DRAG COEFFICIENT RATIO (ϕ) IN GRASHOF - REYNOLDS NUMBERS REPRESENTATION

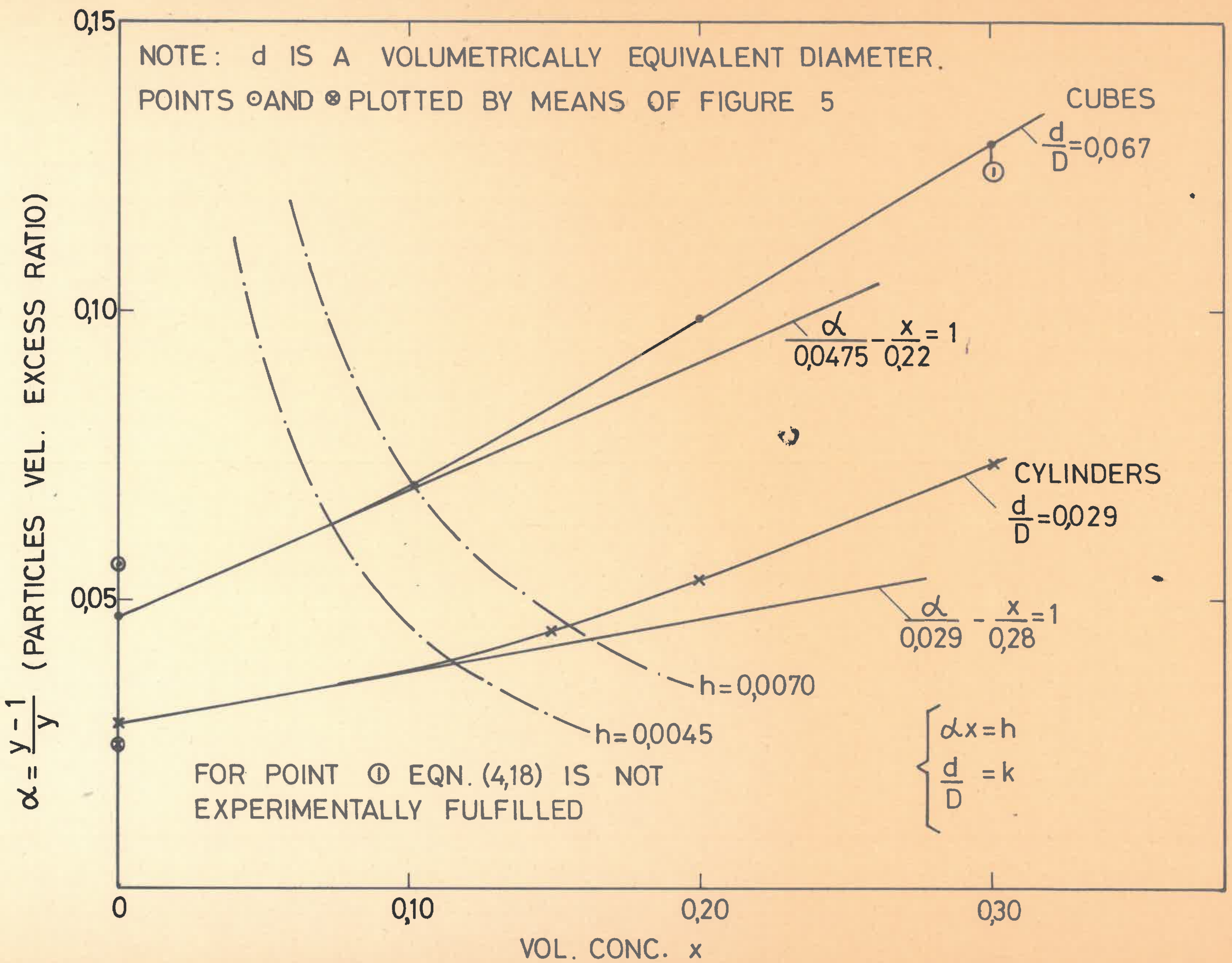

FIGURE 7 PARTICLE VELOCITY EXCESS VERSUS VOL. CONC. (REDUCED FROM REF. 3: TABLE 5,2)