USING GAUSSIAN PROCESS MACHINE LEARNING TO PREDICT DYNAMIC ROAD WEAR OF A RIGID HEAVY VEHICLE

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ABSTRACT

The paved road network is a critical asset to any economy. South Africa has a paved road network that has an estimated value above R2 trillion. This asset is however under threat as there was a backlog in maintenance of more than R416.6 billion in 2018. Heavy vehicles are primarily responsible for road wear, and overloaded vehicles can cause more than 60% of road wear. Most road wear assessments use static axle loads that are assumed to be symmetrical on either side to calculate the road wear caused by a heavy vehicle. Previous work has shown that the effect of crossfall (CF) cannot be ignored when considering the dynamic road wear of heavy vehicles. This paper expands on previous work through the development of a novel Gaussian process machine learning (GPML) model that can predict the dynamic road wear of a rigid heavy vehicle given 15 input parameters. The road wear criteria considered are the first (1^{st}) and fourth (4^{th}) power aggregate forces on the left and right sides using the 95^{th} and 99^{th} percentile conditions. The results show that the model is very accurate and requires comparatively few inputs to train an accurate model. For interpolated results, the average absolute error is less than 1% and for extrapolated results, the average absolute error is less than 3%. The results also include the standard deviation associated with the result which is important for future research to minimise training examples. Using machine learning models to predict dynamic road wear allows for rapid calculation and testing and also does not require expensive multibody dynamics software tools to calculate. This would be very advantageous to the industry, especially when developed for the Smart Truck Pilot Project.

1 INTRODUCTION

1.1 Background

Transport logistics in South Africa is the backbone of the economy, representing 11.8% of Gross Domestic Product (GDP) in 2016 or approximately R499 billion (Havenga, et al., 2016). Road freight transport, in particular, is essential to logistics, as approximately 85% of general freight is transported via road (Havenga, Simpson, King, De Bod, & Braun, 2016). The paved road network in South Africa is therefore a key national asset that has an estimated value above R2 trillion. As at 2014, South Africa had a road maintenance backlog of R200 billion, which has led to a national network where currently 78% of the roads are older than their intended design life. The situation has deteriorated since then and new estimates of the road maintenance backlog are **R416.6 billion in 2018** (Ross & Townshend, 2019)**.** It is, therefore, crucial to minimise the road wear caused by heavy vehicles which, if overloaded, can account for more than 60% of all road damage (Krygsman & Van Rensburg, 2017). Having tools that can accurately calculate or estimate the road wear caused by heavy vehicles is therefore invaluable to the road owners, managers and designers. This paper aims to explore the possibility of using machine learning to accurately and quickly predict the dynamic road wear of a rigid heavy vehicle. This will be beneficial as this method does not require complex and expensive software to calculate nor engineering expertise and long computation times as typically required in these types of analyses.

The equivalent single axle load (ESAL) or load equivalency factor (LEF) is commonly used to quantify road damage due to static loads. The ESAL damage is calculated as shown in Equation 1 (van der Walt, et al., 2018).

$$ESAL = LEF = \left(\frac{Actual\ axle\ load}{Reference\ load}\right)^n \quad (1)$$

Where *n* is an appropriate exponent to quantify the damage on a specific pavement. This value is usually taken as 4 but values greater than 7 have been proposed (van der Walt, et al., 2018).

The aggregate tyre force is a method that can be used to quantify the dynamic road wear of a vehicle. The aggregate tyre force is calculated using Equation 2 (Cebon, 1988; Cebon, 1999).

$$A_k^n = \sum_{j=1}^{N_a} P_{jk}^n \quad k = 1,2,3,\ldots,Ns \quad (2)$$

Where A_k^n is the aggregate n^{th} power force, P_{jk} is the force applied by tyre j to location or station k on the road, N_a is the number of axles on the vehicle and N_s is the number of points or stations of interest along the road. The power (n) is chosen based on the type of road damage that is being considered. For flexible pavements, a value of $n = 1$ is best suited for permanent road deformation and $n = 4$ is best suited for fatigue damage (Cebon, 1988; Cebon, 1999). It has been found that the ratio of the instantaneous fourth power force to the static fourth power force can exceed 3 which indicates how the peak dynamic loads can be substantially higher at certain locations and cause more damage at specific sections (Cebon, 1988; Cebon, 1999).

Equation 2 provides the aggregate tyre force raised to some exponent at every point on the road profile. The road wear of the vehicle is however typically reported as the 95th or 99th percentile of the aggregate tyre force. Of this, the 95th is the most common, but the 99th is also often reported (Cebon, 1988; Cebon, 1999).

The dynamic tyre forces of the vehicles can be obtained through various methods including field testing. This is however more difficult and costly, and the data is most often obtained through computer simulations. The quarter-car model has been the most widely adopted model for simulating dynamic tyre forces. The quarter-car model is not however able to capture complex suspension nonlinearities and the complexities of heavy vehicle body motion, though the frequency content of the quarter-car model is sufficiently accurate (Buhari, et al., 2013). This has been verified by Hardy and Cebon (Hardy & Cebon, 1994). In general, researchers keep models realistic but simple to minimise complexity and reduce computation time (Buhari, et al., 2013).

When higher degree-of-freedom models are developed, researchers often ignore body roll motion or conclude that its influence is negligible. As such, dynamic road wear models are usually limited to either quarter-car or half-car models that utilise pitch-plane motion (Cebon, 1999). Cebon specifically states: “A two-dimensional model (pitch-plane) should be satisfactory for predicting the tyre forces of typical leaf-spring articulated vehicles with well-damped suspension modes, operating under typical conditions of speed and road roughness”. He, however, adds that: “It may therefore be necessary to use a three-dimensional model when the unsprung mass roll modes contribute significantly to dynamic tyre forces”. This would be the case if the crossfall of a road profile is included in the model.

Ihs and Magnusson (2000) state that the effect of road crossfall is negligible. Other previous research projects and guidelines focus primarily on the effect of road roughness, vehicle speeds, loading of the heavy vehicles and suspension characteristics. When crossfall (CF) is referred to, emphasis is placed on its vital importance in preserving road infrastructure by ensuring adequate drainage (Bowen, 2017).

Recent work has however shown that excluding road crossfall in road wear studies leads to a substantial underestimation of the dynamic road wear produced by heavy vehicles. Even crossfall values as low as 1% will produce a notable difference. This study therefore also includes the effect of road crossfall (Steenkamp, et al., 2019; Steenkamp, et al., 2021)

Although the use of computer simulations reduces the cost and time required to obtain these dynamic tyre forces of heavy vehicles, it still requires substantial expertise, often requires proprietary software that can be expensive, and requires substantial time to perform large batch analysis. The need was therefore identified to develop a *machine learning model* that can accurately predict the dynamic road wear of heavy vehicles.

Work by Robert Berman and Joop Pauwelussen has shown that the use of Gaussian process machine learning appears to be the most suitable solution for modelling aspects related to heavy vehicle dynamics (Berman, et al., 2018; Pauwelussen, 2021). A Gaussian process (GP) is a machine learning algorithm that can learn and improve its results by introducing more training examples and prior knowledge (Rasmussen & Williams, 2006). One of the most common applications of Gaussian processes is in regression, where a model is fitted to a dataset and predictions are made by providing input parameters and the model then produces a predicted value based on the information that was provided to train the model. It takes an unknown space with a potentially infinite number of functions and reduces it to a known space by assigning a probability to each of these functions. The mean of the probability distribution thus represents the best or most likely representation of the characterisation of the data (Rasmussen & Williams, 2006). Additionally, using a probabilistic approach allows for the confidence of the predicted value to be determined.

A Gaussian process is a stochastic process that is defined by its mean (μ) and covariance functions (σ^2), where the mean describes the expected value of the distribution. The mean is a vector, where each element describes the mean component in a specific dimension. According to Rasmussen & Williams (2006) a Gaussian Process can be described by a dataset $\mathcal{D}$, which consists of an observation with input x and output y as shown in Equation 3.

$$D = \{(xi, yi) | i = 1, \ldots, n\} \tag{3}$$

In the case of this paper, the input variable x represents the input parameters of the rigid heavy vehicles that are provided and y represents the predicted output parameter (road wear criteria considered). A model is trained for each of the road wear criteria considered, including separate left and right-side models. This led to the creation of 8 models that are used for the prediction of the road wear caused by a test vehicle.

To perform the linear regression that was used in the trained models, Gaussian processes make use of Bayesian linear regression with Gaussian noise as shown in Equation 4 (Rasmussen & Williams, 2006).

$$f(x) = x^T w = y = f(x) + \varepsilon \tag{4}$$

Where x is the input vector, w is the weight vector of the linear model and f is the resulting function value and y is the target value that is observed. ε is the Gaussian noise and also follows a Gaussian distribution with a mean value of zero and a variance of σ^2. This gives the distribution as shown in Equation 5 (Rasmussen & Williams, 2006).

$$\varepsilon \sim N(0, {\sigma_n}^2) \tag{5}$$

Gaussian processes make use of conditional probability which allows for the use of prior probabilities. When the model is first created, the prior probabilities are set to a zero mean Gaussian distribution. When a data point or observation is added to the model, a prior probability is calculated. This calculated prior probability is used to update the information regarding the distribution of the function space, f. The prior probability is then updated with the new information and used when a new data point is entered into the model. This is done using the Bayes Rule as shown in Equation 6 (Rasmussen & Williams, 2006).

$$posterior = \frac{likelihood\ x\ prior}{marginal\ likelihood}, \quad p(y_a|y_b) = \frac{p(y_a).p(y_b|y_a)}{p(y_b)} \tag{6}$$

A visual representation of the learning and space reduction process can be seen in Figure 1-1. The first section (a) shows how in an unknown space, there can be any function present as there is no known information regarding any known points that the functions must pass through. The second section (b) represents how the range of values that functions in the space can assume after some known data points have been introduced to the model and the model uncertainty has been reduced with the aid of posterior knowledge. The four known functions that were entered into the model are denoted by the dotted lines in (b). The grey area represents the variance or 95% confidence interval for the distribution, f(x), at any given

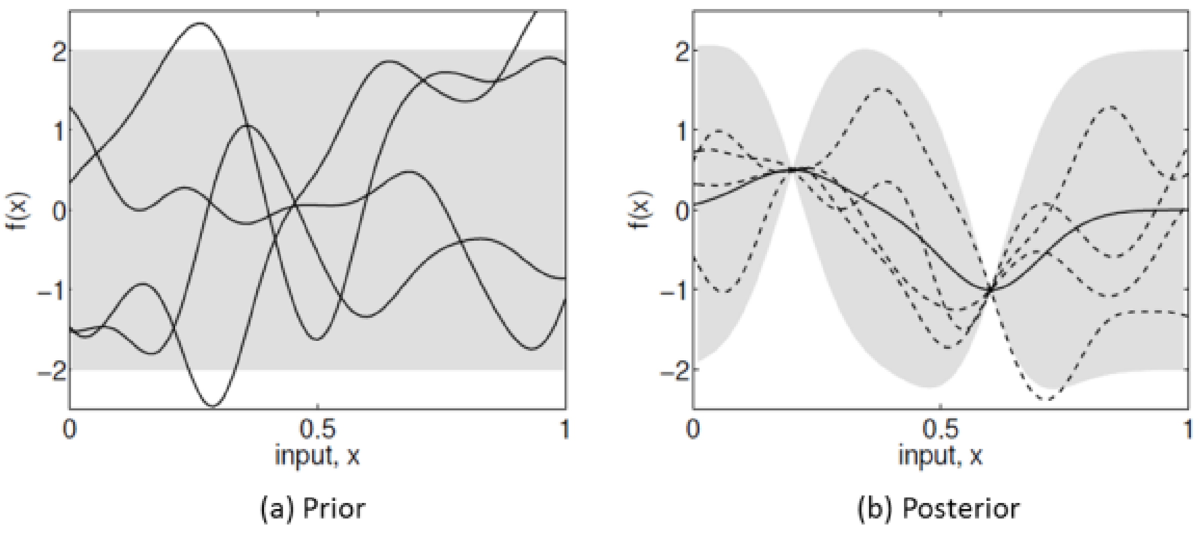

Figure 1-1: Visualisation of the Gaussian Process Showing the Prior Space (Fig a) and Posterior Space (Fig b) (Rasmussen & Williams, 2006)

point x. The function mean of the model is represented by the solid line in (b). After the model has been trained with a number of samples, f(x) provides a mean which is the predicted value at a specific point in the model space and the variance which in turn is the model uncertainty. Where there is a lot of known information for the model, the uncertainty or variance value is very low, but where there is little or no known data, the model has a high level of uncertainty. These areas of high uncertainty are regions where more training data points can be used to better train and fit the model to the training data and reduce the uncertainty for future predictions. The difference between Gaussian Processes and other machine learning techniques such as artificial neural networks (ANN) is that GPML provides a variance value describing the level of uncertainty associated with a prediction whereas other methodologies do not have this capability and simply provide the predicted or mean value as described.

1.2 Aim and Scope

This paper aims to develop a Gaussian process machine learning model that can predict the dynamic road wear produced by a rigid heavy vehicle with selected inputs. An example of such a vehicle is shown in Figure 1.2. The work is exploratory to determine the relative amount of training examples to obtain an accurate result and to also determine whether the model can accurately interpolate and extrapolate based on unseen data.

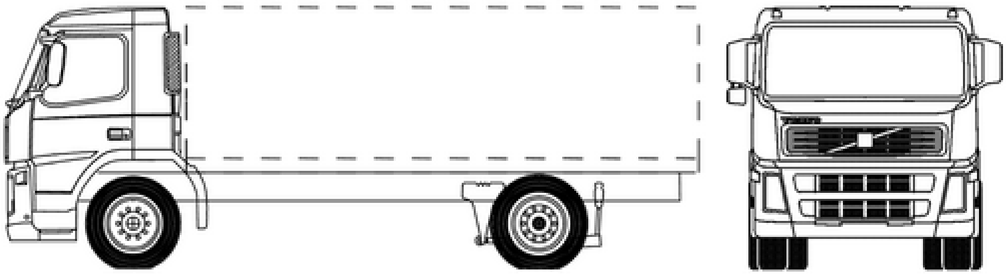

Figure 1.2: Scehmatic of a 4x2 rigid heavy vehicle (Find Blueprints, 2022)

Only Gaussian process machine learning will be considered during this study. The vehicle model will only consist of a rigid heavy vehicle (4x2) with a fixed wheelbase, payload mass, height and inertia, a fixed vehicle speed (80 km/h) and one road roughness value (2 m/km IRI). A total of fifteen other input parameters that are known to influence dynamic road wear when including crossfall are included. These parameters and their ranges are discussed in the next section.

2 METHODOLOGY

It should be noted that the number of variables used in training the model was determined through an iterative process based on the number of examples required to achieve a certain level of accuracy and the time required to train the model. From several iterations, it was found that adding the vehicle speed, vehicle inertia properties (mass, height, inertia, etc), different road roughness values and wheelbase, increases the number of required examples too much. The results did show promise with average absolute errors on the order of 1-5%, but produced extremely high maximum errors (on the order of 100%). Fixed values were used for these inputs as shown in Table 2-1 and are based on various sources and practical experience. The road profiles used were developed using the ISO 8608 (ISO, 2016) standard and was created for 1 km.

Table 2-1: Fixed values used in the heavy vehicle model

Variable	Unit	Value used
Road roughness	m/km	2
Sprung mass	kg	15000
Sprung mass height	mm	2000
Sprung mass longitudinal position	mm	2400
Roll inertia	$kg.m^2$	25000
Pitch and Yaw inertia	$kg.m^2$	35000
Wheelbase	mm	4500
Tyre radius	mm	500
Steer axle mass	kg	700
Steer axle road and yaw inertia	$kg.m^2$	700
Drive axle mass	kg	1000
Drive axle road and yaw inertia	$kg.m^2$	810
Vehicle speed	km/hr	80

This iterative process led to the selection of 15 parameters that affect dynamic road wear of heavy vehicles when including crossfall. The range of the 14 heavy vehicle parameters is shown in Table 2-2. This is based on experience with heavy vehicle assessments and a collation of various sources. Given the input range of the various variables, the number of possible random combinations would exceed several billion. Therefore, the variables were simulated using fixed steps as a first attempt to determine if they sufficiently characterise the vehicle performance over the entire range. This is also shown in Table 2-2.

Table 2-2: Range of variable input parameters of a rigid heavy vehicle

Variable	Unit	Min value	Max Value	Step Size
Trackwidth front	mm	1600	2200	50
Trackwidth rear	mm	1600	2200	50
Vertical tyre stiffness front	N/mm	700	1300	50
Vertical tyre stiffness rear	N/mm	700	1300	50
Spring stiffness front	N/mm	150	1000	50
Spring stiffness rear	N/mm	150	1500	50
Spring trackwidth front	mm	500	1500	50
Spring trackwidth rear	mm	500	1500	50
Auxilliary roll stiffness front	Nm/deg	0	5000	500
Auxilliary roll stiffness rear	Nm/deg	0	10000	500
Damper value front	Ns/m	5	50	5
Damper value rear	Ns/m	5	50	5
Roll Centre Height Front	mm	-300	300	50
Roll Centre height Rear	mm	-300	300	50

To test the ability of the machine learning model to interpolate and extrapolate based on the input data provided, the model was only trained on road crossfall values of 0% and 2%.

The vehicles are simulated in TruckSim®2021.0 (Mechanical Simulation Corporation, 2022) and the variables and automation are completed using Matlab® (MathWorks, 2022)

The total number of vehicles simulated per crossfall is 10 500. This is to allow for 8 000 training examples per crossfall and approximately 2 500 test cases.

After all of the simulations were completed in TruckSim®, the data was post-processed to remove any outliers. The more symmetric the data is, the more accurate the trained model. The skewness for the left side 95th percentile 4th power aggregate force was found to be 46.4 which is extremely high due to some outliers. The data were filtered by removing values that have a standard deviation of more than 3 (based on the left side 95th percentile 4th power aggregate force which is the most important parameter), until the skewness was less than 1, which indicates a minor skewness. This was achieved after 2 iterations for both the 0% and 2% crossfall (CF) and approximately 1.5% of the data was removed over the two filters. Crossfall is the transverse sloping of a roadway toward the shoulder on either side of the road. It should be noted that the majority of vehicles removed are highly unrealistic with very high stiffnesses and track widths on the steer axles and low stiffnesses and track widths on the drive axle.

After the data has been processed, it was sent to Python® (Python, 2022). The data is normalised using built-in functions in “sklearn”, a machine learning library in Python®. The data is split into training and testing data (8 000 examples per crossfall are used for training and the remaining data is used for testing).

The built-in Gaussian process machine learning model in sklearn (GaussianProcessRegressor) was used to train the model. A random seed value of zero was used and the optimisation function was allowed to attempt 10 iterations. The kernel used is the radial basis function with white noise. A model is trained for each of the road wear criteria and for each side of the vehicle (a total of 8 models).

After the model has been trained, the accuracy of the model was tested against the test data. Additional test examples were also created in TruckSim® (2500 examples of each): namely

- 1% CF using the same input parameter steps used to train the model
- 3%CF using the same input parameter steps used to train the model
- 2% CF using a step size of one for all input parameters over each test range.

The simulations were conducted on a laptop with 16 GB of RAM and an Intel(R) Core(TM) i7-10850H CPU @ 2.70GHz processor.

3 RESULTS AND DISCUSSION

The first result of importance for future work is the time required to perform the simulations and the time to train the model. The typical simulation time for each input combination is 15 to 20 seconds.

The training time for each of the machine learning models using 16 000 input examples is approximately 1 to 2 days.

For the two CF values of 0% and 2%, the predicted vs actual values for the left 95th percentile 4th power aggregate tyre force (N^4) is shown in Figure 3-1. The figure shows that the model can accurately predict the dynamic road wear, especially for lower 4th power aggregate tyre forces values for both of the crossfall values used.

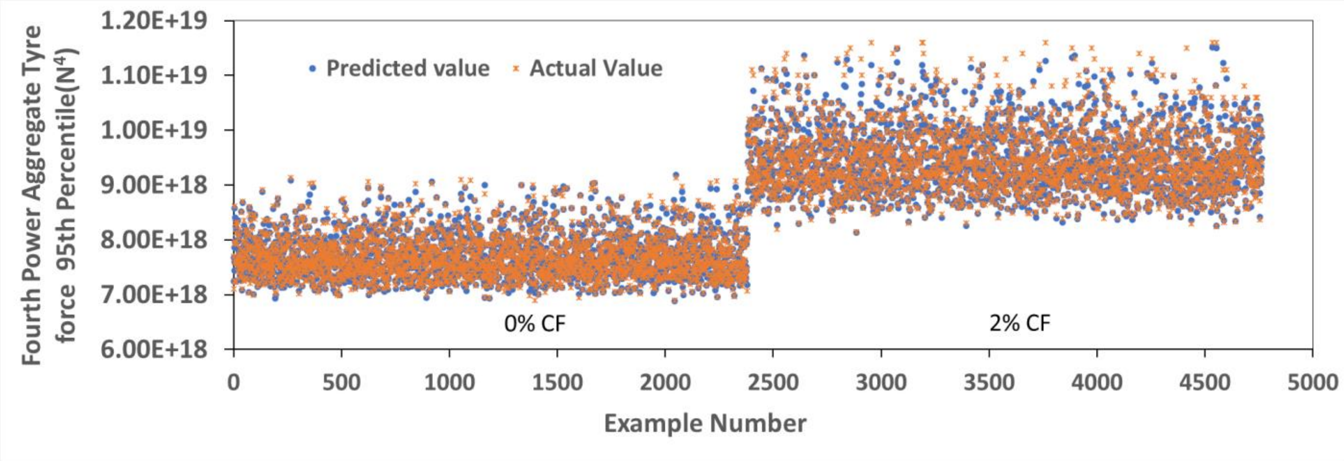

Figure 3-1: Results comparison for trained model

Table 3-1 summarises the results for the test data for the 8 models developed during this study. The results show that the average absolute errors and maximum errors tend to increase slightly for the 95th vs 99th percentile for all the road wear criteria considered. The results also show that the 1st power errors are lower than the 4th power errors as expected. All of the average absolute errors are however still below 1% which indicates that the models can accurately predict all of the road wear criteria over most of the test values used here. The maximum errors for the 1st power road wear are lower than 5% and the maximum errors for the 4th power road wear are less than 20% for all criteria considered.

Table 3-1: Summary of results using the original test data

Road wear criteria	Average absolute error (%)	Maximum error (%)
Left 95th Percentile 1st Power	0.12	3.18
Right 95th Percentile 1stPower	0.12	4.13
Left 99th Percentile 1st Power	0.16	3.78
Right 99th Percentile 1st Power	0.17	4.31
Left 95th Percentile 4th Power	0.53	11.81
Right 95th Percentile 4th Power	0.50	17.59
Left 99th Percentile 4th Power	0.73	14.79
Right 99th Percentile 4th Power	0.71	17.55

An important parameter of consideration is to understand the difference between the left and right road wear when including road crossfall. The results regarding the percentage difference between the left and right road wear for all the criteria considered is shown in Table 3-2. The results show that the average absolute errors only increase slightly compared to the individual left and right models shown in Table 3-1. The maximum errors also increase slightly, for all of the criteria considered but are still reasonable, especially for the 4th power results where small differences in tyre forces can produce relatively large differences in results due to raising it to the 4th power. The errors in Table 3-1 and Table 3-2 are of the same approximate value and this shows that the model is able to accurately capture the difference in the left and right dynamic road wear of a rigid heavy vehicle. There are only a small number of outlier values that produce relatively high errors, but the average absolute errors are very low and confirm that the model is able to accurately capture the underlying phenomena. The results however illustrate that the need exists to include more training examples of vehicles that produce high dynamic road wear values if the maximum absolute errors are to be decreased.

Table 3-2: Summary of results comparing percentage difference between left and right dynamic road wear values

Road wear criteria	Average absolute error (%)	Maximum error (%)
Difference left and right 95th 1st Power	0.17	7.21
Difference left and right 99th 1st Power	0.21	7.31
Difference left and right 95th 4th Power	0.69	24.87
Difference left and right 99th 4th Power	0.85	24.82

These models represent the first attempt documented in literature at building GPML models for dynamic road wear. One of the objectives of this research was to determine whether the model can interpolate and extrapolate for different input data regarding the road crossfall. The three scenarios are discussed in Section 2 and the results are summarised in Table 3-3 for the left 95th percentile 4th power aggregate force (the most important criteria). The results show that the average absolute and maximum errors for the 1% CF remain more or less the same when still using the stepwise inputs (average absolute error of 0.69% and maximum error of 9.69%). The results, therefore, show that the model can accurately interpolate over road crossfall values and this limits the number of training examples required when considering crossfall as a parameter.

Table 3-3: Summary of results for additional test cases using different crossfall (CF) values

Road wear criteria	Average absolute error (%)	Maximum error (%)
% Error Left 95th Percentile 4th Power interpolated for 1 CF	0.69	9.69
% Error Left 95th Percentile 4th Power extrapolated for 3 CF	2.69	14.09
% Error Left 95th Percentile 4th Power interpolated for 2 CF and any input value for parameters (not step wise)	0.50	7.53

Table 3-3 also shows the results when extrapolating to a road crossfall of 3% CF. The results show that the average absolute error increases but is still at a reasonable value of 2.69%. The maximum error has not increased substantially and is only 14%. This also illustrates that the model will be able to extrapolate for unseen crossfall data, but that the accuracy does decrease slightly.

The last test conducted was to use a crossfall of 2% but to specify any input parameter over its test range with a step value of 1 unit for all parameters selected. The results show that the accuracy has essentially not changed when comparing it to the stepwise trained function as seen in Table 3-3 vs Table 3-1. This is once again extremely advantageous as this limits the number of training examples needed, without necessarily affecting the accuracy of the model

GPML has the advantage of providing the posterior probability of a result. This can be used to reduce the number of examples required to train the model to a satisfactory level of accuracy. Because this was the first iteration of the model, the order of magnitude normalised standard deviation (also referred to as the "sigma") required to achieve a certain level of accuracy was not yet known. Figure 3-2 summarises the results for the left 95th percentile 4th power aggregate tyre forces. The results show that the normalised sigma values vary between approximately 0.045 and 0.14 with the maximum absolute error being less than 12%. The results also show that the model seems to be very accurate for lower road wear values (represented by the positive error) and the accuracy decreases for larger road wear values (shown by the negative error). Therefore, it would be more advantageous to train the model over relatively higher expected road wear values than relatively low road wear values.

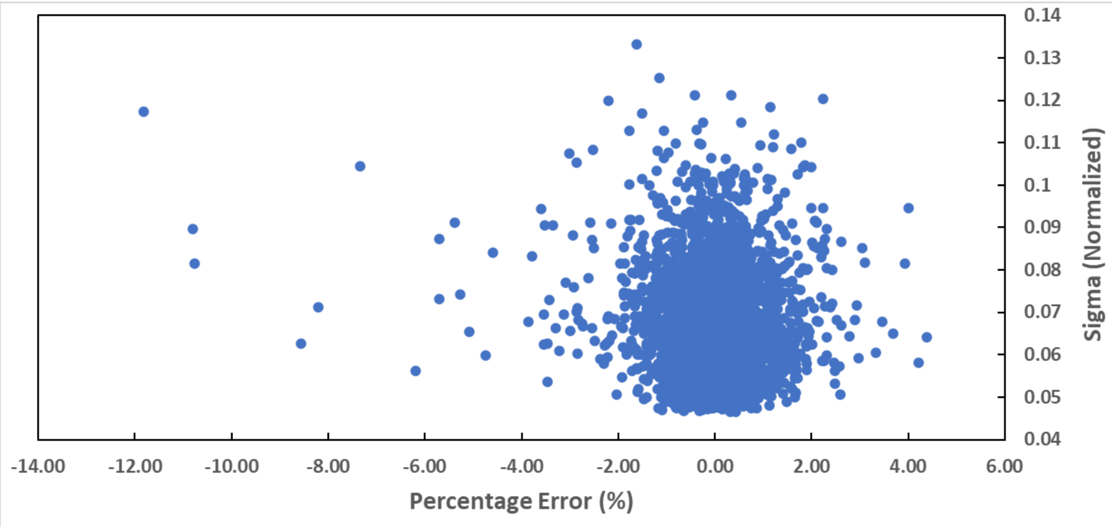

Figure 3-2: Relationship between the percentage error and normalized standard deviation (sigma) of the left 95th percentile 4th power aggregate tyre (N^{4})

4 CONCLUSION

This paper presents the results of the development of the first documented Gaussian process machine learning (GPML) model for the prediction of the dynamic road wear resulted from a rigid heavy vehicle. The accuracy of the models for the different dynamic road wear criteria are all very high (average absolute errors less than 1%) for any interpolated road crossfall value of 0% CF to 2% CF. The accuracy decreases slightly for extrapolated road crossfall values, but remains very accurate (less than 5% average absolute error for all road wear criteria).

This paper proves that the use of GPML appears to be a viable and effective method to develop a model to accurately predict the dynamic road wear expected from a rigid vehicle. It is therefore a viable option to consider for future research when including more input parameters and more complex vehicle designs. The accuracy of the models can also be improved by increasing the number of training examples, especially for vehicle configurations that produce large dynamic tyre forces. Using machine learning models will offer several advantages to users to obtain quick and accurate predictions of values instead of having to perform a detailed study, which often requires substantial expertise and also complex and expensive software. If this methodology is expanded to include more complex vehicle designs, it will be very useful to the Smart Truck Pilot Project.

5 RECOMMENDATIONS

This work represents the first attempt at developing a Gaussian process machine learning (GPML) model for dynamic road wear and as such there are still various improvements and updates that can be done in future work.

The first recommendation is to include more input parameters, especially the mass and inertial properties of the vehicles as this will significantly influence the road wear produced. The second recommendation is to optimise the learning model by including the desired level of standard deviation (sigma) and to sample over the test space while training in such a way as to minimise the standard deviation as quickly as possible (i.e. relatively quick optimisation). A third recommendation is to expand on this model by incorporating more complex heavy vehicle designs such as the B-double combination which is very common in South Africa. This would be of special value to the Smart Truck Pilot Project where a road impact assessment is required. Having tools that can predict dynamic road wear will allow for rapid testing and development and reduce development time and costs.

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