MECHANISTIC DESIGN OPTIMISATION OF THIN ASPHALT SURFACINGS USING MEASURED NON-UNIFORM TYRE-ROAD CONTACT AND INCORPORATING RECTANGULAR CONTACT SHAPES AND LAYER CROSS-ANISOTROPY

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ABSTRACT

Mechanistic design and construction of flexible pavements with relatively thin (< 50 mm) asphalt road surfacings still remain an art today and are aimed especially towards first-class user experiences. Risks for surfacing failures are difficult to assess mechanistically, but poor design and construction will undoubtedly lead to dissatisfied road users. Traditional mechanistic pavement design commonly uses a simple single circular tyre model with uniform vertical contact stress. It is popular because of its relative simplicity (mathematical) and fast speed of solution. Although relatively thin asphalt surfacings are still regarded as a “functional” layer designed and built by experienced designers and contractors, research has pointed towards noticeable effects when designing these layers as “structural” layers with improved tyre-road contact models, such as rectangular shapes, and/or non-uniform contact stresses. Assessment of tyre-road contact shapes from field data (52 895 truck tyres) shows that only 10% of tyre-road contact patches can be classified as circular, with 66% are single rectangular, and 24% are of a triple-rectangular shape. This information points towards the need for an improved mechanistic design of thin surfacings that focuses on optimisation and longevity. Research and associated analysis have led to improved closed-form analysis that is optimised for speed and contact shape. In this paper, improved multi-layer linear elastic analysis that incorporates an option to divide the tyre-road contact into three rectangular shapes (in addition to the conventional circular shape) is demonstrated. The software used in the study was developed for mechanistic-empirical (me) CRoss-anisotropic Analysis of Multi-layer Elastic Systems (*me*CRAMES). It was designed to include options for single/multiple circular and/or rectangular tyre-loading contact shapes, lateral loading, layer cross-anisotropy and interlayer slip. Analyses that were presented indicate potential failure zones inside these thin asphalt surfacings, which are dependent on tyre-road contact shapes, layer cross-anisotropy and/or interlayer slip. It is recommended that these aspects be incorporated during the design stage.

INTRODUCTION AND BACKGROUND

Vehicle-road interaction (VRI) remains one of the most challenging pavement engineering aspects that is difficult to quantify without a list of assumptions. It is, however, the opinion of the authors that as more data is captured, quantified, and turned into knowledge, road managers are better placed to manage their road assets well. Within the context of full-scale accelerated testing of road pavements and surfaces, engineers have demonstrated the difficulties in defining "damage" to surfaces, as well as the mechanistic aspects behind such damage (Jones, 2012; Aguiar-Moya, 2016). It is hypothesised that if pavement structural damage is well understood, the risk for premature failure could be minimised, especially in pavement surfacings. Hence, research and development on VRI is an ongoing process that requires the actual forces within a tyre-road contact patch to be quantified (De Beer et al., 1997, 2012, 2013, 2015, 2016; Himeno et al., 1997; De Beer and Fisher, 2016; Maina et al., 2012, 2013, 2017a, 2017b). With these forces (i.e. contact stresses) known, mechanistic analyses could largely assist in defining "hot spots" in the surfacing layers (or in the rest of pavement as well), which are indicative of potential damage locations. By changing surfacing stiffness (or its thickness or characteristics of layers deeper in the structure), these "hot spots" could be minimised, thus leading to sustainable pavement structural design.

The main aim of this paper is to highlight mechanistic design optimisation of thin asphalt surfacings by using measured non-uniform tyre-road contact that incorporates rectangular contact shapes and layer material cross-anisotropy. Secondly, it demonstrates that the following main factors could potentially influence the structural performance of thin surfacing life:

- The shape of the vertical tyre-road contact patch
- Idealisation of the non-uniformity of vertical contact stress, and associated effects on thin surfacing
- Effect of layer cross-anisotropy (Vertical elastic modulus (E_v) vs Horizontal elastic modulus (E_h))
- Effects of a very thin isotropic interlayer (1 mm, 1 MPa), as well as cross-anisotropic layer of 5 mm thick under the thin asphalt surfacing layer
- The interlayer slip – i.e. interlayers with and without friction

A Stress-In-Motion (SIM) device was used to study tyre-road interactions – under controlled as well as uncontrolled conditions – as the device produces 3D forces (i.e. contact stresses) between the tyre and the rough test surface (De Beer et al., 1997; De Beer and Fisher, 2013). The current study proposes idealisation of both patch dimensions and contact stress uniformity to address this problem in terms of mechanistic-empirical road pavement design. These propositions are demonstrated with simple multi-layer linear elastic (MLLE) analysis using mechanistic-empirical Cross-Anisotropic Analysis of Multi-layer Elastic Systems (meCRAMES) (Maina and Matsui, 2004; Maina et al., 2017a, 2017b).

CHARACTERISING TYRE-ROAD CONTACT SHAPES: LABORATORY AND FIELD DATA

Laboratory data

In an extensive laboratory study of full-scale test truck tyres on the Heavy Vehicle Simulator (HVS), with both tyre load and tyre inflation pressure controlled, the tyre-road contact stresses of a 12R22.5 tyre (most widely used truck tyres in South Africa) were measured with SIM technology (De Beer et al., 2005). The "footprints" of the vertical contact stresses of this tyre are illustrated in Figure 1. The figure indicates that the outside shape of the tyre contact patch changes significantly from basic circular to rectangular when tyre loading is increased (vertically in Figure 1), whereas fewer changes are noticed with a change in tyre inflation pressure (TIP) (horizontally in Figure 1). It should be mentioned that the tyre loading in the full-scale testing is considered to be relatively higher than that of a typical truck. This is important because actual outside contact shapes at various tyre loadings and inflation pressures have an impact on the modelling of pavements for design and analysis purposes.

In order to evaluate tyre-road contact stresses from real vehicles (tyres), a major field study was undertaken which is summarised in the next section. The study focused on the outline shape of the measured vertical tyre-road contact stresses.

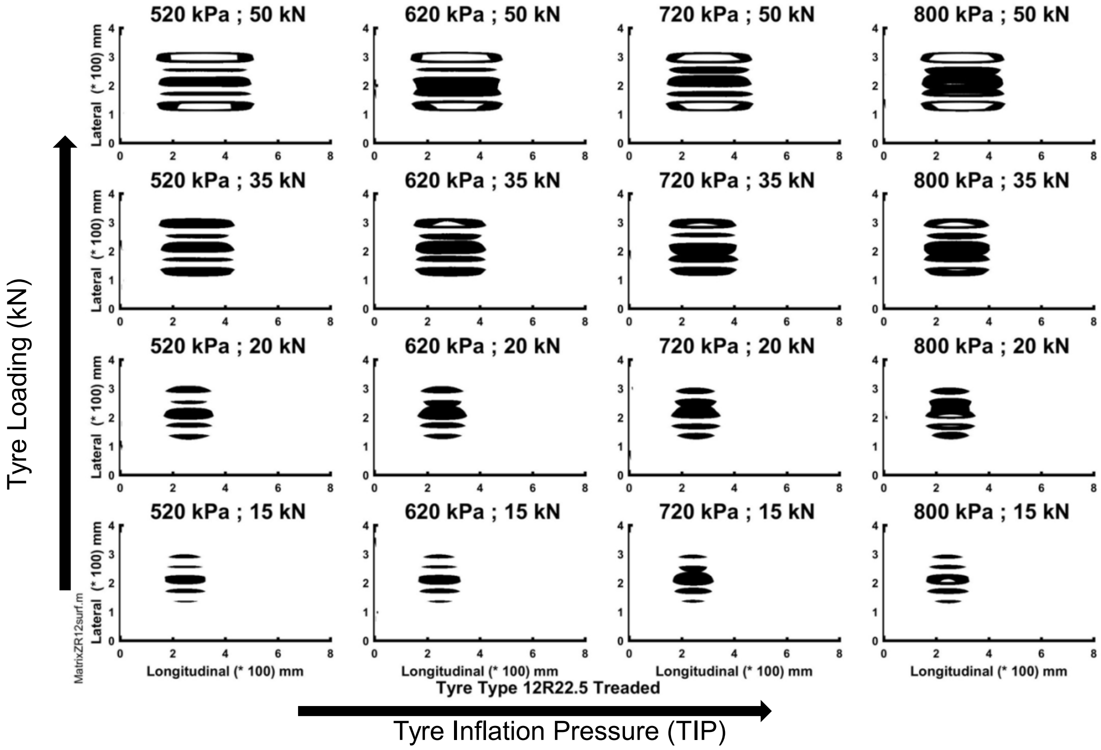

Figure 1. *Example of measured vertical contact stress shapes for a range of test tyre loadings and tyre inflation pressures for a 12R22.5 tyre (laboratory data)*

Field Study

During 2003/4 a major field study was undertaken in which more than 3 000 truck tyres were measured with SIM technology (De Beer, 2006; De Beer et al., 2012, 2016). These trucks were re-routed over a full installation of SIM for the full axle of each truck (see Figure 2). Typical measured vertical stress footprints of the 22 tyres of a six-axle truck on the N3 National Freeway, with best-fit shapes of the vertical contact stress, are illustrated in Figure 3. Analyses have shown that only approximately 10% of these were of best-fit circular shape (CIRC), 66% were of best-fit triple rectangular shape (RECT3), and 24% were of best-fit single rectangular shape (RECT1). The best-fit shapes were determined by maximising the vertical contact area within a particular shape (SARDS, 2013). Since the majority of the best-fit shapes of measured vertical tyre contact is rectangular, effort was needed to develop relatively high-speed closed-form solutions for analysing rectangular shapes as opposed to the traditional standard circular shape for vertical tyre-road contact (Maina and Matsui, 2004; Maina et al., 2013, 2017a, 2017b).

Figure 2. *Example of a truck diverted from N3 Freeway near Heidelberg (Gauteng, South Africa) over the experimental site for Stress-In-Motion measurements for each tyre*

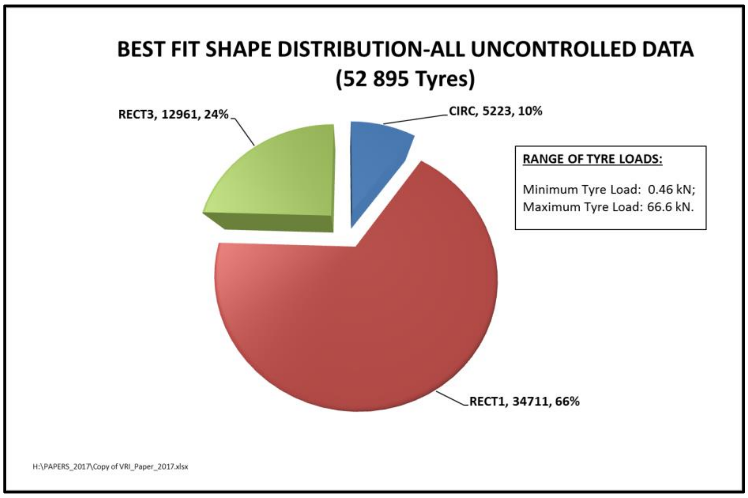

Figure 3. *Distribution of best-fit shapes of Vertical Contact Stress (VCS) – 52 895 tyres from field measurements*

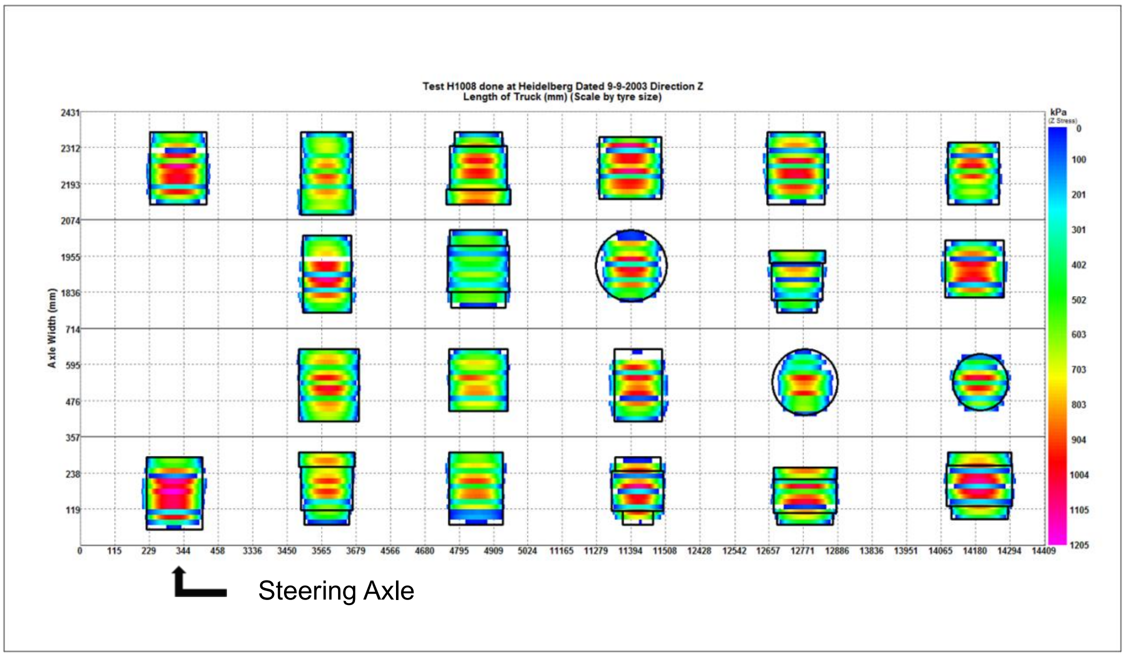

Figure 4. *Typical measured vertical stress footprints of the 22 tyres of a six-axle truck diverted from National Freeway N3, with best-fit contact shapes (field data)*

EFFECT OF VERTICAL TYRE CONTACT SHAPE ON PAVEMENT RESPONSE – THEORY, RESULTS AND DISCUSSION

Damage parameter: Strain Energy of Distortion (SED)

To determine pavement response, the Strain Energy of Distortion (SED) parameter was used. SED is defined as the quantity of strain energy stored per unit volume of the material that can be used as a basis for determining the limiting stress at which failure occurs (Timoshenko and Goodier, 1951; Perdomo and Nokes, 1993). It is the notion in this paper that locations within the pavement structural system that have relatively higher values of SED (so-called ‘‘hot spots’’) will potentially fail first before locations with relatively lower values of SED are damaged.

SED can be expressed as follows (Timoshenko and Goodier, 1951):

$$SED = V_0 - \frac{1-2\nu}{6E}\left(\sigma_x + \sigma_y + \sigma_z\right)^2 \tag{1}$$

where V_0 is total strain energy per unit volume, expressed by using Hooke’s law as follows:

$$V_0 = \frac{1}{2E}\left(\sigma_x^2 + \sigma_y^2 + \sigma_z^2\right) - \frac{\nu}{E}\left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_z\sigma_x\right) + \frac{1}{2G}\left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2\right) \tag{2}$$

with σ_x, σ_y andσ_z being normal stresses in x, y, and z directions respectively, whereas τ_{xy}, τ_{xz} andτ_{yz} are shear stress on x, y, and z planes.

Pavement for analysis

For the analysis done in this paper, a 5-layer pavement was selected, as shown in Table 1 below.

Table 1. Five-layered pavement for multi-layer analysis

Material*	Thick ness (mm)	Vertical (v) E-Modulus, E_v (MPa)	Horizontal (h) E-Modulus, E_h (MPa)	Poisson's Ratio, ν_v	Poisson's Ratio, ν_h	Slip Rate (0=Full Friction)
Asphalt (AG)	40	3000	3000	0.44	0.44	0
Crushed Stone (G2)	150	400	400	0.35	0.35	0
Stabilised (C4)	150	1500	1500	0.35	0.35	0
Selected layer Layer	150	120	120	0.35	0.35	0
Subgrade	∞	70	70	0.35	0.35	0
Aim_1: To demonstrate the effect of tyre-road contact shapes on pavement structure						

* Material Code according to TRH 14 (1985)

Vertical tyre load contact shapes

For the pavement analysis, the following three basic tyre-road contact shapes were considered for illustration purposes of the best-fit shape:

- CIRC: Circular shape with fixed radius at 110 mm (limited to nominal tyre tread width of 220 mm)
- RECT1: Single rectangular shape with fixed width of 220 mm
- RECT3: Triple rectangular shape with fixed width of 220 mm (the tyre contact patch is divided into three rectangular shapes including two outer areas (20% of tyre width), and a central area (60% of tyre width))

Note: Above three basis shapes are also illustrated in Figure 4 above.

Analysis with circular shape (CIRC)

An example of a typical circular model with tyre load of 20 kN and average of 700 kPa contact stress is shown in Figure 5, with SED contour plot profile given in Figure 6. Note in the latter case the SEDmax (314 Nm/m^3) occurred at the bottom of the thin (40 mm) asphalt layer, at the tyre centre X-Y (0, 0) position.

Analysis with rectangular shape (RECT1)

An example of a typical rectangular (square) model with tyre load of 20 kN and average of 700 kPa contact stress is shown in Figure 6, with SED contour plot profile given in Figure 8. Note in this case the SEDmax also occurred at the bottom of the thin (40 mm) asphalt layer, at the tyre centre X-Y (0, 0) position.

Analysis with triple rectangular shape (RECT3)

An example of a typical triple rectangular model with tyre load of 40 kN (overloaded) and maximum contact edge stress of 526 kPa is shown in Figure 9, with the SED contour plot profile given in Figure 10. Note in this case the SEDmax of 124 Nm/m^3 occurred on the surface (Z = 0) of the thin asphalt layer, at the tyre centre X-Y (0, 0) position.

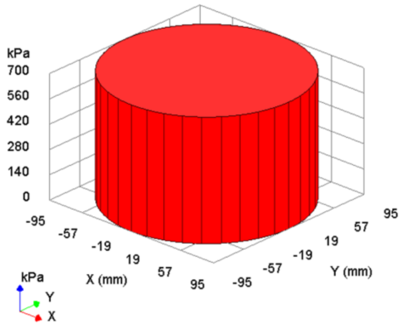

Figure 5. *Circular tyre model (CIRC): 20 kN load and 700 kPa contact stress*

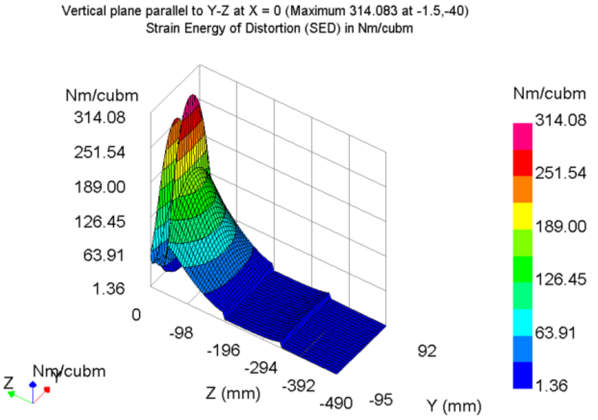

Figure 6. *Profile contour plot of SED with pavement depth (note SEDmax 314 Nm/m^3 at the bottom (Z = - 40 mm) of the asphalt layer at Y = 0*

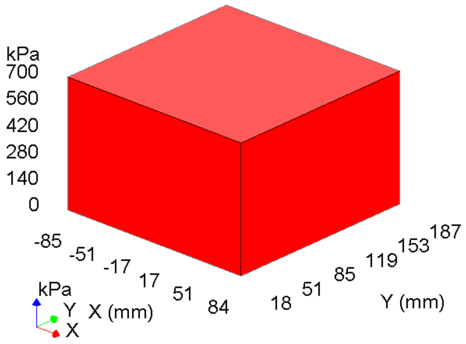

Figure 7. *Rectangular (sq) tyre model (RECT1) with 20 kN loading and 700 kPa contact stress*

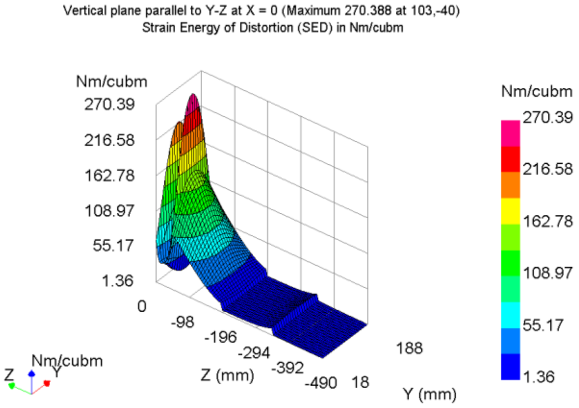

Figure 8. *Profile contour plot of SED with pavement depth (note SEDmax of 270 Nm/m^3 at the bottom (Z = - 40 mm) of the asphalt layer at Y = 0*

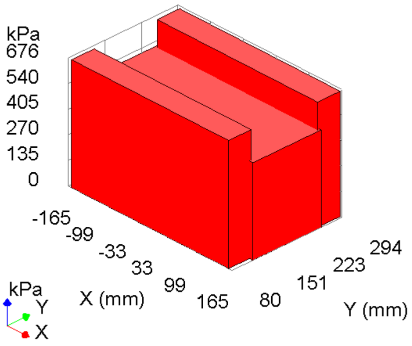

Figure 9. *Triple Rectangular (sq) tyre model (RECT3) with 40 kN loading and 676 kPa maximum contact stress*

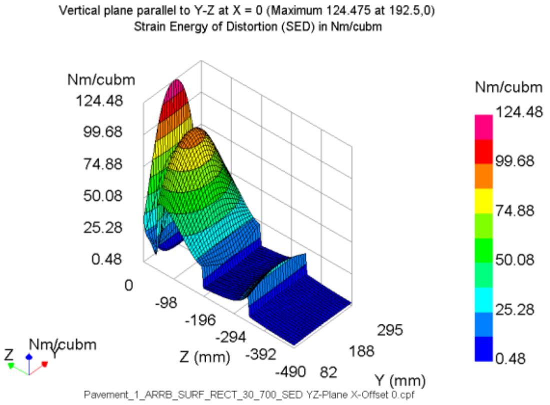

Figure 10. *Profile contour plot of SED with pavement depth (note SEDmax of 124 Nm/m^3 at the top (Z = 0 mm) of the asphalt layer at Y = 0*

Effect of vertical tyre loading and contact shape on SED

In order to estimate the effect of tyre loading and contact shape on pavement damage (as defined here by SED), several MLLE analyses were performed using meCRAMES software. The results are illustrated in Figure 11. Firstly, a relative comparison is shown between tyre models "CIRC" and "RECT1" over a range of single tyre load from 5 kN to 50 kN *at the same assumed vertical contact stress*. For these cases, the SED increased non-linearly to values exceeding 1000 Nm/m^3. In addition, the SED from the RECT1 tyre model is approximately 9% lower compared to the CIRC tyre model. It is therefore the researchers' opinion that a rectangular shape vs. a circular shape tyre-road contact results in somewhat less conservative value for "damage" (i.e. SED) within the thin asphalt surfacing layer. In addition, two sets of triple rectangular (RECT3) tyre models were compared, using measured contact stresses with best-fit rectangular shapes. The one set was analysed with Tyre Inflation Pressure (TIP) at 700 kPa, and the other at 950 kPa. The results indicate that both these tyre models result in much lower SEDs compared with the CIRC and RECT1 models. Furthermore, the effect of TIP is also visible, showing approximately 30% lower SEDs for the model at TIP of 700 kPa, compared with the model at TIP of 950 kPa.

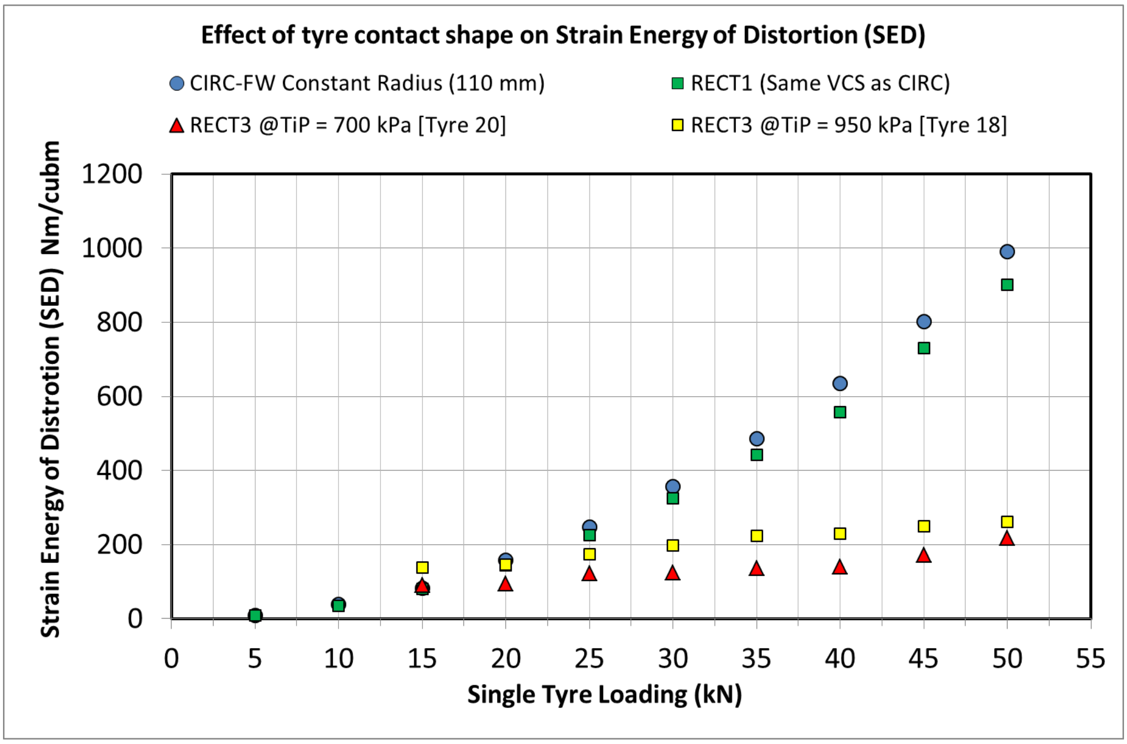

Figure 11. *Effect of vertical single tyre loading and contact shapes on the Strain Energy of Distortion (SED) response.*

Effects of a very thin interlayer under the thin asphalt surfacing layer

Generally, one of the weakest areas in the pavement structure is the contact between the thin asphalt surfacing and the top of the base layer. This may be due to several factors, for instance not enough brooming, or simply poor construction of the final base layer. Also, over time the contact may be lost due to relative movement between thin surfacing and base layer owing to crushing (Litwinowicz and De Beer, 2013). Some deleterious carbonation may also weaken the top of the base by forming an interlayer, if the base is stabilised (De Beer et al., 2012, 2016; Netterberg and De Beer, 2012). In order to quantify the areas of potential distortion (or damage), pavement analyses indicated that the "weaker" the interface, the higher the damage parameter, SED. To demonstrate this effect, very thin (1 mm and 5 mm) layers with very low moduli of elasticity (1 MPa) were inserted between the asphalt surfacing and the top of the granular base. It was found that the pavement system with these very thin interlayers responds drastically differently to pavement loading, irrespective of whether the shape of the tyre contact patch is single circular (CIRC) or single rectangular (RECT1). However, the SEDs from triple rectangular shapes (RECT3) are much lower relative to the single patch shapes (as was illustrated in Figure 11 above and Figure 12).

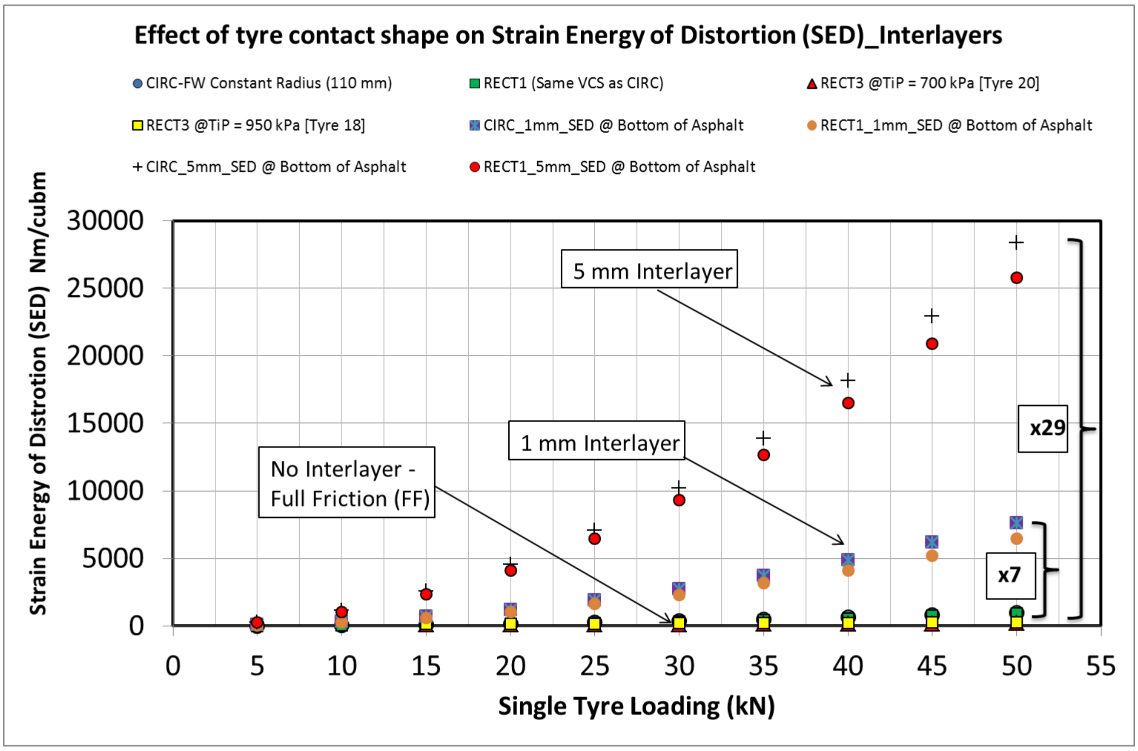

Figure 12. *Effect of vertical single tyre loading and contact shape on Strain Energy of Distortion (SED) with various interlayer conditions at the bottom of asphalt layer*

According to the results of SED indicated in Figure 12, there is an exponential increase in "damage potential" with an increase in tyre loading (under the tyre). In addition, there is a sevenfold increase in SED for the 1 mm interlayer isotropic case, compared to the case with no interlayer, and a 29-fold increase in SED when the thickness of the isotropic interlayer increases to 5 mm. There is also a notable effect when comparing the circular tyre patch with a rectangular patch, especially for tyre loading in excess of 15 kN on pavement with 1 mm interlayer – approximately 8% higher for the circular patch (CIRC), compared with an equivalent single rectangular tyre contact shape (RECT1).

Interlayer slip modelling – (i.e. interlayers with and without friction)

As indicated above, one of the main factors affecting performance of in-service pavements is the condition at layer interfaces. Pavement sections, especially at stop-and-go or turning sections, are prone to layer de-bonding because of the presence of high horizontal (shear) forces. Layer de-bonding causes redistribution of stresses and strains in the pavement structure, which may led to premature pavement failure. A compliance-type model is implemented to express slip at any pavement layer interface. When a slip takes place at the interface between *i*th and (*i*+1)th layers, the phenomenon is modelled as

$$(1-\alpha_i)\left\{u_j^i(h_i)-u_j^{i+1}(0)\right\}=\alpha_i\beta_i\tau_{zj}^i(h_i) \tag{3}$$

in which α_i is called a slip rate and *j* can either be x or y. Complete bonding is represented by $\alpha_i = 0$, while if α_i approaches 1, the interface condition becomes nearly perfect slip. $\tau_{zj}^i(h_i)$ is shear stress at the interface between *i*th and (*i*+1)th layers on the z-plane in either x or y direction. Equations (4) ~ (5) represent the three models for β_i that can be considered to model interface slip, such that α_i becomes a dimensionless parameter.

Slip Model 1: $$\beta_i = b^*\left(\frac{1+\nu_i}{E_i}+\frac{1+\nu_{i+1}}{E_{i+1}}\right) \tag{4}$$

Slip Model 2: $$\beta_i = 2b^*\left(\frac{1+\nu_i}{E_i}\right) \tag{5}$$

Slip Model 3: $$\beta_i = 2b^* \sqrt{\left(\frac{1+\nu_i}{E_i}\right)\left(\frac{1+\nu_{i+1}}{E_{i+1}}\right)} \quad (6)$$

b^* must have a unit of length in order for α_i to be dimensionless. Thus b^* is selected as a maximum dimension among the surface multiple loads.

Equation (4) is the model, which shows the arithmetical means of inverted shear moduli of i^{th} and $(i+1)^{th}$ layers. Equation (5) uses only the inverted shear modulus of upper layer, while Equation (6) is the geometric mean of inverted shear moduli between i^{th} and $(i+1)^{th}$ layers. Currently, Slip Model 1 is implemented for roads pavements and Slip Model 3 for airport pavements (Maina and Matsui, 2004; Maina et al., 2007).

Effect of layer cross-anisotropy

Cross-anisotropy is a material property resulting from the directional rolling compaction of a road construction process where the elastic properties in the horizontal (E_h) and vertical (E_v) directions are considered to be different (Masad et al., 2006). This is the most prevalent but less considered type of material properties in pavement layers. The level of cross-anisotropy is typically characterised by the ratio of the horizontal to vertical modulus (E_h/E_v). Masad et al. (2006) used the Finite Element (FE) program to show how the accuracy of calculated pavement displacements improved when the aggregate base layer was considered to be cross-anisotropic. In their study, Masad et al. (2006) revealed that the displacements calculated by using isotropic material properties tend to be smaller than measured displacements in the field. Further analysis showed that the errors between measured and calculated responses were minimised when the elastic modulus in the horizontal direction was assumed to be 30% of the elastic modulus in the vertical direction (E_h/E_v = 0.3).

In this paper, layer cross-anisotropy ($i.e.\ E_h/E_v$ = *3/10 = 0.3*) of the 5 mm cross-anisotropic interlayer also affects the pavement response in terms of SED, showing an 87-fold increase for both the single circular and single rectangular tyre patch shapes, compared to only a 29-fold increase for the isotropic case. It was interesting to note that the SEDmax occurred at the bottom of the thin layer (Z = - 45 mm). Further analyses are needed to validate this important finding.

SUMMARY AND CONCLUSIONS

In this paper, the authors attempted to optimise the mechanistic design of thin asphalt surfacings by using measured non-uniform tyre-road contact and incorporating rectangular contact shapes, weak interlayers and layer cross-anisotropy. In addition, interlayer weakness was addressed and the following conclusions were reached:

- The majority of measured tyre-road contact shapes are non-circular, and more rectangular. Both single rectangular (RECT1) and triple rectangular (RECT3) tyre models are proposed for multi-layer pavement modelling.
- Analytical methods were developed and used to quantify the differences between circular and rectangular tyre-road contact patches. It was found that for the same amount of uniform vertical contact stress, the rectangular load contact patches (RECT1) produced pavement response (SEDs) approximately *14% lower* compared to the circular (CIRC) tyre model.
- The higher the tyre inflation pressure (TIP), the higher the SED. The effect of tyre inflation pressure (TIP) showed approximately *30% lower SED* values for the tyre model at TIP = 700 kPa, compared with the tyre model at TIP = 950 kPa.
- Weak interlayers and interlayer slip can be modelled using a relatively thin layer (1 mm; 1 MPa) between asphalt surfacing and top of base layer. A 7-fold and a 29-fold increase in SED resulted at the bottom of the asphalt surfacing when these thin layers (i.e. 1 mm and 5 mm respectively, isotropic) were modelled as part of the pavement structure. Therefore, the importance of the ability to model interlayer slip is highly recommended.
- Layer cross-anisotropy (E_h/E_v = *0.3*) also affects the pavement response in terms of SED and shows an 87-fold increase (for both the single circular and single rectangular tyre patch), compared to a 29-fold increase for the isotropic case for the 5 mm interlayer. Further analyses are however needed to further validate this important finding.

RECOMMENDATIONS

Based on the preliminary findings in this paper, especially for thinly (< 40mm) surfaced flexible pavements, it is recommended to model tyre-road interaction with more realistic contact shapes, such as single or multiple rectangular shapes. Weak interlayers (or low interlayer friction) should also receive more attention in order to optimise design and performance of these thinly surfaced pavements.

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