Inaccuracies When Mixing Coordinate Reference Frameworks in a System of Systems Simulation

Bernardt Duvenhage and Jan Jacobus Nel

Abstract—**The modelling of military systems of systems invariably involves the incorporation of existing, often shared, models into larger and more complex simulations. The sharing of models between simulations has long been the goal of the simulation community and although the resulting simulations expand the application domain of the models it comes at a cost. In this paper the authors highlight one of the issues faced, namely the use of different coordinate frameworks on performance and quantifies the errors that could be introduced into the simulation. It starts by providing an overview of the most commonly used coordinate frameworks and then elaborates on some of the experiences gained through its application in the Virtual Ground Based Air Defence System (GBADS) simulator. The focus is on the mixing of flat earth convention and spherical earth convention. The authors conclude with further motivation why a common earth reference model should be used for all simulation entities and, more importantly, why it should be a "real world" earth reference model.**

Index Terms— **Coordinate reference frame, earth reference model, modelling & simulation, model application domain**

I. Introduction

The modelling of a complex simulator, that consists of a number of systems and sub-systems, often requires the use of models that were developed for different application domains. This is because it is more economically viable to re-use existing models and because it is often desirable to use a model supplied by the Original Equipment Manufacturers (OEMs). As a result the developers and users of the simulators end up with a mixture of models with different fidelities and performances.

Organisations such as the Simulation Interoperability and Standards Organisation (SISO) have strived to develop frameworks to alleviate this problem. Not withstanding these efforts, in practice it still occurs.

In this paper one aspect, namely the mixture of coordinate reference frameworks is explored. The issues surrounding coordinate reference frameworks and the translation between these are not new and are widely discussed in the literature ([1], [2], [3], [4] and [5]). The authors felt, however, that benefit could be gained by quantifying some of the errors as discovered in the Virtual Ground Based Air Defence (GBADS) Simulator, specifically; the impact of converting between flat earth and spherical earth reference models is discussed with some suggestions for the mitigation thereof.

II. Earth Reference Model

A coordinate reference frame is a description of the coordinate axis and reference point required for the specification of position, relative direction and orientation. For a coordinate reference frame to be useful for geo-positioning and orientation it must include a spatial reference model of some kind for the earth.

An Earth Reference Model (ERM) represents an approximation to the geometrical shape of the earth and the gravitational potential associated with the earth. Different ERMs exist of which the most commonly used are flat and spherical earth. Another well known ERM is the World Geodetic System from 1984 (WGS84). For the purposes of this paper the authors however focus on the simpler flat and spherical earth models with example coordinate reference frames.

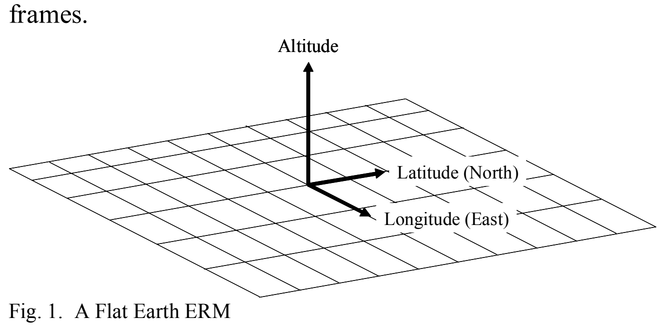

Fig. 1. A Flat Earth ERM

A flat ERM (as depicted in Fig. 1) consists of a plane on which longitude and latitude lines may conceptually be drawn. The latitude and longitude lines are parallel lines on a plane that represents the mean sea level. The altitude of a coordinate is the distance above or below this plane with the gravity vector being normal to it and pointing down. Such a system can be called an augmented ERM as it has no real physical basis. It distorts the geometrical properties of the real ball-like earth. Due to this distortion the conversion from

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latitude and longitude angles to North and East in meter is an arbitrary linear one based on the earth's equatorial circumference. In effect the latitude circles become all of similar length and equal to the longitudinal (polar) great circles.

A spherical ERM (as depicted in Fig. 2) consists of a perfectly spherical earth with a gravity vector always pointing towards the centre of the earth. This ERM ignores the flattening of the real earth at the poles. In other words, the equatorial and pole to pole diameters of the earth, as modelled in this ERM, are equal. This paper will use the term 'real world' ERMs to refer to spherical, WGS84 and better ERM approximations.

A. The Earth-Centred, Earth-Fixed, Cartesian Reference Frame

The Earth-Centred, Earth-Fixed (ECEF) reference frame (as depicted in Fig. 2) is a Cartesian coordinate system that represents a position as a point in X, Y, Z space, measured in meters along each axis.

The ECEF Cartesian coordinate (0,0,0) represents the centre of the earth. The Z-axis represents the earth's rotation axis with north being in the direction of positive z. The X- and Y-axis both rotate with the earth and always go through latitude and longitude (0˚, 0˚) and latitude and longitude (0˚, 90˚East) respectively.

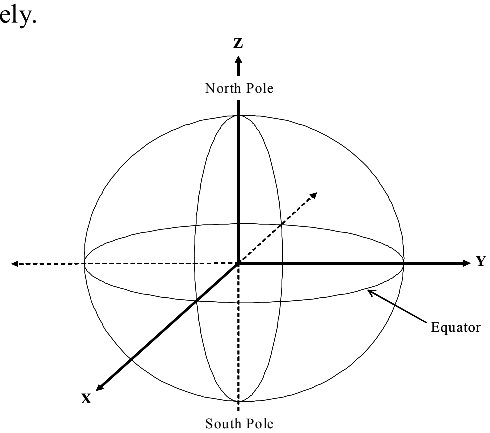
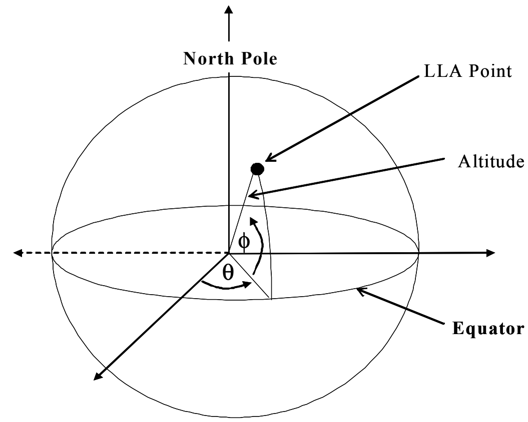


Fig. 2. The ECEF Cartesian coordinate reference frame for a spherical ERM

B. The ECEF Latitude-Longitude-Altitude Reference Frame

The Latitude-Longitude-Altitude (LLA) reference frame (as depicted in Fig. 3) indicates that a point's position is specified using spherical coordinates with an altitude equal to the height above mean sea level in meter. Latitude 0˚ refers to the equator and longitude 0˚ refers to the semi-great circle going through the Greenwich meridian. The latitude of a point is measured as the offset from the equator and longitude as the offset from the Greenwich meridian.

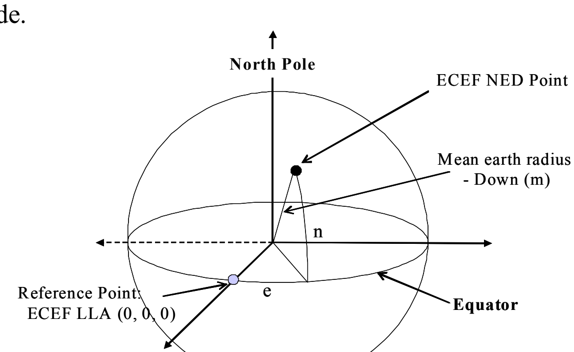


θ = Longitude (angle)
ϕ = Latitude (angle)
Altitude = - (Mean Earth Radius + Altitude) (meters)

Fig. 3. The ECEF LLA coordinate reference frame for a spherical ERM.

C. The ECEF North East Down Reference Frame

The North-East-Down (NED) reference frame (as depicted in Fig. 4) indicates a point's position by measuring the distance north (measured on a mean earth radius circle) and the distance east (measured on the relevant line of constant latitude) relative to some reference point which is usually ECEF LLA (0,0,0). Down is equal to the negative of the altitude.



n = North (m)
e = East (m)
Down = -(Altitude) (m)

Fig. 4. The ECEF NED coordinate reference frame for a spherical ERM.

D. The Local-Level, Local-North (LLLN) NED Reference Frame

The LLLN NED reference frame (as depicted by Fig. 5) indicates that there is a local reference frame (local to the reference point) and that it is a Cartesian system, on a flat plain. The Cartesian system's axes are rotated such that the North axis is a tangential to the circle through the reference point, towards positive latitude. The Down axis points towards the earth's centre and the East axis is tangential to a line of constant latitude through the reference point, towards the positive longitude. It is required to fix a LLLN NED system to some ECEF reference coordinate before it is usable in a simulation. The reference coordinate is often specified in ECEF LLA.

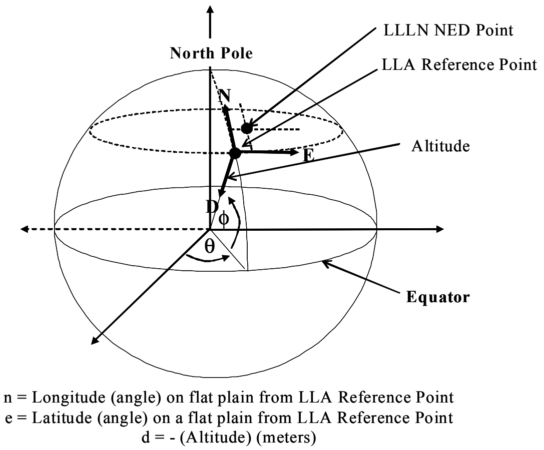


Fig. 5. The LLLN NED coordinate reference frame for a spherical ERM.

E. *The LLLN Azimuth, Elevation and Range Reference Frame*

The LLLN Azimuth, Elevation and Range (AER) reference frame (as depicted in Fig. 6) is a coordinate system that indicates the local direction and range (within LLLN; Fig. 5) from the reference point to some other position. In the case of a radar sensor the azimuth, elevation and range values of detections are calculated using the sensor's position as the reference point in the LLLN NED coordinate reference frame. Azimuth represents the position of the direction vector around the down axis relative to North and positive towards East. Elevation represents the direction vector's angle above the local level and range is the slant range.

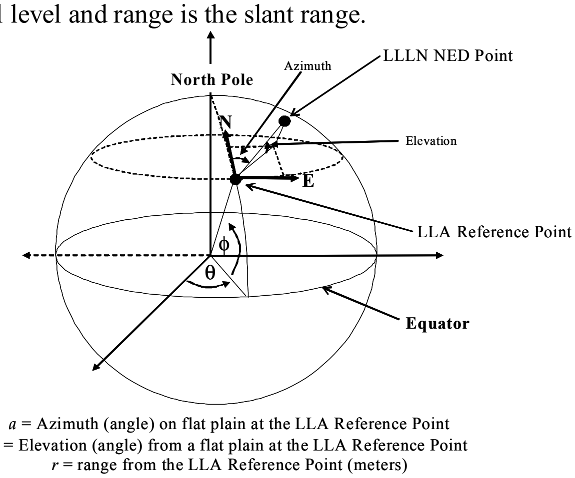


Fig. 6. The LLLN AER coordinate reference frame for a spherical ERM.

III. The Virtual GBADS Simulator

The Virtual GBADS Simulator is a system of systems simulator developed to support the acquisition of a Ground Based Air Defence System (GBADS) by the South African National Defence Force (SANDF).

The simulation architecture provides a generic distributed communication backbone that provides simulation time and frame synchronisation services only. The architecture consists of a number of different nodes, with each hosting a number of models.

A. *Models*

The convention used in the GBADS simulator was that all models should accept and give exact model position information in spherical ECEF LLA and modelled sensor observations in LLLN AER with the observer as the reference ECEF LLA coordinate. Coordinate conversion from LLA and AER to the model's internal coordinate reference frame and back is therefore the model's responsibility.

As the coordinate conversion is the model's responsibility, each model can 'operate' in any internal coordinate reference frame as long as they properly convert information to and from this internal representation. This standardized model interface also allows models with different internal ERMs to be used together. Problems experienced with mixing ERMs are addressed in the section titled *'Flat Earth Conversions Used On Spherical Earth'*.

B. *Simulator Viewers*

Two types of viewers are used in simulator namely a 3D viewer with terrain, making use of an internal spherical ERM, visibly curved horizon, etc., and a plan view (2D) viewer with a flat augmented ERM. Viewers are often used as developer tools when designing and debugging a simulation, but are also used during feedback sessions or for capturing video sequences of simulation runs.

Inaccuracies when using different ERMs in the simulation viewers and the simulation is as problematic as between the simulation and models if not more so. Confidence in a simulation is quickly lost when the visual feedback shows even small errors in spatial referencing such as target tracking or model positioning. Inaccuracies when mixing ERMs in the context of simulation viewers are discussed further in the section titled, *'Integration of Plan View and 3D Viewers'*.

IV. Flat Earth Conversions Used On Spherical Earth

In the interaction with model developers contributing to the Virtual GBADS Simulator it became apparent that the misconception exists that the differences in spatial referencing results between a flat earth LLA ERM and a spherical earth is purely range dependent and usually negligible. This may well be the case for scenarios near the equator, but for cases where scenarios play out in other locations, the difference is significant. Around 0˚North the latitude circles have approximately the equatorial circumference, but at latitudes closer to the poles this is definitely not the case and the flat earth LLA ERM becomes severely inaccurate for spatial referencing.

For example, the systematic (not random or Gaussian) target designation error of a flat earth radar sensor model located at 0˚N and 0˚E on a spherical earth is shown in Fig. 7, Fig. 8 and Fig. 9 below. These errors are introduced when converting from LLA to the sensor's local Cartesian coordinate reference frame, without applying corrections for the spherical ERM.

The figures show the sensor errors relative to the position of the sensor. The sensor is indicated by the radar icon, and a low altitude target is positioned as an example at a randomly selected position relative to it, indicated by the '+'. The sensor is therefore located at the origin of the figure which is showing a top view of the earth north (y-axis) and east (x-axis) of the sensor.

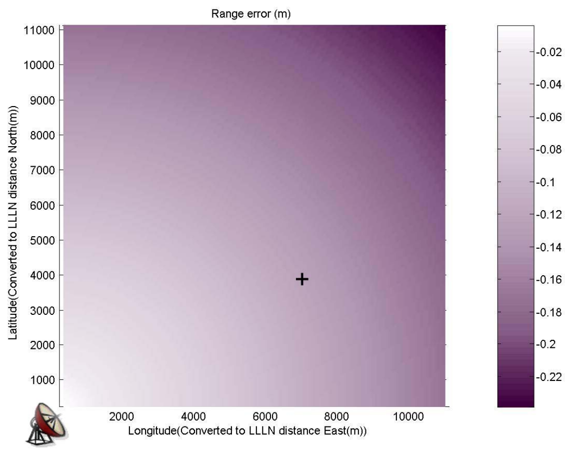


Fig. 7. Range error in meter made by a flat earth sensor over a 0.1 by 0.1 degree grid. Sensor located 0˚S and 0˚E.

The errors that are present represent the best case (smallest achievable errors) of a flat earth approximation to a spherical ERM.

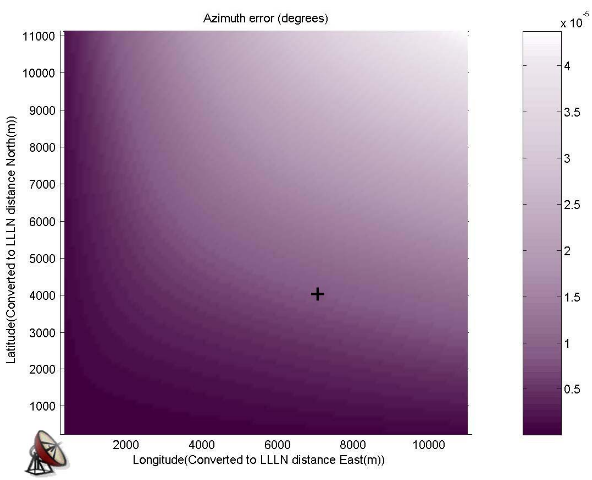


Fig. 8. Azimuth error in degrees made by a flat earth sensor over a 0.1 by 0.1 degree grid. Sensor located 0˚S and 0˚E.

For example, the measurement located at the '+', which can be read from the graph to be 4km North and 7km east of the radar and at a distance of approximately 8km, has systematic range, azimuth and elevation errors of 0.12km, 0.9˚ and 0.04˚ respectively.

As a second example, a flat earth radar sensor model is located at Overberg Test Range in South Africa and positioned at 30˚S and 23˚E. The systematic target designation error of the flat earth LLA sensor model when used in a spherical ERM, without correction, is shown in Fig. 10, Fig. 11 and Fig. 12 below.

An interesting point to note from Fig 11 is that the azimuth error is a function of the target's azimuth and the sensor's latitude. In both flat and spherical earth a target directly East of a sensor has been defined as having an azimuth of 90˚ and similarly a target directly North would be at an azimuth of 0˚.

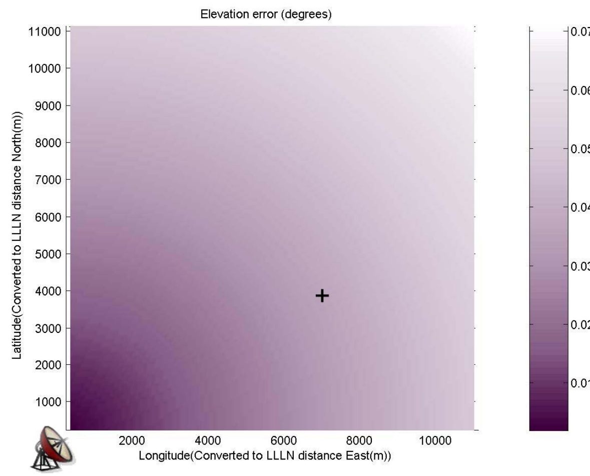


Fig. 9. Elevation error in degrees made by a flat earth sensor over a 0.1 by 0.1 degree grid. Sensor located 0˚S and 0˚E.

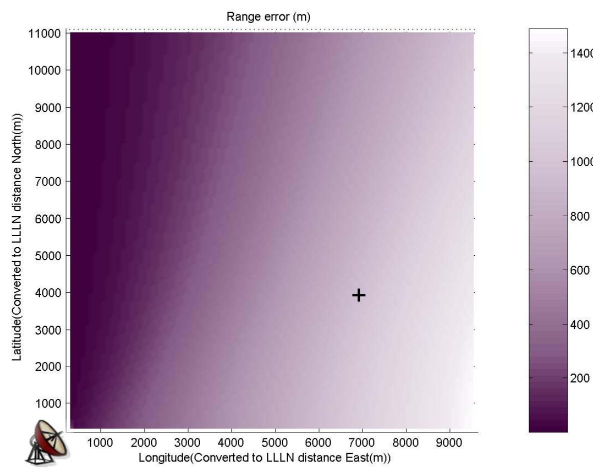


Fig. 10. Range error in meter made by a flat earth sensor over a 0.1 by 0.1 degree grid. Sensor located 30˚S and 23˚E.

The azimuth error thus approaches zero towards North-South and East-West with the maximum error being dependent on the sensor latitude as already explained. The azimuth error is also clearly more dependent on target azimuth than it is on target range which is contrary to the original misconception mentioned earlier.

For an example measurement located at the '+', which can be read from the graph to be 4km North and 7km east of the radar, the systematic range, azimuth and elevation errors are 1.2km, 3.5˚ and –0.2˚ respectively. It is clear that such measurements are not accurate enough for a tracking radar model that's part of an automatic gun system model where accurate target measurement is essential.

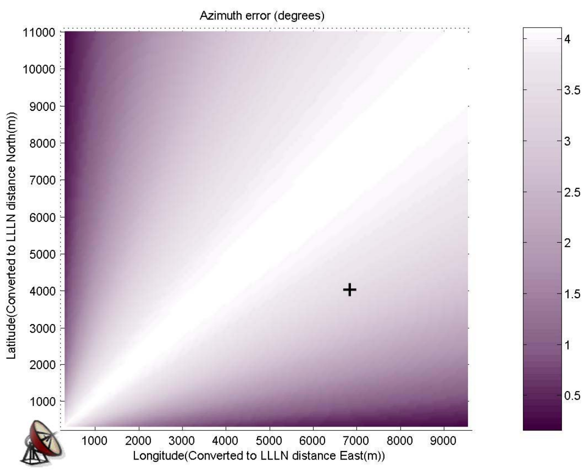


Fig. 11. Azimuth error in degrees made by a flat earth sensor over a 0.1 by 0.1 degree grid. Sensor located 30˚S and 23˚E.

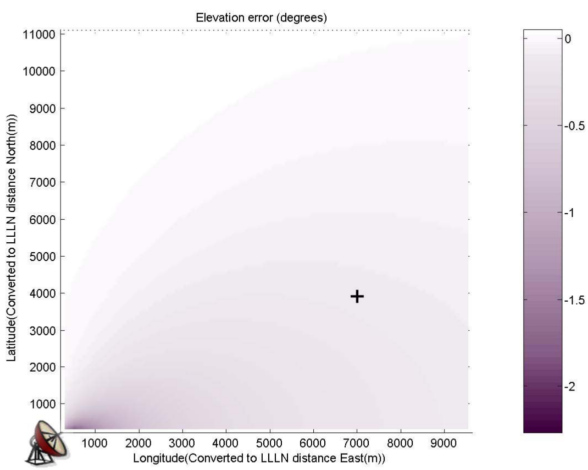


Fig. 12. Elevation error in degrees made by a flat earth sensor over a 0.1 by 0.1 degree grid. Sensor located 30˚S and 23˚E.

V. Using a Flat Earth Model In A Spherical Earth Simulation

From the previous section it is clear that merely using a flat earth model's existing LLA to LLLN conversion without correction in a spherical earth simulation results in unacceptable errors. In the following paragraphs it is shown that the required spherical earth correction can be added to a legacy model. Such a correction must be prescribed to the developer responsible for encapsulating the model in the simulator.

By decree the simulation infrastructure communicates model positions in the external (external to the model) coordinate reference frame F_E which is, as mentioned, LLA coordinates in a spherical ERM. All model position interfaces will therefore be expected to interact with the simulation in LLA and the model must convert F_E coordinates to its own internal ERM and coordinate reference frame F_I with its internal conversion procedure C_I. In effect $F_E \cdot C_I = F_I$. In a legacy model, such as a flat earth sensor model, C_I unfortunately incorrectly converts between *flat* earth LLA and F_I instead of between spherical earth LLA and F_I. A spherical earth correction must thus be applied to the model for it to be usable in the simulation.

F_I is a modelling assumption and must be known. An external conversion procedure C_E may be created to convert F_E directly to F_I. The model is then wrapped in a compound conversion $C_E C_I^{-1}$ to convert F_E to a coordinate reference frame which the flat earth model expects the external reference frame to be., which is flat earth LLA. In effect $F_E \cdot \left(C_E C_I^{-1}\right) C_I = F_I$. It may be noted that C_I and C_I^{-1} are the inverse of one another which reduces the coordinate conversion to the desired $F_E \cdot C_E = F_I$, but with no internal modification to the original legacy model.

Taking a flat earth legacy sensor model as an example, the added correction procedure $C_E C_I^{-1}$ (shown in Fig. 13) can then be done by applying the inverse of the model's LLA to internal LLLN conversion, thus converting the intermediate F_I LLLN north, east and down distances to *flat earth* LLA for use by the model. These conversions and their inverse conversions are often provided with the model in directly accessible utility classes resulting in very short development and debugging times. This newly calculated flat earth LLA position can then be used as input to the original model.

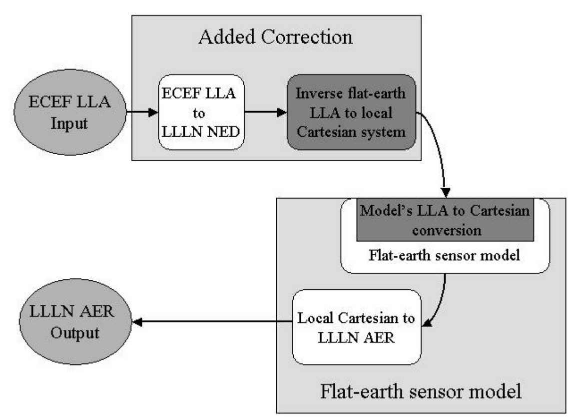


Fig. 13. Range error in meter made by a corrected flat earth sensor over a 0.1 by 0.1 degree grid. Sensor located 30˚S and 23˚E.

Doing the described correction (shown in Fig. 13) for a flat earth sensor model results in a LLLN spherical ERM sensor model and solves the flat to round earth conversion errors as shown in Fig. 14, Fig. 15 and Fig. 16. The errors for the measurement at '+' is now negligible compared to the uncorrected case. The remaining errors are due to floating point round off errors.

The flat ERM approximation for the gravity vector, unfortunately, cannot be corrected for so easily and will still be normal to the local level of the model. The impact of gravity vector errors is most prominent on the modelling of physical entities such as long-range projectiles. The parabolic like flight path in a flat ERM, for example, is behaviourally correct, but in a spherical ERM an elliptical flight path is required.

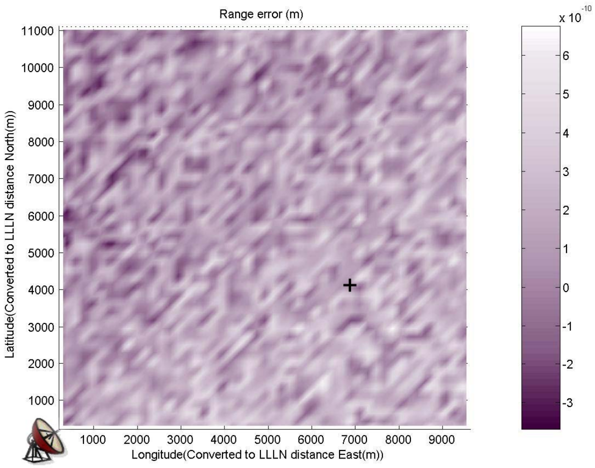


Fig. 14. Range error in meter made by a corrected flat earth sensor over a 0.1 by 0.1 degree grid. Sensor located 30˚S and 23˚E.

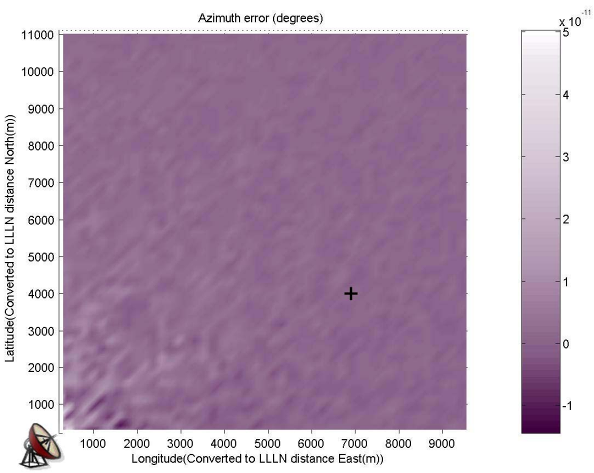


Fig. 15. Azimuth error in degrees made by a corrected flat earth sensor over a 0.1 by 0.1 degree grid. Sensor located 30˚S and 23˚E.

The simulator architect or developer therefore still has some responsibility in deciding whether a model that behaves correctly in flat earth can be used in a local level coordinate reference frame on a spherical ERM. Depending on various aspects, such as the type of model and the approximations made, modifications and/or additional correction techniques might be required. The complexity of this problem merits additional work to be done on finding appropriate correction techniques for all situations.

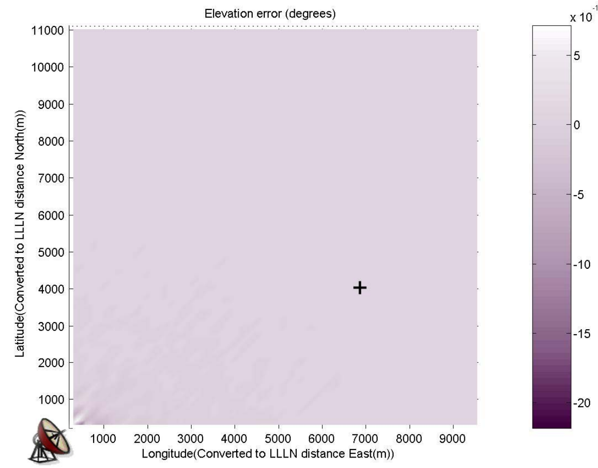


Fig. 16. Elevation error in degrees made by a corrected flat earth sensor over a 0.1 by 0.1 degree grid. Sensor located 30˚S and 23˚E

VI. Summary of Conversion Errors

The findings about the above mentioned conversion errors are summarised in Table I. The error values are rounded to the same number of decimal places for easy comparison.

TABLE I
SUMMARY OF CONVERSION ERRORS

Error Dimension	Uncorrected Conversion @(0˚S, 0˚E)	Uncorrected Conversion @(30˚S, 23˚E)	Corrected Conversion @(30˚S, 23˚E)
Azimuth	0.9000˚	3.5000˚	0.0000˚
Elevation	0.0400˚	-0.2000˚	0.0000˚
Range	0.1200km	1.2000km	0.0000km

VII. Integration of Plan View and 3D Viewers

A viewer's internal representation of data can also be a real earth or a 'flat' augmented ERM. If a 2D viewer uses a flat ERM internally the simulation builders are faced with a problem. Even though LLA positions may be converted to flat earth easily enough (by scaling and drawing latitude and longitude directly), Azimuth, elevation and range from a sensor to a target can't always be realistically drawn.

A direction line (fixed length arrow pointing in the required direction) on a 2D viewer making use of a flat ERM must be drawn as a curved line for the same reason that the longitude and latitude lines on a map of the world are drawn curved. Drawing the direction line curved looks intuitively wrong though. Observers mentally extend direction lines expecting them to intersect the target position and extending curved lines does not come naturally to most observers.

If one has all three dimensions of the measured target position the target may be presented as a LLLN AER direction in the sensor's local level reference frame. The target position can then be converted to LLLN NED with the sensor as reference and then to ECEF LLA. One then draws a 'straight' direction line between the 2D flat earth latitude and longitude of the sensor and the latitude and longitude of the just calculated target position. Following this new 'straight' direction line from the sensor will then exactly intercept the measured target position. The drawback of course being that such a direction line can now no longer be used for doing LOS evaluation for example. The 'straight' direction line is only valid at its beginning and at its end, while the original 'curved' direction line would be spatially accurate along its entire length.

If one or more of the components of the target position are not known, as is the case for a 2D Az-El sensor for example, the unknown dimension(s), range in this case, must be estimated in order to draw a 'straight' direction line, decreasing its accuracy and value to the observer.

VIII. Future Model And Viewer Development Considerations

The authors propose that when a model is developed it should always be done for a real world ERM and not an augmented flat ERM. The model may still use any convenient coordinate reference frame for the chosen ERM. If performance is a problem the modelling equations may be approximated within the real world ERM, but the approximations must be apparent in terms of gravity vector direction for LLLN coordinate reference frames. Such a modelling approximation made in a real earth ERM would give approximate results within the model's application domain, but the model will have an application domain of all real world ERMs instead of one specific augmented ERM as is the case for a flat earth model. A real world ERM ensures realistic interaction of models in current and future system simulations that operate over a number of application domains.

The authors propose that ALL viewers should use a real world internal representation for positions of objects and direction lines. To calculate the actual screen positions of objects and direction lines an orthographic or perspective projection of a simulation frame may then be done by an eye-in-the-sky looking down at a 2D map viewer or from an observer's, possibly on the ground, point of view for a 3D type viewer. Such a projection will also correctly display the latitude and longitude lines on the earth's surface as being curved and not a grid of parallel lines. The required projections are relatively inexpensive when implemented on the CPU and are effectively for free when a graphics accelerator is used to do the drawing.

IX. Conclusion

This paper captured some of the experiences gained from using legacy models in a complex simulator and quantifies some of the errors which may result when flat ERMs are used, as is, in a real-world environment.

This paper only addressed spatial error corrections and specifically looked at the corrections necessary when converting between a flat ERM and real-world ERM. It is clear that a mixture of models may have significant effects on the results obtained. Further work in identifying a representative set of coordinate reference frame and ERM combinations that require spatial correction would be very valuable. Such a set may be applied to predict when, and possibly with what magnitude, referencing errors will occur.

The considerations when developing 2D viewers were also addressed and the ergonomic issues when a viewer using a flat ERM is used to present real-world ERM data. Selecting the correct map projection for easy and intuitive spatial referencing user interaction is a potential point for further study

Acknowledgment

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