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Abstract. In this paper, we experimentally demonstrate selective excitation of high-radial-order Laguerre-Gaussian (LG_p or $LG_{p,0}$) modes with radial-order, $p = 1 - 4$ and azimuthal-order, $l = 0$, using a diode-pump solid-state laser (DPSSL) that is digitally controlled using Spatial Light Modulator (SLM). We encoded amplitude mask containing p-absorbing rings, of various incompleteness (segmented), on grey-scale computer generated digital holograms (CGDH), and displayed them on an SLM, that acted as an end mirror of the diode-pumped solid-state digital laser (DPSSDL). The various incomplete (α) p-absorbing rings were digitally encoded to match the zero intensity nulls of the desired LG_p mode. We illustrate that the creation of LG_p, for $p = 1$ to $p = 4$, only requires an incomplete circular p-absorbing ring that have a completeness of $\approx 37.5\%$, which makes the DPSSL resonator to have the lower pump threshold power, while maintaining the same laser characteristics (such as beam propagation properties).

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1. Introduction

High-order Laguerre-Gaussian ($LG_{p,l}$) beams are used in many applications such as in industry, medicine, military, communication, microscopy and sensing [1, 2, 3, 4]. Applications such optical trapping [5], which are also termed optical tweezers, have been shown that they would be inconceivable without higher-order Laguerre-Gaussian ($LG_{p,l}$) beams. Especially those that only contains the azimuthal order, l, component as they can transfer orbital momentum (OM) to a trapped particle (this is mainly used for manipulating biological cells). By creating a vortex field around the particle thereby "forcing" the particle to rotate around the optical axis of the beam and trapping the cell [6]. The interest over the years for $LG_{0,l}$ beams that have only the azimuthal index have even lead to the optical tweezers to be known as optical vortices [7].

The current demand for high brightness lasers, especially in military and applications that require high brightness lasers, has sparked an interest to study further high order LG beams, especially those that have only the radial order, p, index. The notion is to intra-cavity generate LG_p mode to be a fundamental mode that will have high energy [8]. Over the years it has been shown that LG_p modes can be created using extra-cavity and intra-cavity methods [9, 10, 11]. The major disadvantage of LG_p modes in many applications has been using the laser that produces the fundamental Gaussian beam.

In this article, we utilised a new technology called the diode-pumped solid-state digital laser (DPSSDL) that was recently invented at the Council for Scientific and Industrial Research in South Africa (CSIR), Pretoria, Republic of South Africa [11]. We used this technology to generate high-order Laguerre-Gaussian modes (LG_p) of the radial index, p, and zero azimuthal order index, l. It has been already shown that these modes can be generated by inserting of a mask made-up of p-absorbing rings having a radius coinciding with the zeros of the desired LG_p mode within the cavity [12]. However, doing that introduces supplementary losses and consequently increases the laser threshold. In this article, we will show that it is possible to force the fundamental mode of the laser to be a single high-order LG_p mode by using incomplete absorbing rings allowing the reduction of inserting losses [13, 14]. It is worthwhile to recall that an LG_p beam is made up of a central peak surrounded by p-rings of light separated by a p-zeros of intensity. The laser resonator that we opted to use was a diode-end-pumped system due to the ease of controlling the pump spot size, the wavelength of the pump, the divergence and incident angle of the pump beam onto the gain medium [15, 16, 17].

Forcing the fundamental mode to be an LG_p mode can offer some advantages:

- An improved energy extraction from the laser medium due to a great lateral extent compared to the usual Gaussian beam. The LG_p mode can be transformed in the focal plane of a lens into a single-lobed beam by using a binary diffractive optical element playing the role of the rectifier [18].
- A simple diaphragm can transform a LG_1 beam into an optical bottle beam [19] or a flat-top intensity profile [20].

2. Radial-order Laguerre-Gaussian modes intensity

Since the propagation of the Gaussian beam is well-known and can be derived analytically [21], one can be able to transfer the understanding to higher-order modes. Radial Laguerre-Gaussian (LG_p) beams have an intensity distribution that is made up of the central bright lobe and concentric rings, thus the mathematical representation can be written as shown in Eq. 1.

$$I = \frac{2}{\pi} \times \frac{1}{w_p^2} \times \left[L_p^0 \left(\frac{2r^2}{w_p^2} \right) \right]^2 \times e^{\frac{-2r^2}{w_p^2}}, \tag{1}$$

where r is the radial coordinates and L_p^0 is the Laguerre polynomial. All other parameters have their usual meaning defined for Gaussian beams [22].

The maximum intensity occurs at central peak while the intensity associated with the ring is lower. The radial intensity distribution of LG_p beams becomes broader as mode order p increases and be characterised by a width (w_p) based on the second-moment of the intensity pattern and the width (w_p) is given by:

$$w_p = w_0\sqrt{2p+1}, \tag{2}$$

where w_0 is the width of the Gaussian beam. In addition to Eq. 2, the propagation property of LG_p beams is the propagation factor (M_p^2) that is defined by as:

$$M_p^2 = 2p + 1, \tag{3}$$

the propagation factor (M_p^2) is used to measure the beam quality of the laser beam according to ISO Standard 11146 [23]. We can numerically analyse LG_p propagation by starting with Laguerre Polynomial part of Eq. 1, taking radial-order $p = 1 - 4$. The Laguerre polynomial and r_i the position of the zeroes of intensity are given in Tab. 1 of Ref. [12]. To force the laser to generate LG_p modes, a digital mask containing p-absorbing rings that have a geometry which closely follows the location of the Laguerre polynomial p-zeros as illustrated in Tab. 2 of Ref. [12], were digitally encoded and displayed on to intra-cavity SLM screen of the laser resonator [11], which then allowed for the generation of LG_p modes.

3. Experimental Methodology and Concept

We considered a planoconcave solid-state laser (DPSSL) resonator that is diode-pumped with a multi-mode fibre coupled diode laser, for the generation of radial order Laguerre-Gaussian (LG_p) modes. The Hamamatsu spatial light modulator (X10468-03) was encoded with an amplitude mask that operates as an end mirror of the solid-state digital laser [11]. The amplitude mask was encoded to have p-absorbing rings of varying width thickness that will nearly 98% match each null of the LG_p mode, and varying the completeness of the ring for radial-order $p = 1 - 4$. A schematic of the experimental setup is presented in Fig. 1.

The gain medium is a Nd: YAG rod crystal of 25 mm in length with a 1.1% neodymium concentration, and it was pumped with a diode laser operating at 808 nm.

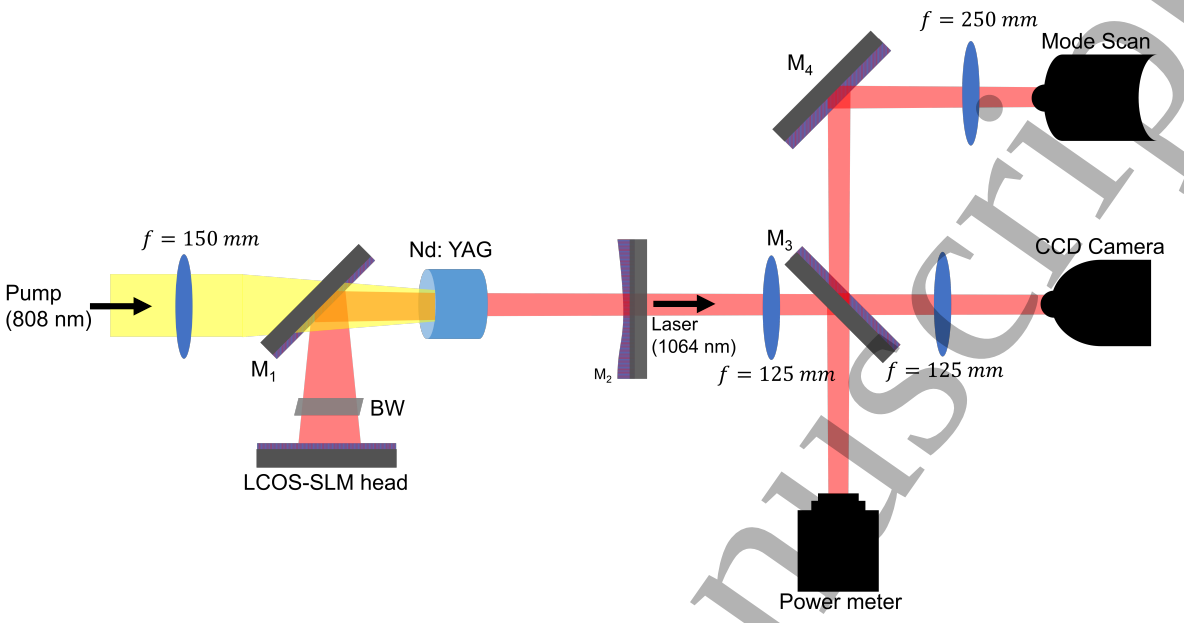

Figure 1. Schematic of a diode-pumped solid-state digital laser. The laser beam was transmitted out of the cavity through an output coupler mirror (M_2), and was 1:1 relay imaged using two lenses with focal length of 125 mm to the USBeamPro CCD Camera. In addition, the transmitted beam was relay imaged and expanded using two lenses with focal length 125 mm and 250 mm, respectively, to a ModeScan for measuring the beam quality factor M_p^2. Mirror M_3 is used as a flip and twist mirror that allows one to transmit the beam to the CCD or ModeScan or Power meter.

The laser crystal was mounted inside a 21 °C water-cooled copper block. The diode laser pump (Jenoptic, JOLD-75-CPXF-2P) have a maximum output power of 75 Watts at an emission wavelength of 808 nm (at an operating temperature of 25 °C). A gain area with a radius of 2 mm was then excited within the centre of the Nd: YAG rod crystal by a focusing (by a lens with 150 mm focal length) the pump beam.

The planoconcave resonator cavity is comprised of a phase-only SLM which acted as a plane end mirror of the cavity with a reflectivity of 95% and a curved output coupler mirror (M_2) with a radius of curvature of 400 mm and a reflectivity of 90%. Since the SLM is a phase-only device, yet many of the desired holograms require both amplitude and phase change to the field. In order to achieve this, we make use of the well-known methods of complex amplitude modulation [24, 25], because this is suitable for implementation on the SLMs. Between the SLM and the mirror (M_1), the brewster window, (BW), was used to force the polarisation of the laser to match the eigen polarisations of the SLM.

The resonator was designed to form an L-shape (in order to avoid illuminating the SLM with the residual pump light) by including a 45° mirrors (M_1) within the cavity that was highly reflective for 1064 nm and highly transmissive for 808 nm. The resonator length was chosen to be 160 mm. The laser beam was transmitted out of the cavity through an output coupler mirror (M_2), and was 1:1 relay imaged using two lenses to the

USBeamPro CCD Camera Beam Profiler (Photon Inc.). The CCD camera was used to capture images that are normalised to 1 and was also used to measure the beam width. In addition, the transmitted beam was relay imaged and expanded to a ModeScan-1780 Laser M^2 Measuring System (Ophir Photonics) for measuring the beam quality factor M_p^2. The power meter is used measure the output power of the laser.

4. Numerical simulations for complete ring

We performed a numerical calculation of the fundamental mode of the resonator that will contain an intra-cavity amplitude mask that will match an appropriate ring mask. The simulation is based on the expansion of the resonant field on the basis of the eigenmodes of the bare cavity (without any diffracting object). This method will not be given here since it has been already described elsewhere for the case of a planoconcave cavity including an absorbing ring on the plane mirror [26]. The simulation given in Ref. [26] can be easily adapted to the case of an amplitude mask made up of concentric absorbing rings just by evaluating the overlapping integral (Eq. A10 of Ref. [26]) upon all the regions of transparency of the mask.

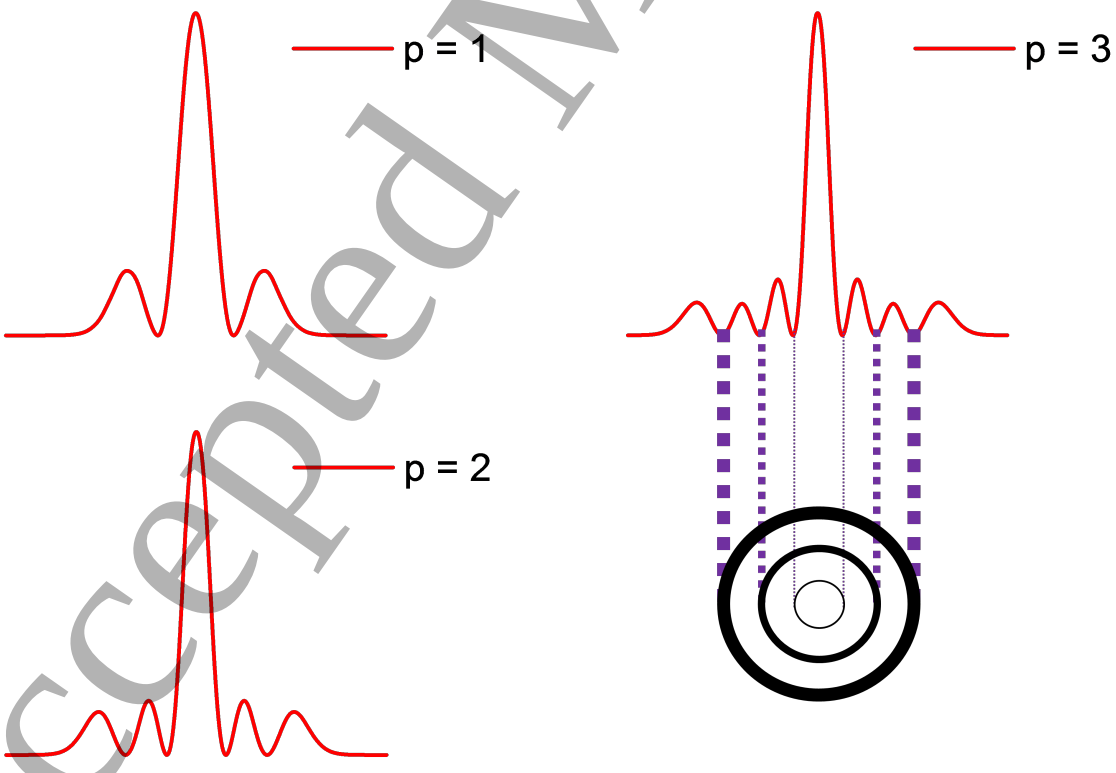

Figure 2. Intensity cross section of numerically simulated lowest-loss eigenmodes for $p = 1$ to 3, when a mask containing an absorbing p-rings is inserted inside the resonator. The mask is inserted such that the high-loss absorbing rings coincide with the intensity p-zero nulls for an example for $p = 3$, LG_p mode.

The width thickness of absorbing rings is designed to be 98% the theoretical width thickness the LG_p mode p-zero nulls that will be oscillating inside the resonator (Fig. 1). The simulated LG_p modes of $p = 1 - 3$ cross-section profiles are shown in Fig. 2. The width thickness of the p-absorbing rings for $p = 3$ mode, increases from the inner ring to the outer ring. The minimum width thickness of the ring is 20 μm which corresponds to the pixel pitch of the SLM. It is worthwhile to note that the fundamental mode of the laser cavity is the mode reaching first the oscillation threshold and which is not Gaussian in shape in the presence of the mask. The fundamental mode is an LG_p with p adjustable from $p = 1$ to 3 depending on the number of absorbing rings making up the mask.

5. Numerical simulations for incomplete ring

From Sec. 3, the planoconcave cavity has been shown to have a length $L = 160$ mm, and the concave mirror (M_2) have a radius of curvature $R = 400$ mm. Now, we will focus on the generation of an LG_p mode imposed by the insertion of a single absorbing ring of radius r_A. The ring is set against the plane mirror (SLM of Fig. 1) and a diaphragm of radius r_0 is set against the plane mirror (SLM of Fig. 1). For our case, the p-absorbing

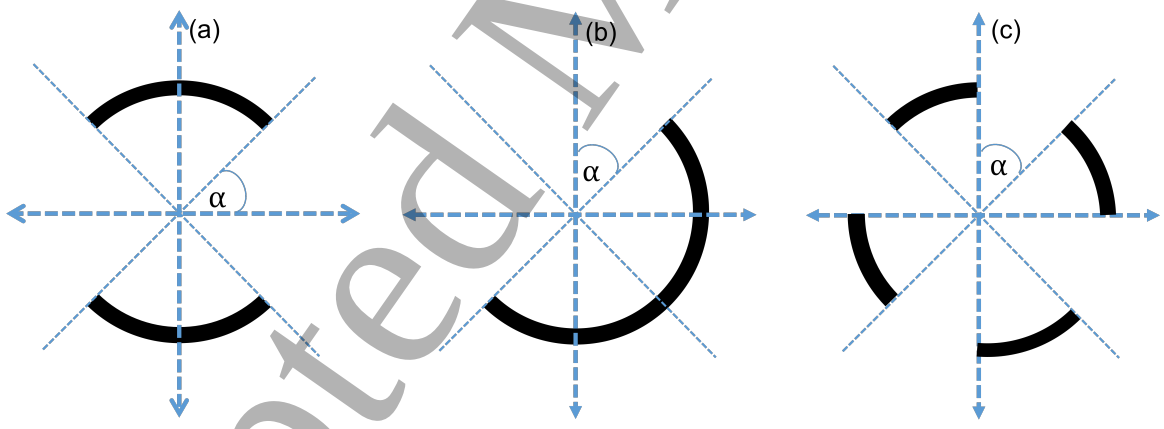

Figure 3. Schematic diagram illustrating a 50% complete p-absorbing ring, regardless of the symmetry. With α representing the incompleteness of the ring.

ring diaphragm was encoded to the SLM. Part of the fundamental mode loss level is due to the circular absorbing ring mask, and we expect to reduce the losses by using an incomplete absorbing ring mask shown in Fig. 3.

The losses associated with the incompleteness of the absorbing ring does not depend on how many times the absorbing ring has been segmented as shown in fig. 3. The losses of the laser depend on the overall effect of each incomplete absorbing ring. i.e. For a full ring, $\alpha = 0$, the fundamental mode of the resonator is expected to be Laguerre-Gaussian mode of radial order $p = 1$ (LG_1), with a beam propagation factor $M^2 = 3$ (by choosing $p = 1$ and $l = 0$ in Eq. 3).

The ring is almost non-existent, at $\alpha > 0.95\pi/2$, thus the Gaussian mode will oscillate. However, the LG_p of order p=1 will oscillate provided that the angle α of the absorbing ring is kept less than $0.95\pi/2$ as shown in Fig. 4a. The propagation factor M^2 of the fundamental mode remains equal to 3 as shown in Fig. 4b, up until the angle α is close to $\pi/2$ when the laser resonator will then selects p=0 mode and the M^2 will then become 1. The losses of the fundamental mode, δ, are indicated in Fig. 5a,

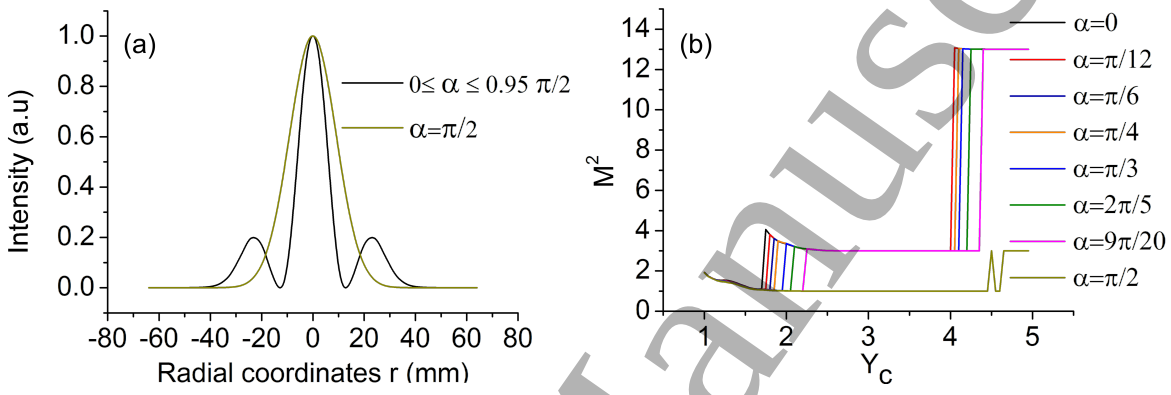

Figure 4. (a) Numerically simulated fundamental mode shape at the far-field. (b) Numerically simulated propagation factor M^2 as a function of truncation ratio Y_C.

as a function of the beam truncation ratio Y_c for different values of angle α . When $\alpha = \pi/2$, the cavity is expected to generate a Gaussian fundamental mode of $p = 0$, since the cavity losses will be the lowest as shown in Fig. 5a. When $\alpha = 0$, the cavity is expected to generate a $p = 1$ as the fundamental mode of the cavity since the ring will be complete and the losses will be the highest. The simulation of the losses of the laser

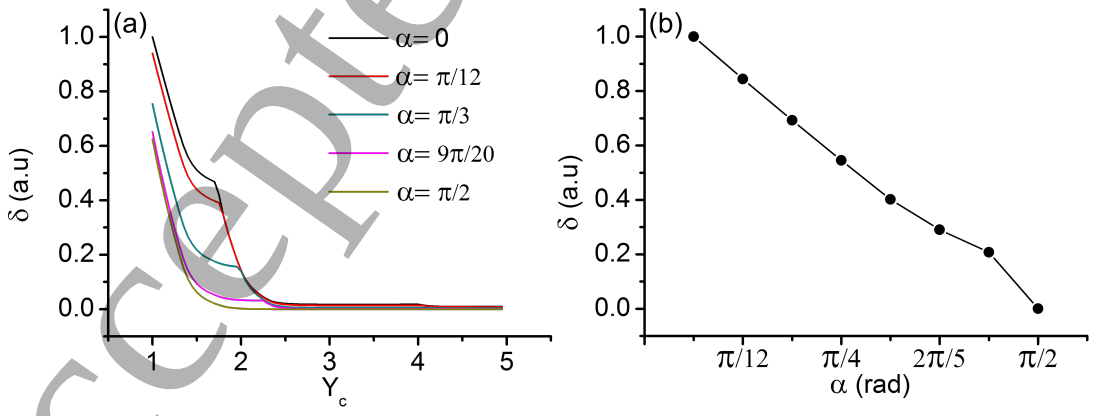

Figure 5. (a) Numerically simulated intra-cavity losses, δ, of the fundamental mode as the function of the beam truncation ratio, (Y_C). (b) Numerically simulated intra-cavity losses, δ, of the fundamental mode as the function of ring incompleteness (α).

resonator as a function of the absorbing ring incompleteness angle, α, is shown in Fig.

5b. The simulation shows that varying angle, α, from a full ring of 100% completeness of $\alpha = 0$ to $\alpha = 9\pi/20$, the cavity will select $p = 1$ mode as the fundamental mode of the cavity; and between $\alpha = 9\pi/20$ and $\alpha = \pi/2$, the simulation shows that the losses will be decreasing at a fast rate towards zero and the laser resonator will then select the $p = 0$ as the Gaussian fundamental mode. The concept is tested by varying of completeness of the ring, by varying α on the CGDH. The experimental results are shown in Sec. 6, where we measured the pumping threshold, ϵ_{th}, the propagation factor, M_p^2, the generated LG_p beam width, w_p, and the slope efficiency, η_p, of the output LG_p beams.

6. Experimental Results and Discussion

The intensity distributions profiles results for selected laser modes with the varying completeness of the p-absorbing ring are shown in Fig. 6. The absorbing ring completeness was varied from 12.5% to 100%. The method used to excite the modes from the cavity was by employing CGDH amplitude masks which were encoded as pixelated grey images and displayed onto an SLM that also acted as a flat end mirror of the resonator of the diode end-pumped solid-state laser (DPSSL) resonator [11]. The CGDH amplitude masks contained p-absorbing rings of varying circular completeness and their corresponding intensity profiles are shown in Fig. 6a, 6b, 6c and 6d, for LG_p mode of order $p = 1$ to 4, respectively.

The results in Fig. 7a shows that when the completeness of the ring is less than 37.5%, the cavity is not generating LG_p mode but different laser beams with varying propagation factor, M^2 values. The results further confirm that when the completeness of the ring is greater or equal to 37.5%, the resonator produces the expected LG_p modes with a propagation factor M^2 values close to $(2p{+}1)$ the theoretical values given by Eq. 3, for all generated LG_p modes from $p = 1$ to $p = 4$. Therefore our stable resonator cavity conforms to the ABCD matrix [17] theory when the completeness of the ring is greater and equal to 37.5% as shown by the results in Fig. 7b; where the generated LG_p mode width, w_p, follow closely the theory given in Eq. 2.

Since the output power from the laser resonator is defined to be linearly proportional to the mode volume, V_p, where the volume of the p^{th} radial mode is given as:

$$V_p = V_0 M^2 \left(1 + \frac{l_0^2}{3z_r^2}\right), \tag{4}$$

where l_0 is the length of the gain medium and V_0 is the mode volume of the Gaussian mode. From Eq. 4 it is clear that the mode volume, V_p, is directly proportional to the propagation factor, M^2, of the fundamental LG_p mode; and therefore we can deduce that the output power extracted from the laser is also directly proportional to both the mode volume, V_p, and the propagation factor, M^2, of the oscillating LG_p mode [27]. The experimental results shown in Fig. 8a suggest that the mask with a completeness comprised between 37.5% to 100% is able to select an LG_p mode, as proven by the slope efficiency of the laser to be constant for each LG_p laser mode. Furthermore, the

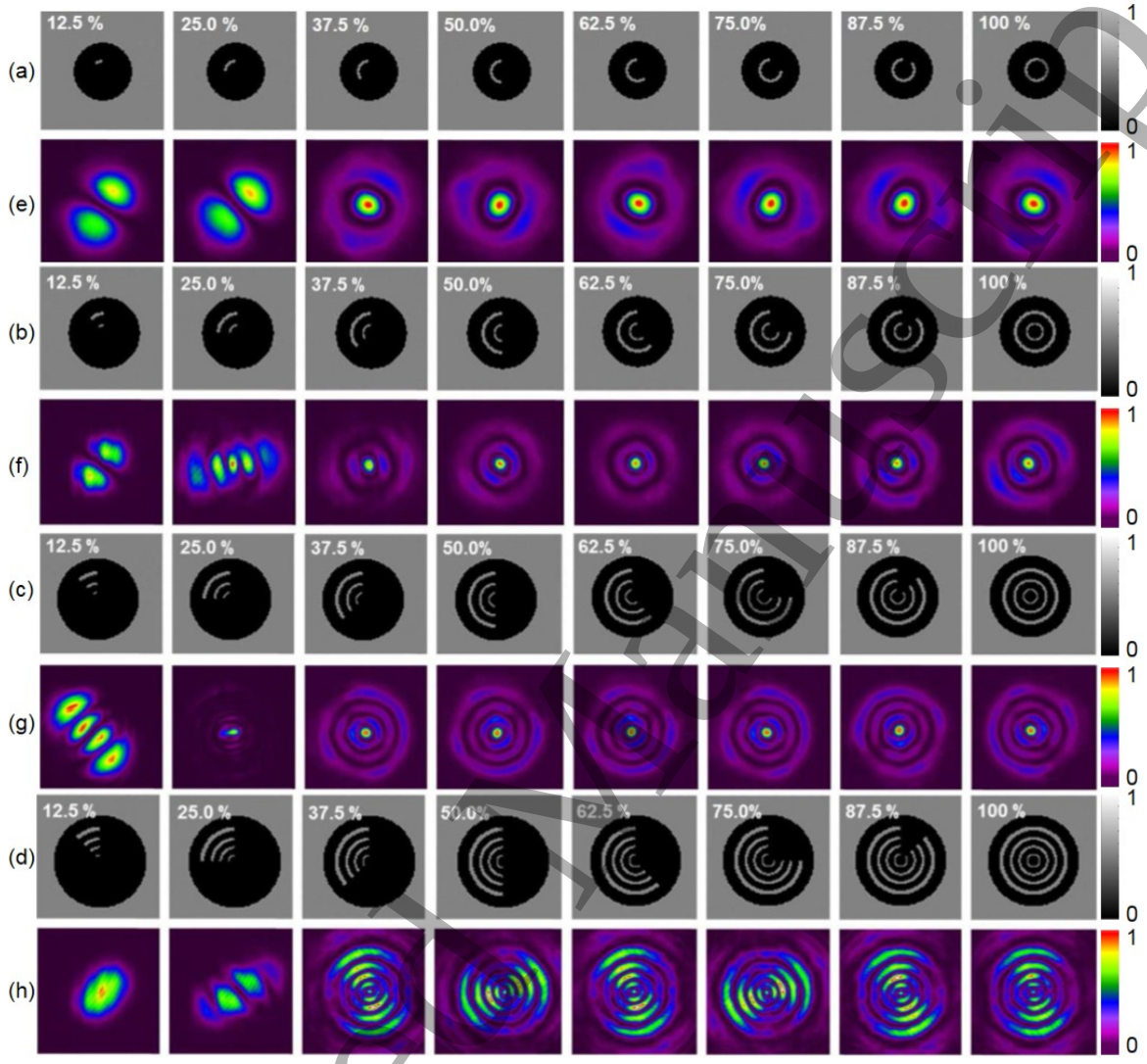

Figure 6. The top row of (a)-(d) shows computer generated digital holograms encoded as pixelated greyscale (0 to 1) images on the SLM where the completeness of the p-absorbing ring is varied. Bottom rows of (e)-(h) show the generated intensity profiles (normalised to 1) from the laser output coupler mirror for the different corresponding p-absorbing ring digital holograms on the SLM.

threshold of the laser resonator with respect to the pump power, P_ϵ, and the absorbed pump power, $P_{abs,\epsilon}$, can be mathematically described as follow [15]:

$$P_\epsilon = \frac{\pi h \nu_\epsilon}{4\eta_{(\varepsilon-q)}\sigma_{em}\tau}(w_p^2 + w_\epsilon^2)(T+L) \quad \text{and;} \tag{5}$$

$$P_{abs,\epsilon} = \eta_{(\epsilon-q)}\frac{\nu_l}{\nu_\epsilon}\frac{T+L}{T}, \quad \text{for} \quad w_\epsilon < w_p, \tag{6}$$

where w_p and w_ε are the laser LG_p mode radius and pump radius; ν_l and ν_ϵ are the laser and pump frequencies; σ_{em} is the effective stimulated-emission cross-section; τ is the lifetime of the laser transition; $\eta_{(\varepsilon-q)}$ is the pump quantum efficiency which is the

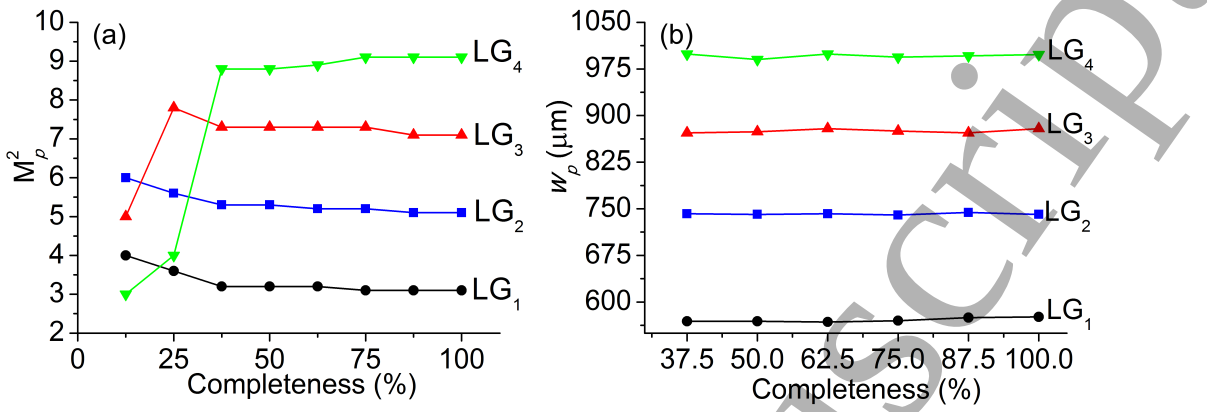

Figure 7. (a) Beam propagation factor, M_p^2, and (b) beam radius, w_p, at the output coupler for LG modes of $p = 1 - 4$ with varying completeness of the absorbing rings.

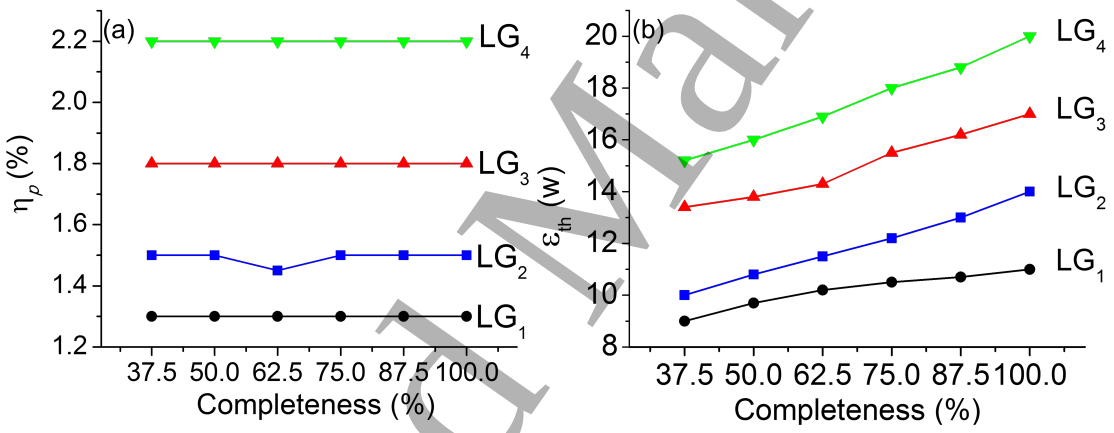

Figure 8. The graphs showing the slope efficiency, (a) η_p, and the pump power threshold, (b) ϵ_{th}, for LG_p modes of $p = 1 - 4$ with varying completeness of the absorbing circular ring.

average number of ions in the upper manifold created per absorbed pump photon; T is the output coupler transmission and L is the laser resonator losses, such as scattering, absorbing rings, aberrations due to thermal effects. The only major resonator loss, L, that we are interested is to study the laser resonator absorbing rings. Since the slope efficiency, η_p, of the laser resonator for each LG_p mode generated is given as:

$$\eta_p = \eta_{(\varepsilon - q)} \frac{\nu_l}{\nu_\epsilon}. \tag{7}$$

Using Eq. 6, then η_p can be represented as:

$$\eta_p = P_{abs,\epsilon} \frac{T}{T + L}. \tag{8}$$

It is important to note that Eq. 5 and Eq. 6 are only valid for $T << 1$ and $L << 1$, otherwise T must be replaced with $-ln(1 - T)$, and $L = \sum L_i$, where L_i are all the separate cavity losses other than the output coupler, which in out case it will be

absorbing rings losses, δ, for each LG_p mode. However, up to few percent loss, the transmission approximations are very good ($-ln(1-T) = 0.051$ for $T = 0.050$ and $-ln(1-T) = 0.105$ for $T = 0.100$). Therefore inserting Eq. 8 into Eq. 5 and Eq. 6, and solving for slope efficiency, η_p, the equation will become:

$$\eta_p = \frac{\pi h \nu_l}{4\sigma_{em}\tau} \times \frac{(w_p^2 + w_\varepsilon^2)}{P_\epsilon} \times (T + L). \tag{9}$$

From Eq. 9, it is also clear that the slope efficiency, η_p, is directly proportional to the LG_p mode volume, V_p, since the absorbing ring losses L and the pump power, P_ϵ, cancel each other for each LG_p amplitude mask as they always simultaneously increase concurrent. This effectively makes the slope efficiency, η_p, to be constant as shown in Fig. 8a for various ring completeness, from ring completeness equal or greater than $\approx$ 37.5%, when the laser resonator is generating LG_p modes.

It is observed in Fig. 8b that the pump threshold, ϵ_{th} is shown to increases linearly as the absorbing rings completeness or losses increases, for each LG_p mode. The lowest pump threshold, ϵ_{th}, for the generation of LG_p modes is shown to occur when the completeness of the absorbing rings is at $\approx 37.5\%$ and this is $\approx 2\pi/5$ shown by Fig. 8b. These results are 93% ($(2\pi/5)/(9\pi/20)$) in good agreement with the simulation shown in Fig. 5b where the simulation shows that the laser starts operating on high LG_p mode when $\alpha > 9\pi/20$.

7. Conclusion

Forcing the fundamental mode of a laser to be an LG_p mode, can be obtained by inserting a mask made up of p-absorbing rings having radii coinciding with the zeros of the desired radial Laguerre-Gaussian mode. However, doing that increases the loss level and the laser threshold. In this work, we have experimentally demonstrated that it is possible to reduce the additional losses by using a mask having a completeness equal or greater than 37.5%. The better results in terms of the laser threshold are obtained for a completeness of 37.5%. We have successfully demonstrated that high-radial-order Laguerre-Gaussian, LG_p, modes can be generated using circular absorbing rings that are $\approx$ 37.5% to 100% complete. The results also indicate that we can digitally excite high-radial-order LG_p modes with the lowest excitation pump threshold, ϵ_{th}, when the absorbing rings are $\approx$ 37.5% complete or when $\alpha = 2\pi/5$, while maintaining all the other resonator characteristics (such as the generated LG_p mode size, w_p, beam quality factor, M_p^2, and the slope efficiency, η_p, of the laser).

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