A Spreadsheet Tool for the Analysis of Flows in Small-scale Water Piping Networks

Kazeem B. Adedeji[1], Yskandar Hamam[1], Bolanle T. Abe[1] and Adnan M. Abu-Mahfouz[1,2]
[1]Department of Electrical Engineering, Tshwane University of Technology
Pretoria, South Africa
[2]Meraka Institute, Council for Scientific and Industrial Research
Pretoria, South Africa
Email:adedejikb@tut.ac.za

Abstract—The analysis of water piping system have been presented by several authors in the past and in recent years proposing several solution algorithms. Among the notable methods are the Hardy cross method, linear approximation method, Newton Raphson method and the hybrid method to mention but a few, to solve a system of partly linear, and partly non-linear hydraulic equations. In this paper, the authors demonstrate the use of Excel solver to verify the Hardy Cross method for the analysis of flow in water piping networks. A single-loop water network derived from real situation was used as numerical example and case study. Detailed numerical data are presented to explain the results of the studied network.

I. INTRODUCTION

Spreadsheets are efficient tools widely used in engineering profession. Several engineering applications of this tool include it usage in rainfall-runoff, hydraulic and thermal systems [1, 2]. More so, its application for water resource concept such as the design of dendritic distribution network systems has been reported [3–5]. Of particular importance is its application in water distribution networks analysis.

Water distribution network (WDN) analysis provides the basis for the design of new systems and the extension of existing systems. The design of an optimal water distribution systems involves an extensive simulation of flows in the system for every trial set of design parameters [6]. Thus, pipe network analysis involves the computation of the flows rates/discharges in each pipe or link as well as the pressure head at each node in the network. This will give an insight on the condition of the network for leakage analysis, pressure control and state estimation purposes [7–17].

Hydraulic models [18, 19] for the solution of this system are formulated based on the pipe-node formulations in which the pipes with unknown flow rates are interconnected at nodes with unknown nodal pressure heads. The equations governing the hydraulic analysis of the flow in any water piping system is based on the mass continuity at each nodes of the network as well as the energy conservation around the hydraulic loop of the network [6, 7, 20–22].

II. WATER DISTRIBUTION NETWORK FORMULATION

A water distribution network with n_j number of junction nodes, n_p number of pipes and l number of loops, the mass conservation at the jth node revealed that the sum of the inflow and outflow in each connected pipe at a junction node must be balance by the external demand at the node. The continuity equation for each node j, with $j = 1, 2, ..., n_j$, can be expressed as;

$$\sum_{k=1}^{np(j)} (Q_{in}(jk) - (Q_o(jk)) = Q_j^{ext} \quad (j = 1, 2, ...n_j) \quad (1)$$

where, $Q_{in(jk)}$ represent internal flow rate in pipe k into node j, $Q_{o(jk)}$ is the flow rate in pipe k out of node j, Q_j^{ext} the external demand/consumption at node j, while $n_p(j)$ is the number of pipes connected to node j. In a more compact form,

$$Q_j^{ext} = \sum_{k}^{np} C_{jk} Q_k \quad (j = 1, 2, ...n_j) \quad (2)$$

In (2), Q_k represents the flow rate in the kth pipe and C_{jk} is the node-pipe connectivity matrix connecting the kth pipe to the jth node. The element of this matrix is given by

$$C_{jk} = \begin{cases} +1 & \text{if the flow in pipe } k \text{ enters node } j \\ 0 & \text{if pipe } k \text{ is not connected to node } j \\ -1 & \text{if the flow in pipe } k \text{ leaves node } j \end{cases} \quad (3)$$

The energy conservation around a loop states that the sum of the pipe head losses (or pressure drop), together with any heads generated by in-line booster pumps (pumps connected to pipes) around any close loop or path must be zero. That is,

$$\sum_{k=1}^{np} (H_l(k) - (H_p(k)) = 0 \quad (4)$$

where, $H_l(k)$ denotes the head loss in pipe k and $H_p(k)$ represents the head generated by booster pump connected to pipe k. The frictional head loss due to the flow of water in the kth pipe with the pipe wall is related to the pipe discharge through the relation

$$\left. \begin{aligned} h_l(k) &= r_k Q^n \\ &= r_k Q |Q|^{n-1} \end{aligned} \right\} \quad (5)$$

where r_k is the hydraulic resistance of the kth pipe which depends on the pipe length, diameter and the pipe roughness

coefficient)[6]. The exponent n varies from 1 to 2 depending on the head loss model used ($n = 1.85$ for Hazen-Williams and $n = 2$ for both Darcy-Weisbach or Manning head loss model [23]. Using the Darcy-Weisbach (DW) equation, the hydraulic resistance of the kth pipe is expressed as

$$r_k = \frac{8 f_k L_k}{g \pi^2 D_k^5} \tag{6}$$

where, f denotes the frictional factor of the kth pipe (a function of the pipe roughness coefficient), L_k represents the length of the kth pipe, g is the acceleration due to gravity and D, the diameter of the kth pipe.

If one substitutes (4) and (5) into (1) for every nodes in the network, the whole equations becomes a set of partly linear and partly non-linear equations to be solved. Each of the equations can be expressed in terms of the unknown flow rates or the unknown nodal pressure heads. Thus, WDNs hydraulic models are able to compute nodal heads and pipe flows for a fixed set of tank level by solving simultaneously the mass continuity equations at each node as well as the head losses or pressure drop (due to the friction of the flow of water with the pipe wall) relationship for each pipe in the network.

Various methods of solution of these system of equations exist in the literature each with different degree of convergence. These include the famous Hardy Cross method, the Newton-Raphson method, linear approximation method, the hybrid method, the gradient algorithm and the global gradient algorithm [24–29]. In this paper, the use of excel solver to demonstrate the Hardy Cross method for a single-loop network is presented and discussed. We demonstrated herein that the excel numeric solver can be used to obtain the flow correction factor necessary to sufficiently satisfying the energy conservation around loop, that is, making the sum of the head losses zero without the need to use the Hardy Cross flow correction factor equation. The rest of the paper is organised as follows. Section II discusses the water piping network analysis techniques. In section III, Hardy Cross technique is briefly discussed. Section IV entails the results obtained from the use of the excel solver for the case studied networks while section V concludes the paper.

III. Pipe flow analysis techniques

The analysis for the formulation of water piping networks are categorised based on the variables used to solve such a network. The type of pipe flow analysis technique choosing depends on the available water distribution network data, the required output and the computational tool to be used. These are classified if the following parameters are used as;

- Q-system (using pipe flow as unknown);
- H-system (using nodal pressure head as unknown);
- $\triangle Q$-system (using corrective flow rates as unknown); and
- $\triangle H$-system (using corrective nodal head as unknown).

A. The Q-system

The Q-system uses the unknown pipe discharge as the major variable for solving the system. Thus, the number of equations

to be solved comprises of $n_n - 1$ independent continuity equations and l loop equations assuming a balanced network. The system is a mixture of the continuity equation (linear) and the loop equation (non-linear).

B. The H-system

In the H-system, the nodal pressure heads (H) is used as a variable for solving the system. Thus, the number of unknown is n_n and is less than the Q-system. Therefore, by expressing the discharges through each pipe, only $n_n - 1$ continuity equations are written for each node as;

$$Q_{ij} = \left(\frac{H_i - H_j}{r_k} \right)^{1/n} \tag{7}$$

where, H_i and H_j represent the pressure head at the upstream and downstream of the pipe. Thus, this results in a system of non-linear equations.

C. The $\triangle Q - system$

In the $\triangle Q$-system, the corrective flow factor $\triangle Q$ is the variable required to satisfy the energy conservation equation around the hydraulic loops. The solution of the system can be obtained by considering one loop at a time by written the loop equation in the form;

$$\sum_{k=1}^{np(l)} r_k (Q_{0,k} \pm \triangle Q)^n = 0 \tag{8}$$

where, $Q_{0,k}$: initial estimate/guess of flow for the kth pipe, $\triangle Q$: flow correction factor in the loop and $n_p(l)$ is the number of pipes in the loop l.

D. The $\triangle H - system$

In the $\triangle H$-system, the corrective nodal pressure head $\triangle H$ is the variable used and required to satisfy the continuity equation at a node. The continuity equation can be written for any node as;

$$\sum_{k=1}^{np(j)} \left(\frac{1}{r_k} \right)^{1/n} (H_{0,k} \pm \triangle H)^{1/n} - Q_{ext} = 0 \tag{9}$$

IV. Pipe flow solution method

The focus of this paper is the use of Microsoft excel solver to demonstrate the Hardy Cross method for the solution of simple loop water distribution network. Thus, in this section, we briefly discus the Hardy Cross solution method.

A. Hardy Cross solution method

The Hardy Cross method (HCM) is an iterative method which involves a series of successive guessing and adjustment of discharges in pipes. It is a popular procedure for water piping network analysis [29]. The HCM requires an initial guesses of all pipe flows under the condition that such guesses satisfy the conservation of mass at each node. The following steps describe the iterative procedure for the Hardy Cross method;

1) Compute the pipe resistance value r_k for each pipe using the (6);
2) Assume a reasonable flow direction and distribution (Q_o) in each pipe satisfying the mass continuity at each node;
3) Estimate the first approximation of the flow correction factor $\triangle Q$ for each loop using (18) (derived later in this section);
4) Add the correction to the initial estimated flow (the initial flow guess) of each pipe. That is, $Q = Q_o \pm \triangle Q$;
5) Repeat step 3 and 4 until $\triangle Q$ is reasonably small or until $\sum h_l \cong 0$.

The loop correction factor is derived from the head loss equation as; by making an initial guess/estimate of the flow in the kth pipe connected to node j, the true value of the flow in each pipe is computed as

$$Q_k = Q_{o,k} + \triangle Q \qquad (10)$$

where, Q_k denotes the true value of the flow in the kth pipe, $Q_{o,k}$ represents the initial guess of the flow in pipe k and $\triangle Q$ is the flow correction factor. Substituting (10) into the head loss equation, (5), can be re-written as

$$h_l = r_k(Q_{o,k} + \triangle Q)^n \qquad (11)$$

Applying Binomial expansion to the system of equation in (11), the head loss h_l results to

$$h_l = r_k(Q_{o,k}^n + nQ_{o,k}^{n-1}\triangle Q + \frac{n(n-1)!}{2!}Q_{o,k}^{n-2}\triangle Q^2 + .. + \triangle Q^n) \qquad (12)$$

Assuming $\triangle Q$ is small, the higher times of $\triangle Q$ will be $\lll \triangle Q$ and can be neglected. Thus, (12) reduced to;

$$\left.\begin{array}{l} h_l = r_k(Q_{o,k}^n + nQ_{o,k}^{n-1}\triangle Q) \\ = r_kQ_{o,k}^n + nr_kQ_{o,k}^{n-1}\triangle Q \end{array}\right\} \qquad (13)$$

Thus, (13) can be re-written as

$$h_l = r_kQ_{o,k}|Q_{o,k}|^{n-1} + nr_k|Q_{o,k}|^{n-1}\triangle Q \qquad (14)$$

This equation can be inserted into the energy conservation equation (4) neglecting the head generated due to pump resulting into

$$\sum_{k=1}^{np}(r_kQ_{o,k}|Q_{o,k}|^{n-1} + nr_k|Q_{o,k}|^{n-1}\triangle Q) = 0 \qquad (15)$$

which can be further expressed as

$$\sum_{k=1}^{np}r_kQ_{o,k}|Q_{o,k}|^{n-1} + \sum_{k=1}^{np}nr_k|Q_{o,k}|^{n-1}\triangle Q = 0 \qquad (16)$$

Therefore, the flow correction factor $\triangle Q$ is expressed as

$$\triangle Q = \frac{-\sum\limits_{k=1}^{np}r_kQ_{o,k}|Q_{o,k}|^{n-1}}{n\sum\limits_{k=1}^{np}r_k|Q_{o,k}|^{n-1}} \qquad (17)$$

$$\triangle Q = \frac{-\sum\limits_{k=1}^{np}r_kQ_{o,k}^n}{n\sum\limits_{k=1}^{np}r_k|Q_{o,k}|^{n-1}} = \frac{-\sum\limits_{k=1}^{np}h_l}{n\sum\limits_{k=1}^{np}r_k|Q_{o,k}|^{n-1}} \qquad (18)$$

B. Using Microsoft Excel solver for HCM

Microsoft Excel numeric solver can be used to obtain a suitable flow correction factor $\triangle Q$ for each loop required to make the summation of the head losses across any loop equal zero without necessary estimating and using the Hardy Cross flow correction equation (18). The following steps was used to achieve such.

1) Set up an Excel spreadsheet (see Fig. 2) and enters the pipe network data (L, D, f).
2) Assume an initial flow Q_o for each pipe satisfying the continuity equation at each node.
3) Compute the hydraulic resistance r_k for each pipe and the head losses h_l in each pipe using the desire equations.
4) Assume $\triangle Q$ to be initially zero and add to the assumed initial flow guess. That is, $Q_o + \triangle Q$.

The objective/target function is to set the sum of the head losses $\sum h_l$ to zero, subject to varying $\triangle Q$.
Thus, the value of $Q_o + \triangle Q$ is adjusted immediately. Once the solver find a solution, this column contains the correct pipe flows $(Q_o + \triangle Q)$. This reduces the number of iteration required to perform in order to achieve energy conservation around the loop.

V. NUMERICAL EXAMPLE

In this section, the use of the Excel solver for the hydraulic analysis of some water distribution networks (single-loop network) is demonstrated. The studied single-loop water distribution network shown in Fig. 1 consist of four pipelines interconnected at junction nodes, where water demand occurs.
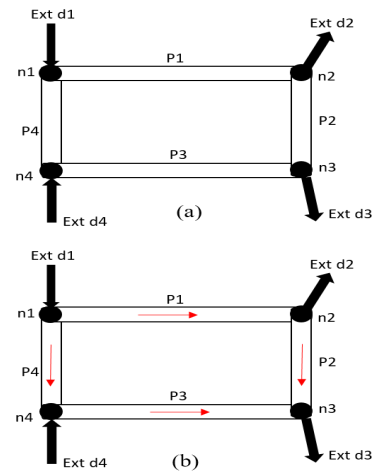

Fig. 1. Single-loop WDN (a) without flow direction (b) with assume flow direction.

Each pipe in the network has its own data such as length, diameter and frictional factor as illustrated in Table I. Likewise, the demand at each node of the network is presented in Table I. The length, diameter and the frictional factor was used to compute the hydraulic resistance of each pipe in the network using (6) as display in the Excel sheet in Fig. 2.

As shown in Fig. 2, cells F6:F9, an initial guess of the flows in each pipes was made satisfying the mass continuity equation at each node of the network using the known nodal demands. Thereafter, by initially setting the flow correction factor in the loop (cell B3) to zero, the adjusted flows in each pipe is computed (cells G6:G9). Thus, the head loss in each pipe is estimated via cells H6:H9.

Fig. 3. A layout of the Excel sheet for the studied network showing the excel numeric solver tool.
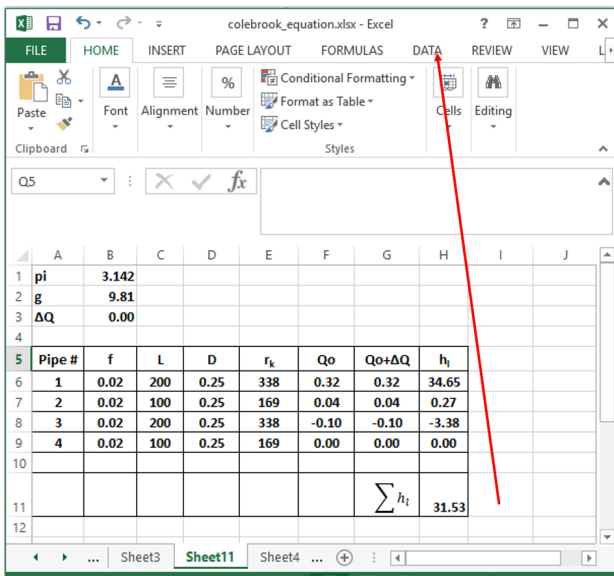

Fig. 2. A layout of the Excel sheet for the studied network.

Ordinarily, these steps will be repeated until the sum of the head losses becomes zero. However, to avoid going through these iterative procedure, the use of the Excel solver (obtained from the *Data menu* of Excel sheet as in Fig. 2 and Fig. 3 can be used to set the sum of the head losses to zero by changing the value of the flow correction factor in cell B3. Fig. 4 shows the layout to achieve such. From the layout in Fig. 4, the objective function is to set the sum of the head losses in cell H11 to a value of zero by changing the value of the flow correction factor in cell B3. This was achieved using the solver parameter displayed in Fig. 4. The solver works with a group of cells that participate in computing the value of the objective cell (cell H11), the sum of the head loss in
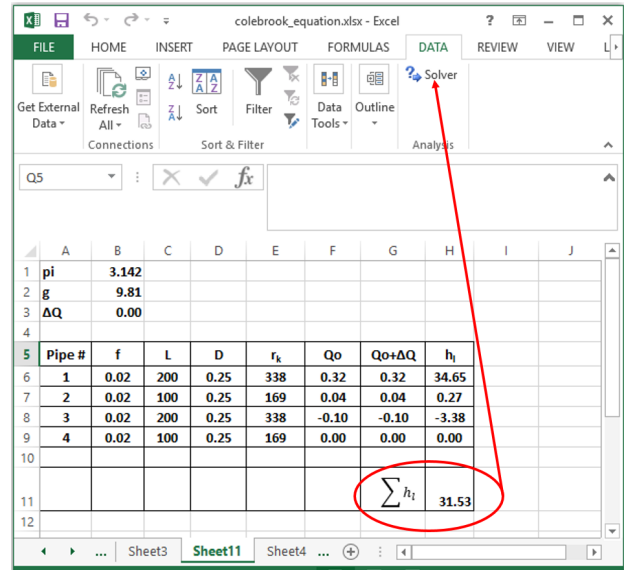
each pipe, and the constraint cell (cell B3) as shown in Fig. 4. Thus, the solver adjusts the values in the group of cells that is (cells G6:G9; H6:H9) to satisfy the limits on the constraints cell (cell B3) and produce the desire result for the objective cell (cell H11). In this example, the target is to set the sum of the pipe head losses to zero as in cell H11 (the objective cell) by changing the values of the flow correction factor in cell B3, (the constraint cell). After a click of the solver button, and some inherent iteration is performed, the solver find an optimal value for the flow correction factor in cell B3 for which the sum of the pipe head losses in cell H 11 is zero.

In Fig. 5, it is observed that the cells G6:G9 and H6:H9 has been updated after the solver finds a solution. The optimal value of the flow correction factor $\triangle Q$ for which the objective function (the sum of the head losses is zero) is met is given in cell B3 of Fig. 5.
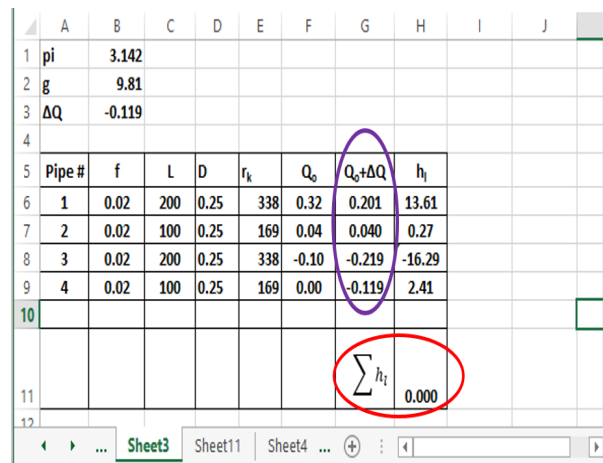

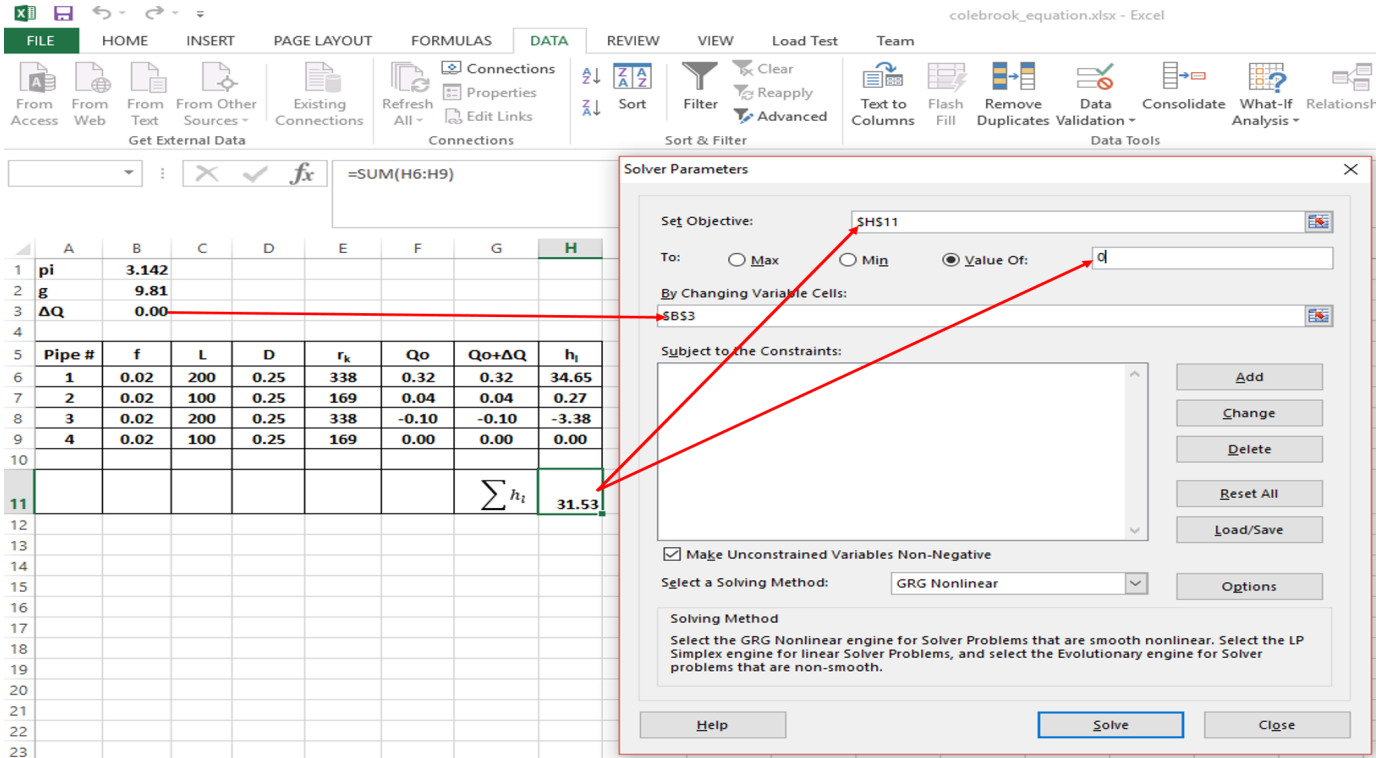Fig. 5. The results after the application of the solver tool.

Fig. 4. A layout of the numeric solver tool using the solver parameter to set head losses to zero.

The cells containing the updated flows in Fig. 5, that is, $Q_o + \triangle Q$ (cells G6:G9) become the true value of the flows in each of the pipes. A negative sign indicate the flow is in opposite direction to the assume flow direction.

VI. CONCLUSION

In this paper, we demonstrate the use of Excel spreadsheet powerful tool, *the excel solver* for the analysis and solution of simple loop water piping network. The analysis was based on its usage for Hardy Cross method for solution of simple loop water network. We shows that the Microsoft Excel numeric solver can be used to obtain a suitable flow correction factor for each loop required to make the summation of the head losses across any loop equal zero without necessary estimating and using the Hardy Cross flow correction equation. The tool can also be extended for other WDN solution methods such as the Hybrid methods, the global gradient algorithm for simpler networks.

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